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# **Chiral Effective Field Theory Beyond the Power-Counting Regime**

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## Abstract

Chiral effective field theory complements numerical simulations of quantum chromodynamics on a spacetime lattice. It provides a model-independent formalism for connecting lattice simulation results at finite volume, and at a variety of quark masses, to the physical region. Knowledge of the power-counting regime of chiral effective field theory, where higher-order terms of the expansion may be regarded as negligible, is as important as knowledge of the expansion. Through the consideration of a variety of renormalization schemes, techniques are established to identify the power-counting regime. Within the power-counting regime, the results of extrapolation are independent of the renormalization scheme.

The nucleon mass is considered as a benchmark for illustrating this approach. Because the power-counting regime is small, the numerical simulation results are also examined to search for the possible presence of an optimal regularization scale, which may be used to describe lattice simulation results outside of the power-counting regime. Such an optimal regularization scale is found for the nucleon mass. The identification of an optimal scale, with its associated systematic uncertainty, measures the degree to which the lattice QCD simulation results extend beyond the power-counting regime, thus quantifying the scheme-dependence of an extrapolation.

The techniques developed for the nucleon mass renormalization are applied to the quenched  $\rho$  meson mass, which offers a unique test case for extrapolation schemes. In the absence of a known experimental value, it serves to demonstrate the ability of the extrapolation scheme to make predictions without prior phenomenological bias. The robustness of the procedure for obtaining an optimal regularization scale and performing a reliable chiral extrapolation is confirmed.

The procedure developed is then applied to the magnetic moment and the electric charge radius of the isovector nucleon, to obtain a consistent optimal regularization scale. The consistency of the results for the value of the optimal regularization scale provides strong evidence for the existence of an intrinsic energy scale for the nucleon-pion interaction.



## **Statement of Originality**

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