A Petrophysical Joint Inversion of Magnetotelluric and Gravity Data for Enhanced Subsurface Imaging of Sedimentary Environments

by

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Contents

Abstract

An emerging field in geophysics is that of joint inversions, in which multiple technique data sets are analysed and inverted simultaneously. This helps to integrate the complementary data sets and reduce model ambiguity, common in single technique inversions. In this thesis a new implementation of a magnetotelluric (MT) and gravity 2D joint inversion scheme is developed based on a petrophysical approach. In sedimentary rock environments, electrical conductivity (which underpins the MT technique) can be approximated by Archie's Law, whereas density (which underpins the gravity technique) can be derived from the porosity-density relationship. Since both expressions are themselves dependent on porosity, this petrophysical property provides the crucial link exploited by the 2D joint inversion. The 2D joint inversion approach devised here inverts directly for a porosity model, which is converted to resistivity and density models through Archie's Law and the porosity-density relationship, then constrained (fitted) by the MT and gravity data. Thus, a single porosity model is produced that satisfies both data sets.

By means of synthetic data inversions, it was established that the joint inversion is more effective in reproducing the true subsurface model than can be achieved by an MT or gravity inversion alone. Models produced by the joint inversion show improved placement of subsurface features and a greater accuracy of reconstructing the original subsurface (physical property) values. For optimal joint inversion results, broadband MT data should be used in favour of long period MT data, and the number of gravity stations should be greater than or equal to the number of MT stations. The joint inversion is particularly useful in extracting coherent information from noisy MT data when combined with good quality gravity data. While evaluating the MT and gravity compatibility, a new method was developed for evaluating the information contained in the MT Jacobian (sensitivity) matrix.

The Renmark Trough in South Australia is an area of current geothermal interest for which multi-technique data (seismic, gravity, MT) exists. These field data were used to demonstrate and verify the effective use of the joint inversion in a practical real-world example. The Renmark Trough is a half graben structure with the Hamley Fault delineating the north-east boundary. At the Hamley Fault, the base of the trough is 3.5 km deep and rises gradually in a south-west direction. The inversion of the MT data alone produced a model inconsistent with seismic knowledge of the basement depths and geometries. In contrast, the joint inversion yielded a more geologically accurate image of the trough and faithfully reconstructed the basement depths and geometries.

In the process of developing the joint inversion scheme, a 2D gravity inversion algorithm, based on the Occam maximum smoothness approach, was produced. This inversion algorithm demonstrated the inherent non-uniqueness of gravity interpretation by only placing strong density contrasts at the surface. Attempts to improve the gravity inversion results, such as the use of depth weighting functions and fixing structure locations in parts of the model, were not as effective as the joint inversion in producing an accurate representation of the subsurface.

Statement of Originality

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SIGNED: ....

DATE: ....

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List of Symbols

Throughout this thesis, numerous symbols will be used repeatedly to represent specific quantities or parameters. In this section, a list of symbols and a short description of each is given for the readers convenience. Every effort has been made to maintain conformity of symbols used here and wherever possible, standard symbols and notations have been used. However, standards have dictated a single symbol be used to represent more than one quantity. As a result, variations in symbol type, which are non-standard, have been used. The most significant double use of a symbol is ρ, which is typically used to represent both density and resistivity. To resolve this double use of this symbol, the slightly modified typeface ϱ has been used to represent resistivity.

α	...	MT RMS weighting
β	...	Gravity RMS weighting
β_*	...	Gravity depth weighting constant
γ	...	Complex propagation constant (wave number)
δ	...	Skin depth
δm	...	Small amount a model parameter is changed
$\underline{\partial}$	...	Differential operator
$\underline{\partial}_y$	...	Horizontal roughness matrix
$\underline{\partial}_z$	...	Vertical roughness matrix
ε	...	Dielectric permittivity
ε_0	...	Dielectric permittivity of free space
ε_r	...	Relative dielectric permittivity or dielectric constant

$\widehat{\zeta}$	...	Scale of the step length in the steepest decent inversion scheme
η	...	Expansion coefficient for data space inversion scheme
ϑ	...	Width of the cells in the Occam regularisation grid
κ	...	Real component of the complex propagation constant
λ	...	Tikhonov regularisation parameter
λ_c	...	Compaction coefficient
μ	...	Lagrange multiplier
μ_*	...	Magnetic permeability
μ_0	...	Magnetic permeability of free space
μ_r	...	Relative magnetic permeability
ν	...	Volume
ϖ	...	Weighting term in the model norm
ρ	...	Density
ρ_{air}	...	Density of air
ρ_{bulk}	...	Bulk rock density
ρ_{fluid}	...	Density of the formation fluid
ρ_{matrix}	...	Density of the rock matrix
ρ_w	...	Density of fresh water
σ	...	Electrical conductivity
σ_{bulk}	...	Bulk rock conductivity
σ_{fluid}	...	Conductivity of the formation fluid
σ_w	...	Conductivity of water
σ_{w_o}	...	Conductivity of water at zero temperature and fixed salinity
σ^*	...	Standard deviation
ϕ	...	Porosity
ϕ_o	...	Surface porosity
φ	...	Phase
χ	...	Chi distribution
χ_*	...	Target misfit

ω	...	Angular frequency
ϱ	...	Electrical resistivity
ϱ_a	...	Apparent resistivity
ϱ_{bulk}	...	Bulk rock resistivity
ϱ_{fluid}	...	Resistivity of the formation fluid
ϱ_w	...	Resistivity of water
ϱ_{w_o}	...	Resistivity of water at zero temperature and fixed salinity
$\Delta\phi$	...	The change in porosity due to incorrect petrophysical parameters
Δg	...	Gravity offset term
Δm	...	Perturbation in the model space
A	...	Cross-sectional area of a regularisation grid cell
$\mathbf{a}$	...	Acceleration
a	...	Tortuosity factor
$\mathbf{B}$	...	Magnetic induction
B_0 B_1	...	Components of the surface magnetic induction
B_x B_y B_z	...	Components of the magnetic induction
b	...	Width of a rectangular prism
C_d	...	Data covariance matrix
c	...	Cut in the Occam inversion step size
$\mathbf{D}$	...	Electric displacement current
$\mathbf{d}$	...	Data vector
d	...	Data vector component
$\mathbf{E}$	...	Electric field
E_0 E_1	...	Components of the surface electric field
E_x E_y E_z	...	Components of the electric field
$\mathbf{F}$	...	Forward model operator
$\mathbf{F}_g$	...	Gravitational force
$\mathbf{F}_{GV}$	...	Gravity forward model operator
$\mathbf{F}_{MT}$	...	MT forward model operator

$\mathbf{F}_{new}$	...	New set of models after the Occam step size has been cut
f	...	Frequency
$\mathbf{G}$	...	Linear forward model operator
G_c	...	Gravitational constant
$\mathbf{g}$	...	Gravitational acceleration
g_z	...	Vertical component of the gravitational acceleration
$\mathbf{H}$	...	Magnetic field
h	...	Number of linearly independent equations
$\mathbf{I}$	...	Identity matrix
$\mathbf{J}$	...	Jacobian matrix
$\mathbf{J}_c$	...	Electric current density
J_{cy}	...	Component of the electric current density
$\mathbf{J}_{GV}$	...	Gravity component of the joint inversion Jacobian
$\mathbf{J}_{MT}$	...	MT component of the joint inversion Jacobian
k	...	Imaginary component of the complex propagation constant
l_1	...	l_1-norm
l_2	...	Euclidean norm or l_2-norm
M	...	Mass
$\mathbf{m}$	...	Model parameter vector
m	...	Component of the model parameter vector
$\mathbf{m}_\phi$	...	Model parameter vector containing only the porosity model components
m_{cf}	...	Cementation factor
$\mathbf{m}_0$	...	Reference or starting model
n	...	Number of model parameters
$\widehat{\mathbf{n}}$	...	Unit normal vector
n_s	...	Saturation exponent
$\widehat{o}$	...	Vector of the direction of maximum decent in the steepest decent inversion scheme
$\mathbf{P}$	...	A point in space

P	...	Pressure
p	...	Number of MT data points
q	...	Number of data points
q_f	...	Free charge density
$\mathbf{R}$	...	Rotation matrix
r	...	Distance between points
$\widehat{\mathbf{r}}$	...	Unit distance vector
S	...	Fractional saturation
Sal	...	Salinity
SA	...	Sensitivity vector
s	...	Number of gravity data points
s_f	...	The surface over which an itergral is performed
T	...	Temperature
t	...	Time
U	...	Objective function
U_d	...	Data norm
U_g	...	Gravitational Potential
U_m	...	Model norm
U_{Tik}	...	Objective function of the Tikhonov Regularisation
v	...	Height of the cells in the Occam regularisation grid
$\mathbf{W}_d$	...	Data weighting matrix
$\mathbf{W}_{GV}$	...	Data weighting matrix used to implement the gravity data weighting
$\mathbf{W}_m$	...	Model weighting matrix
$\mathbf{W}_{MT}$	...	Data weighting matrix used to implement the MT data weighting
w	...	Gravity depth weighting function
w_y	...	Number of elements in the y-direction in the Occam regularisation grid
w_z	...	Number of elements in the z-direction in the Occam regularisation grid

X_{xx} X_{yy} X_{xy} X_{yx}	...	Real components of the impedance tensor
x	...	Spatial direction in Cartesian coordinates
Y_{xx} Y_{yy} Y_{xy} Y_{yx}	...	Imaginary components of the impedance tensor
y	...	Spatial direction in Cartesian coordinates
$\mathbf{Z}$	...	Impedance tensor
Z_{xx} Z_{yy} Z_{xy} Z_{yx}	...	Components of the impedance tensor
z	...	Spatial direction in Cartesian coordinates
$\widehat{\mathbf{z}}$	...	Unit vector vertically downwards
z_0	...	Gravity depth weighting constant
z_1	...	Depth to the top of a rectangular prism
z_2	...	Depth to the bottom of a rectangular prism

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