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THE APPLICATION OF TIKHONOV REGULARISED INVERSE FILTERING
TO DIGITAL COMMUNICATION THROUGH MULTI-CHANNEL ACOUSTIC
SYSTEMS

Pierre M. Dumuid

School of Mechanical Engineering
The University of Adelaide
South Australia 5005

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Abstract

Communication between underwater vessels such as submarines is difficult to achieve over long distances using radio waves because of their high rate of absorption by water. Using underwater acoustic wave propagation for digital communication has the potential to overcome this limitation. In the last 30 years, there have been numerous papers published on the design of communication systems for shallow underwater acoustic environments. Shallow underwater acoustic environments have been described as extremely difficult media in which to achieve high data rates. The major performance limitations arise from losses due to geometrical spreading and absorption, ambient noise, Doppler spread and reverberation from surface and seafloor reflections (multi-path), with the latter being the primary limitation. The reverberation from multi-path in particular has been found to be very problematic when using the general communication systems that have been developed for radio wave communication systems.

In the early 1990s, the principal means of combating multi-path in the shallow underwater environment was to use non-coherent modulation techniques. Coherent techniques were found to be challenging due to the difficulty of obtaining a phase-lock and also that the environment was subject to fading. Designs have since been presented that addressed both of these problems by using a complex receiver design that involved a joint update of the phase-lock loop and the taps of the decision feedback filter (DFE). In recent years a technique known as time-reversal has been investigated for use in underwater acoustic communication systems. A major benefit of using the time-reversal filter in underwater acoustic communication systems is that it can provide a fast and simple method to provide a receiver design of low complexity.

A technique that can be related to time-reversal and possibly used in underwater acoustics is Tikhonov regularised inverse filtering. The Tikhonov regularised inverse filter is a fast method of obtaining a stable inverse filter design by calculating the filter in the frequency domain using the fast Fourier transform, and was originally developed for use in audio reproduction systems. Previous research has shown that the Tikhonov regularised inverse filter design outperformed time-reversal when using a Dirac impulse

transmission within a simulated underwater environment. This thesis aims to extend the previous work by examining the implementation of Tikhonov regularised inverse filtering with communication signals. In addressing this goal, two topics have been examined: the influence of the sensitivities in the filter designs, and an examination of various design implementations for Tikhonov regularised inverse filtering and similar filtering techniques.

The influence of transducer sensitivities on the Tikhonov regularised inverse filter

During the implementation of the Tikhonov regularised inverse filter it was observed that both the Tikhonov regularised inverse filter and the time-reversal filter were influenced by the sensitivity of the transducers to the acoustic signals, which is determined by the transducer design and the amplifying stages. Unlike single channel systems, setting the sensitivities of the transducers to their maximum value for multi-channel systems does not always maximise the coherence between the input and output of the entire system consisting of the inverse filter, the sensitivities and the electro-acoustic system where the channel is the electro-acoustic transfer function between the transmitter and receiver. The influence the sensitivities have on the performance of the multi-channel Tikhonov regularised inverse filters and the time-reversal filter was examined by performing a mathematical examination of the system. An algorithm was developed that adjusted gains to compensate for the decrease in performance that results from the poor sensitivities. To test the algorithm, a system with an inappropriate set of sensitivities was examined. The performance improvement of the communication system was examined using the generated gains to scale the signal. The algorithm was found to reduce the signal degradation and cross-talk. If the gains were used in the digital domain (after the analog to digital and before the digital to analog converters) then the quality of the signal was improved at the expense of the signal level.

During this examination it was found that the time-reversal filter is equivalent to the Tikhonov regularised inverse filter with infinite regularisation.

Variations of the Tikhonov regularised inverse filter and performance comparisons

In this thesis, various design structures for the implementation of the Tikhonov inverse filter were proposed and implemented in an experimental digital communication system that operated through an acoustic environment in air. It was shown that the Tikhonov inverse filter and related filter design

structures could be classified or implemented according to three different classifications. The Tikhonov inverse filter was implemented according to each of these classifications and then compared against each other, as well as against two other filter designs discussed in the literature: time-reversal filtering, and the two-sided filter developed by Stojanovic [2005]. Due to the number of parameters that could be varied, it was difficult to identify the influence each parameter had on the results independently of the other parameters. A simulation was developed based on a model of the experiment to assist in identifying the influences of each parameter. The parameters examined included the number of transmitter elements, carrier frequency, data rate, and the value of the regularisation parameter.

When the communication system consisted of a signal receiver, the Stojanovic two-sided filter generally outperformed the Tikhonov regularised inverse filter designs when communicating. However, at higher data rates, the Stojanovic two-sided filter required the addition of a regularisation parameter to allow it to continue to operate. However, given an appropriately selected regularisation parameter, the difference between the performance of the Tikhonov filter and the Stojanovic two-sided filter was minimal.

When performing multi-channel communications, the full MIMO implementation of the Tikhonov regularised inverse filter design was shown to have the best performance. For the environment considered, the Tikhonov regularised inverse filter was the only design that was able to eliminate all symbol errors.

Statement of originality

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution to Pierre Dumuid and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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