Gluon polarization in the proton

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We combine heavy-quark renormalization group arguments with our understanding of the nucleon's wave function to deduce a bound on the gluon polarization Δg in the proton. The bound is consistent with the values extracted from spin experiments at COMPASS and RHIC.

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Polarized deep inelastic scattering (pDIS) experiments have revealed a small value for the nucleon's flavor-singlet axial-charge, $g_A^{(0)}|_{\rm pDIS} \sim 0.3$, suggesting that the quarks' intrinsic spin contributes little of the proton's spin. The challenge to understand the spin structure of the proton [1–6] has inspired a vast program of theoretical activity and new experiments. Why is the quark spin content $g_A^{(0)}|_{\rm pDIS}$ so small? How is the spin $\frac{1}{2}$ of the proton built up from the spin and orbital angular momentum of the quarks and gluons inside?

A major topic of investigation has been the role of polarized glue in the nucleon, both in terms of its contribution to the nucleon's spin and possible suppression of the nucleon's singlet axial charge through the QCD axial anomaly. Key experiments to measure gluon polarization are COMPASS at CERN and PHENIX and STAR at RHIC. In this paper we investigate gluon polarization via the charm-quark axial charge, matching the results of the heavy-quark renormalization group with what we know of the proton's wave function. We suggest a bound $|\Delta g(m_c^2)| \lesssim 0.3$, which is consistent with the results of the present experiments.

We start by recalling the g_1 spin sum rules, which are derived from the dispersion relation for polarized photon-nucleon scattering and, for deep inelastic scattering, the light-cone operator product expansion. At leading twist the first moment of the g_1 spin structure function measures a linear combination of the nucleon's scale-invariant axial charges $g_A^{(3)}$, $g_A^{(8)}$, and $g_A^{(0)}|_{\rm inv}$ plus a possible subtraction constant β_∞ in the dispersion relation [1]

$$\int_0^1 dx\, g_1(x,Q^2) = \left(\frac{1}{12}g_A^{(3)} + \frac{1}{36}g_A^{(8)}\right)c_{\rm NS}[\alpha_s(Q^2)] + \frac{1}{9}g_A^{(0)}|_{\rm inv}c_{\rm S}[\alpha_s(Q^2)] + \beta_\infty. \quad (1)$$

Here $c_{\rm NS}$ and $c_{\rm S}$ are the nonsinglet and singlet Wilson coefficients. In terms of the flavor dependent axial charges $2Ms_\mu \Delta q = \langle p,s|\overline{q}\gamma_\mu\gamma_5 q|p,s\rangle$ the isovector, octet, and singlet axial charges are $g_A^{(3)} = \Delta u - \Delta d$, $g_A^{(8)} = \Delta u + \Delta d - 2\Delta s$, and $g_A^{(0)}|_{\rm inv}/E(\alpha_s) \equiv g_A^{(0)}(Q^2) = \Delta u + \Delta d + \Delta s$. Here $E(\alpha_s) = \exp\int_0^{\alpha_s} d\tilde{\alpha}_s\, \gamma(\tilde{\alpha}_s)/\beta(\tilde{\alpha}_s)$ is a renormalization group factor which corrects for the (two-loop) nonzero anomalous dimension $\gamma(\alpha_s)$ of the singlet axial-vector current [7], $J_{\mu5} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s$, which is close to one and which goes to one in the limit $Q^2 \to \infty$; $\beta(\alpha_s) = -(11 - \frac{2}{3}f)(\alpha_s^2/2\pi) + \cdots$ is the QCD beta function and $\gamma(\alpha_s) = f(\alpha_s/\pi)^2 + \cdots$, where f is the number of active flavors. The singlet axial charge, $g_A^{(0)}|_{\rm inv}$, is independent of the renormalization scale μ and corresponds to $g_A^{(0)}(Q^2)$ evaluated in the limit $Q^2 \to \infty$. The flavor nonsinglet axial charges are renormalization group invariants.

The isovector axial charge is measured independently in neutron β decays ($g_A^{(3)} = 1.270 \pm 0.003$ [8]) and the octet axial charge is commonly taken to be the value extracted from hyperon β decays assuming a two-parameter SU(3) fit ($g_A^{(8)} = 0.58 \pm 0.03$ [9]). The uncertainty quoted for $g_A^{(8)}$ has been a matter of some debate [10,11]. There is considerable evidence that SU(3) symmetry may be badly broken and some have suggested that the error on $g_A^{(8)}$ should be as large as 25% [10]. Indeed, prompted by the work of Myhrer and Thomas [12], which showed that the effect of the one-gluon-exchange hyperfine interaction [13] and the pion cloud [14] of the nucleon was to reduce $g_A^{(0)}$ calculated in the cloudy bag model to near the experimental value, a reevaluation of these effects on $g_A^{(3)}$, $g_A^{(8)}$, and $g_A^{(0)}$ including kaon loops led to the value $g_A^{(8)} = 0.46 \pm 0.05$ [15]. Here the reduction from the SU(3) value came primarily from the pion cloud.

Assuming no twist-two subtraction constant, polarized deep inelastic scattering experiments have been interpreted in terms of a small value for the flavor-singlet axial charge: $g_A^{(0)}|_{{\rm pDIS},\,Q^2\to\infty} = 0.33 \pm 0.03\ ({\rm stat.}) \pm 0.05\ ({\rm syst.})$ [16] if one uses the SU(3) value for $g_A^{(8)}$. On the other hand, using the value $g_A^{(8)} = 0.46 \pm 0.05$ from SU(3) breaking, the corresponding experimental value of $g_A^{(0)}|_{\rm pDIS}$ would increase to $g_A^{(0)}|_{\rm pDIS} = 0.36 \pm 0.03 \pm 0.05$. In the naive parton model $g_A^{(0)}|_{\rm pDIS}$ is interpreted as the fraction of the proton's spin which is carried by the intrinsic spin of its quark and antiquark constituents.

Historically, the wish to understand the suppression of $g_A^{(0)}$ relative to $g_A^{(8)}$ led to considerable theoretical efforts to understand the flavor-singlet axial charge in QCD. QCD theoretical analysis leads to the formula [1,17–20]

$$g_A^{(0)} = \left(\sum_q \Delta q - 3\frac{\alpha_s}{2\pi}\Delta g\right)_{\rm partons} + \mathcal{C}_\infty. \quad (2)$$

Here $\Delta g_{\text{partons}}$ is the amount of spin carried by polarized gluons in the polarized proton ($\alpha_s \Delta g \sim$ constant as $Q^2 \to \infty$ [17,18]) and $\Delta q_{\text{partons}}$ measures the spin carried by quarks and antiquarks carrying "soft" transverse momentum $k_t^2 \sim P^2, m^2$ where P is a typical gluon virtuality and m is the light quark mass. The polarized gluon term is associated with events in polarized deep inelastic scattering where the hard photon strikes a quark or antiquark generated from photon-gluon fusion with $k_t^2 \sim Q^2$ [19,20]. It corresponds to the QCD axial anomaly in the flavor-singlet axial-vector current. $\mathcal{C}_\infty$ denotes a potential nonperturbative gluon topological contribution [1] associated with the possible subtraction constant in the dispersion relation for g_1 and possible spin contributions at Bjorken $x = 0$, that is outside the range of polarzed deep inelastic scattering experiments. The measured singlet axial charge is $g_A^{(0)}|_{\text{pDIS}} = g_A^{(0)} - C_\infty$.

In the parton model $\Delta q_{\text{partons}}$ is associated with the forward matrix of the partially conserved axial-vector current J_{+5}^{con} evaluated in the light-cone gauge $A_+ = 0$ and corresponds to the quark spin contribution extracted from experiments using the JET or AB factorization schemes [21]. For each flavor q, this term and the possible topological term $\mathcal{C}_\infty$ are renormalization group invariants [1]. All of the renormalization group scale dependence induced by the anomalous dimension, $\gamma(\alpha_s)$, is carried by the polarized glue term:

$$\left\{\frac{\alpha_s}{2\pi}\Delta g\right\}_{Q^2} = \left\{\frac{\alpha_s}{2\pi}\Delta g\right\}_{\infty} - \frac{1}{f}\,\{1/E(\alpha_s) - 1\}\, g_A^{(0)}\big|_{\text{inv}}, \tag{3}$$

where all quantities in this equation are understood to be defined in the f-flavor theory. Flavor nonsinglet combinations of the Δq are renormalization group invariant so that each flavor evolves at the same rate, including heavy-quark contributions ($q = c, b, t$). The growth in the gluon polarization $\Delta g \sim 1/\alpha_s$ at large Q^2 is compensated by growth with opposite signs in the gluon orbital angular momentum. The quark and gluon total angular momenta in the infinite scaling limit are given by [22] $J_q(\infty) = \frac{1}{2}\{3f/(16+3f)\}$ and $J_g(\infty) = \frac{1}{2}\{16/(16+3f)\}$. There is presently a vigorous program to disentangle the different contributions involving experiments in semi-inclusive polarized deep inelastic scattering and polarized proton-proton collisions [3,23].

Heavy-quark axial charges have been studied in the context of elastic neutrino-proton scattering [24,25] and heavy-quark contributions to g_1 at Q^2 values above the charm production threshold [26–29]. Charm production in polarized deep inelastic scattering is an important part of the COMPASS spin program at CERN [30].

Following Eq. (2) we can write the charm-quark axial-charge contribution as

$$\Delta c(Q^2) = \Delta c_{\text{partons}} - \left\{\frac{\alpha_s}{2\pi}\Delta g\right\}_{Q^2, f=4}, \tag{4}$$

where $\Delta c_{\text{partons}}$ corresponds to the forward matrix element of the plus component of the renormalization group invariant charm-quark axial current with just mass terms in the divergence (minus the QCD axial anomaly), viz. $(\bar{c}\gamma_\mu\gamma_5 c)_{\text{con}} = (\bar{c}\gamma_\mu\gamma_5 c) - k_\mu$ with k_μ the gluonic Chern-Simons current, and we neglect any topological contribution.[1] For scales $Q^2 \gg m_c^2$, $\Delta c_{\text{partons}}$ corresponds to the polarized charm contribution one would find in the JET or AB factorization schemes.

The heavy-quark contributions to the nonsinglet neutral current axial charge measured in elastic neutrino-proton scattering have been calculated to next-to-leading order (NLO) in Ref. [24]. For charm quarks, the relevant electroweak doublet contribution at leading order (LO) is

$$\begin{aligned}(\Delta c - \Delta s)_{\text{inv}}^{(f=4)} = &-\frac{6}{27\pi}\alpha_s^{(3)}(m_c^2)\, g_A^{(0)}\big|_{\text{inv}}^{(f=3)} \\ &- \Delta s_{\text{inv}}^{(f=3)} + O(1/m_c^2).\end{aligned} \tag{5}$$

For the LO contribution this is made up from

$$\begin{aligned}\Delta s_{\text{inv}}^{(f=4)} &= \Delta s_{\text{inv}}^{(f=3)} + \left(\frac{6}{27\pi} - \frac{6}{25\pi}\right)\alpha_s^{(3)}(m_c^2)\, g_A^{(0)}\big|_{\text{inv}}^{(f=3)}, \\ &\quad + O(1/m_c^2), \\ \Delta c_{\text{inv}}^{(f=4)} &= -\frac{6}{25\pi}\alpha_s^{(3)}(m_c^2)\; g_A^{(0)}\big|_{\text{inv}}^{(f=3)} + O(1/m_c^2).\end{aligned} \tag{6}$$

Here $\Delta c_{\text{inv}} = \Delta c(Q^2)$ evaluated in the limit $Q^2 \to \infty$, where the charm-quark axial-charge contribution is $2Ms_\mu \Delta c = \langle p, s|\bar{c}\gamma_\mu\gamma_5 c|p, s\rangle$. The change in Δs_{inv} between the four- and three-flavor theories in Eqs. (6) comes from the different number of flavors in $E(\alpha_s)$ for the four- and three-flavor theories.

Equations (6) contain vital information about $\{(\alpha_s/2\pi)\,\Delta g\}_\infty$ in Eq. (4) if we know the renormalization group (RG) invariant quantity $\Delta c_{\text{partons}}$. Indeed, if the latter were zero and if we ignore the NLO evolution associated with the two-loop anomalous dimension $\gamma(\alpha_s)$, then Eqs. (6) would imply (at LO)

$$\Delta g^{(f=4)}(Q^2) = \frac{12}{25}\frac{\alpha_s^{(f=3)}(m_c^2)}{\alpha_s^{(f=4)}(Q^2)}\, g_A^{(0)}\big|_{\text{inv}}^{(f=3)}, \tag{7}$$

or $\Delta g \sim 0.23$ when $\alpha_s(Q^2) \sim 0.3$. The following discussion is aimed at assessing the possible size of $\Delta c_{\text{partons}}$ plus the NLO evolution associated with $\gamma(\alpha_s)$, and hence the error on this value.

Canonical (anomaly free) heavy-quark contributions to the proton wave function are, in general, suppressed by powers of $1/m_c^2$, so we expect $\Delta c_{\text{partons}} \sim O(1/m_c^2)$. The RG invariant quantity $\Delta c_{\text{partons}}$ takes the same value at all momentum scales. We may evaluate it using quark-hadron duality in a hadronic basis with meson cloud methods [32]. Experimental studies of the strange quark content of the nucleon over the past decade have given us considerable confidence that both the matrix elements of the vector and scalar charm-quark currents (which are anomaly free) in the proton are quite small [33,34]. This gives us confidence in estimating the polarized charm contribution through its suppression relative to the corresponding polarized strangeness -0.01 [15] by the factor

[1]Any topological contribution will be associated with some of the $\Delta c_{\text{partons}}$ being shifted to Bjorken $x = 0$. In general, topological contributions are suppressed by powers of $1/m_c^2$ for heavy-quark matrix elements [31].

$\sim(m_\Lambda - m_N + m_K)^2/4m_c^2 < 0.1$ so that $|\Delta c_{\rm partons}| < 0.001$. QCD four-flavor evolution and Eqs. (6) then enable an estimate of Δg at scales relevant to experiments at COMPASS and RHIC.

In perturbative QCD the LO contribution to heavy-quark production through polarized photon-gluon fusion yields

$$\int_0^1 dx\, g_1^{(\gamma^* g\to h\bar{h})} \sim 0, \quad Q^2 \gg m_h^2, \tag{8}$$

where m_h is the heavy-quark mass ($h = c, b, t$). The anomalous $-\alpha_s/2\pi$ term is canceled against the canonical term when $m_h^2 \gg P^2$ (the typical gluon virtuality in the proton) [19]. If the gluon polarization were large so that $-\frac{\alpha_s}{2\pi}\Delta g$ made a large contribution to the suppression of $g_A^{(0)}$, at this order one would also find a compensating large canonical polarized charm contribution in the proton. To understand this more deeply, we note that the result in Eq. (8) follows from the complete expression [20]

$$\int_0^1 dx\, g_1^{(\gamma^* g)} = -\frac{\alpha_s}{2\pi}\left[1 + \frac{2m^2}{P^2}\frac{1}{\sqrt{1+\frac{4m^2}{P^2}}}\ln\left(\frac{\sqrt{1+\frac{4m^2}{P^2}}-1}{\sqrt{1+\frac{4m^2}{P^2}}+1}\right)\right]. \tag{9}$$

Here m is the mass of the struck quark and P^2 is the gluon virtuality. We next focus on charm production. The first term in Eq. (9) is the QCD anomaly and the second, mass-dependent, canonical term gives $\Delta c_{\rm partons}^{(\rm gluon)}$ for a gluon "target" with virtuality P^2. Evaluating Eq. (9) for $m_c^2 \gg P^2$ gives the leading term $\int_0^1 dx\, g_1^{(\gamma^* g)} \sim -\frac{\alpha_s}{2\pi}\frac{5}{8}\frac{P^2}{m_c^2}$, hence the result in Eq. (8).

It is interesting to understand Eqs. (8) and (9) in terms of deriving the QCD axial anomaly via Pauli-Villars regularization (instead of the usual dimensional regularization derivation used in [19]). The anomaly corresponds to the heavy Pauli-Villars "quark," which will cancel against the heavy charm quark for a charm-quark mass much bigger than gluon virtualities in the problem (there are no other mass terms to set the scale). When the axial-vector amplitude is evaluated at the two-loop level there will be gluon loop momenta between m_c and the ultraviolet cut-off scale generating a small scale dependence so that the cancellation between canonical heavy-quark and anomalous polarized glue terms is not exact in full QCD. This scale dependence corresponds to the two-loop anomalous dimension $\gamma(\alpha_s)$ in $E(\alpha_s)$. The result in Eq. (8) was previously discussed in Refs. [10,27] in the context of the phenomenology that would follow if there were large gluon polarization in the proton. Nonperturbative evaluation of $\Delta c_{\rm partons}$ allows us to constrain the size of Δg given what we know about charm and strangeness in the nucleon's wave function.

There is a further issue that the derivation of Eqs. (6) involves matching conditions where the spin contributions are continuous between the three- and four-flavor theories at the threshold scale m_c modulo $O(1/m_c^2)$ corrections, which determine a theoretical error for the method. Using QCD evolution with the renormalization group factor $E(\alpha_s)$, the results in Eqs. (6) are equivalent to the leading twist term $\Delta c(m_c^2)$ vanishing at the threshold scale m_c modulo $O(1/m_c^2)$ corrections, viz. $\Delta c(m_c^2) = O(1/m_c^2)$ [10]. The leading $O(1/m_c^2)$ term is estimated using effective field theory in Refs. [10,25,27]. For polarized photon-gluon fusion, this is the $-(\alpha_s/2\pi)(5/8)(P^2/m_c^2)$ leading term in the heavy-quark limit of Eq. (9). The heavy charm quark is integrated out at threshold to give the matrix element of a gauge-invariant gluon operator with dimension 5 and the same quantum numbers as the axial-vector current, viz. $\Delta c(m_c^2) \sim O[\alpha_s(m_c^2)/4\pi](M^2/m_c^2)$ [27] or $\Delta c(m_c^2) \sim O[\alpha_s(m_c^2)\Lambda_{\rm QCD}^2/m_c^2] \sim 0.017$ [10].[2] Taking this as an estimate of the theoretical error gives $\Delta c(m_c^2) = 0 \pm 0.017$.

We next combine this number for $\Delta c(m_c^2)$ with our estimate of the canonical charm contribution $|\Delta c_{\rm partons}| < 0.001$ in quadrature to obtain a bound including theoretical error on the size of the polarized gluon contribution: $|-(\alpha_s/2\pi)\Delta g(m_c^2)| \lesssim 0.017$ or

$$|\Delta g(m_c^2)| \lesssim 0.3 \tag{10}$$

with $\alpha_s(m_c^2) = 0.4$. Values at other values of Q^2 are readily obtained with Eq. (3) or $\alpha_s \Delta g \sim$ constant for large values of Q^2.

It is interesting to extend this analysis to full six-flavor QCD. The values of $\Delta c_{\rm inv}^{(f=6)}$, $\Delta b_{\rm inv}^{(f=6)}$, and $\Delta t_{\rm inv}^{(f=6)}$ were derived in Ref. [24] to NLO in the heavy-quark expansion. Taking just the leading-order contribution plus the heavy-quark power correction according to the recipe [10,27] described above gives $\Delta c^{(f=5)}(m_b^2) = -0.006 \pm 0.017$ and $\Delta c^{(f=6)}(m_t^2) = -0.009 \pm 0.017$ for polarized charm. For the bottom and top quarks one obtains $\Delta b^{(f=5)}(m_b^2) = 0 \pm 0.001 \pm 0.017$, $\Delta b^{(f=6)}(m_t^2) = -0.003 \pm 0.001 \pm 0.017$, and $\Delta t(m_t^2) = 0 \pm 2\times 10^{-7} \pm 0.017$. Here the first error comes from the $O(1/m_h^2)$ mass correction for the heaviest quark of c, b, and t. The second error comes from the other heavy-quarks with lesser mass as we evaluate these heavy-quark contributions in terms of the measured value of the light-quark quantity $g_A^{(0)}|_{\rm inv}^{(f=3)}$. These numbers overlap with a zero value for $(\alpha_s/2\pi)\Delta g$ in the relevant f-flavor theories. The QCD scale dependence of $(\alpha_s/2\pi)\Delta g$ starts with NLO evolution induced by Kodaira's two-loop anomalous dimension $\gamma(\alpha_s)$. The combination $(\alpha_s/2\pi)\Delta g$ is scale invariant at LO. This means that if we work just to LO and Δg vanishes at one scale, it will vanish at all scales (in LO approximation). The LO QCD evolution equation for gluon orbital angular momentum in the proton [22] then simplifies so that $L_g(\infty) = \frac{1}{2}\{16/(16+3f)\}$. In practice, the two-loop anomalous dimension generates slow evolution of $(\alpha_s/2\pi)\Delta g$. Dividing the finite value of this combination at large scales by the small value of α_s gives a finite value for the gluon polarization Δg, which can readily be the same order of magnitude as the gluon

[2]These $O(1/m_c^2)$ terms associated with the full Δc are manifest in polarized photon-gluon fusion through the heavy-quark limit of Eq. (9), $\int_0^1 dx\, g_1^{(\gamma^* g)} \sim -(\alpha_s/2\pi)(5/8)(P^2/m_c^2)$, and are to be distinguished from the model evaluation of $\Delta c_{\rm partons}$.

total angular momentum (or larger with cancellation against a correspondingly larger gluon orbital angular momentum contribution).

It is interesting that the value of Δg deduced from present experiments COMPASS at CERN and PHENIX and STAR at RHIC typically give $|\Delta g| < 0.4$ with $\alpha_s \sim 0.3$ corresponding to $|-3(\alpha_s/2\pi)\Delta g| < 0.06$ [3]. This experimental value is extracted from direct measurements of gluon polarization at COMPASS in the region around $x_{\text{gluon}} \sim 0.1$, NLO QCD motivated fits to inclusive g_1 data taken in the region $x > 0.006$, and RHIC spin data in the region $0.02 < x_{\text{gluon}} < 0.4$. The theoretical bound, Eq. (10), is consistent with this experimental result.

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