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Financial Risk Measures

— The Theory and Applications of Backward Stochastic Difference/Differential Equations with respect to the Single Jump Process

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Abstract

This thesis studies financial risk measures which dynamically assign a value to a risk at a future date which can be interpreted as the present value of a future monetary value. In particular, the theories of backward stochastic difference equations in discrete time and differential equations in continuous time (BSDE) with respect to a single jump process are developed. Based on these theories, some associated dynamic risk measures are defined.

Chapter 1 is an introduction to the background of BSDEs, risk measures, and the single jump process, and also outlines the structure of this thesis.

Part I considers backward stochastic difference equations related to a discrete finite time single jump process (Chapter 2) and backward stochastic differential equations related to a finite continuous time single jump process (Chapter 3). We prove the existence and uniqueness of solutions of these BSDEs under some assumptions. Comparison Theorems for these solutions are also given. Applications to the theory of nonlinear expectations are then investigated.

Part II considers some applications of the theories established in Part I. In Chapter 4, risk measures related to the solutions of backward stochastic difference equations with respect to a discrete time single jump process are defined and some simple numerical examples are given. In Chapter 5, we consider the question of an optimal transaction between two investors to minimize their risks. We define a dynamic entropic risk measure using backward stochastic differential equations related to a continuous time single jump process. The inf-convolution of dynamic entropic risk measures is a key transformation in solving the optimization problem.

Signed Statement

I, Bin Shen (Leo Shen) certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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Published works within this thesis

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Shen, Leo and Elliott, R.J., Backward Stochastic Differential Equations With A Single Jump Process, *Stochastic Analysis and Applications* **29**(4)(2011), 654–673.

Shen, Leo and Elliott, R.J., How To Measure Risk, *Expert Systems With Applications* **39**(5)(2012), 6111–6115.

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Dedication

To Amy