

Gluonic Profile and Confining String in Static Mesons and Baryons at Finite Temperature

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Contents

Statement of originality	11
Dedication	12
Abstract	13
1 Quantum Chromodynamics	14
2 Lattice Gauge Theory	20
2.1 Outline	20
2.2 Introduction	20
2.3 Path integral formulation	20
2.4 Gauge field	21
2.5 Fermion field	22
2.6 Correlation functions	24
2.7 Continuum limit	25
2.8 Quenched approximation	27
2.9 Numerical simulation errors	27
2.10 Markov chain	28
2.11 Metropolis algorithm	29
2.12 Hybrid heat bath/Overrelaxation	30
2.12.1 Heat-bath algorithm	30
2.12.2 Overrelaxation	34
3 Systematic Uncertainties in Wilson-loop	35
3.1 Outline	35
3.2 Introduction	35
3.3 Flux distribution calculation	35
3.4 Source-shape dependence	36
3.5 Stringless operators	40
3.6 Lattice QCD at finite temperature	45
3.7 Conclusion	48
4 Bosonic String Model	49
4.1 Outline	49
4.2 Introduction	49
4.3 Mesonic string	51
4.3.1 Quark–antiquark potential	51

4.3.2	Width of the string	52
4.4	The Baryonic string picture	54
4.4.1	Width of the junction	57
4.5	Summary and Conclusion	64
5	Noise Reduction in Loop Correlators	65
5.1	Outline	65
5.2	Four-dimensional smearing of the gauge field	65
5.2.1	Smearing and cooling	66
5.2.2	Smearing radius	66
5.2.3	Calibration of smearing algorithms	68
5.2.4	Quark–antiquark potential	68
5.3	Link integration	74
5.4	Multi-level algorithm	77
5.5	Conclusion	78
6	Gluonic Profile of Static Mesons	80
6.1	Outline	80
6.2	Introduction	80
6.3	Colour field measurements	82
6.4	Action density	82
6.4.1	Tube profile (qualitative picture)	82
6.4.2	Tube growth in width	92
6.5	Conclusion	94
7	Bosonic Strings and the UV Filtering of QCD	96
7.1	Outline	96
7.2	Introduction	96
7.3	Quark–antiquark potential	98
7.4	The Gluonic profile	101
7.5	Conclusion	107
8	On The Ground State of Yang-Mills Theory	109
8.1	Outline	109
8.2	Introduction	109
8.3	Wilson loop operator	111
8.4	Numerical results and discussions	114
8.5	Conclusion	119
9	Gluonic Profile of Static Baryons	121
9.1	Outline	121
9.2	Introduction	122
9.3	Measurements	124
9.4	Statistics	124
9.5	Forces in the static baryon	125
9.6	Action Density	129
9.6.1	Flux iso-surface profile	129
9.6.2	Flux radius profile	139

9.6.3	Flux amplitude profile	142
9.6.4	The broadening of the flux width	144
9.6.5	Planes aspect ratio	147
9.7	Delocalization of the junction	149
9.8	Conclusion	154
10	Summary and Conclusion	157
A	Transition Form Factors	161
A.1	Introduction	161
A.2	The Vector-current-Hadron vertex function	161
A.2.1	Transition form factors	165
A.2.2	The ratio method	167
	Bibliography	169

NOTE: Pagination of the digital copy does not correspond with the pagination of the print copy.

List of Figures

3.1	The three-quark Wilson loop operator.	36
3.2	Spatial points [1] where vacuum field fluctuations are maximally suppressed in the three-quark system as measured by the correlation function $C(\vec{y})$.	37
3.3	The rendered gluonic field distribution via Eq. (3.3.3) mimics the form of the spatial link configurations in Wilson loop [2].	37
3.4	The ground-state overlap of the 3Q system [3]. $C_0 = \langle W_{3Q}(T) \rangle^{T+1} / \langle W_{3Q}(T+1) \rangle^T$, with the smeared link (upper data) and unsmeared link (lower data) [3]. The horizontal axis has been taken as L_{min} , with the minimal length of the flux tubes linking the three quarks. For each 3Q configuration C_0 is largely enhanced as $0.8 < C_0 < 1$ by smearing.	39
3.5	The static mesonic state is constructed via two Polyakov loops winding around the time in opposite directions.	43
3.6	(a) Average Polyakov loop over several successive Monte Carlo measurements [4], below the transition temperature (in the confined phase), (b) above the transition temperature.	44
3.7	$g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ plotted against temperature T from Ref. [5]. The vertical dashed line denotes the critical temperature $T_c = 280$ MeV in quenched QCD.	47
4.1	The mean-square width, Eq. (4.3.17), of the flux tube evaluated at all planes ξ_1 perpendicular to the quark–antiquark line. The separation distance between the pair is $Ra^{-1} = 10$.	54
4.2	Fluctuating flux tubes of three static color sources Q. The junction position ϕ is measured relative to the classical location which minimizes the total strings length (Fermat point).	55
4.3	World sheet spanned by one of the strings during time evolution up to the junction.	56
4.4	The world sheets of the strings in a baryon and a meson. The string in the static meson is modelled as being composed of two strings connected by a junction in the middle.	60
4.5	The Δ and Y baryonic strings configurations. The delocalization of the junction in the Y-shaped string system trace a filled Δ shaped energy distribution with maximal effect at the junction of the system. The Δ string configuration implies an energy distribution of maximum effect at the edges.	62

5.1	(a) Plot of the logarithm of the average value of $n(\rho = 0.06)/n(\rho)$ versus $\ln(\rho/0.06)$. The line corresponds to a fit to a straight line passing through the origin.	69
5.2	The quark–antiquark potential measured at each depicted smearing level. The lines correspond to fits of the potential obtained from the string picture of Eq. (4.3.7) for each data set as described in the text. The upper plot is at $T = 0.8 T_c$ while the lower plot is at $T = 0.9 T_c$	71
5.3	The $q\bar{q}$ force measured for all the smearing levels up to a distance of 1.4 fm. The temperature is $T = 0.8 T_c$, and $\beta = 6$. The line denotes the force as predicted by the string model at finite temperature, Eq. (4.3.7).	72
5.4	The diameter of smearing $2 R_s$ versus the number of sweeps n_s for the improved stout-link algorithm with $\rho = 0.06$ as in Eq. (5.2.9).	73
5.5	The Polyakov loop correlators measured using the heatbath integration (above), and fast link integration (below).	75
6.1	The flux-distribution $\bar{\mathcal{C}}(\vec{\rho}, \vec{r}_1, \vec{r}_2)$ as given by the characterization Eq. (6.3.3) in the plane of the quark–antiquark pair $\vec{\rho}(x, y, z = z_0)$, for separation distances R (a) 0.5 fm, (b) 0.6 fm, to (d) 0.8 fm at $T = 0.8 T_c$. The spheres refer to the positions of the quark and antiquark.	83
6.2	The flux-contour-line distribution in the plane of the quark–antiquark pair z_0 , for separation distances of (a) 0.9 fm, (b) 1.0 fm. The spheres denote the positions of the $q\bar{q}$ pair, $T = 0.9 T_c$	85
6.3	The flux iso-surface passing through the quarks, plotted together with a surface plot for the density distribution in the $q\bar{q}$ plane (inverted). The measurements are taken on 80 sweeps of smearing for separation distance $R = 9 a$, and $T = 0.8 T_c$. The lattice spatial extent is 36^3 at $\beta = 6$, $a = 0.1$ fm.	86
6.4	The density distribution $\mathcal{C}(\vec{\rho})$ for separation distance of $R = 12 a$, $T = 0.9 T_c$, plotted for the transverse planes $x = 1$, $x = 3$, and $x = 6$. The lines correspond to the Gaussian fits to the density in each plane $\vec{\rho}(x_i, y, z_0)$	87
6.5	The width difference $\delta W^2 = W^2(x_i) - W^2(x_0)$ for $q\bar{q}$ separations (a) 0.7 fm, (b) 0.8 fm, (c) 0.9 fm, and (d) 1 fm, $\beta = 6$, $T = 0.9 T_c$ for each depicted smearing level. The line denotes the width difference δW^2 as predicted by the string model Eq. (4.3.17). The lowest smearing level provides the best estimate of the width difference.	89
6.6	Similar to Fig. 6.5, the change in width is plotted for $q\bar{q}$ separations (a) 1.1 fm, (b) 1.2 fm, (c) 1.3 fm, and (d) 1.4 fm.	90
6.7	The width $W^2(x_i)$ for $q\bar{q}$ separations $R = 6 a$ to $R = 13 a$ at four consecutive planes (a) $x = 1$, (b) $x = 2$, (c) $x = 3$, and (d) $x = 4$. $\beta = 6$, $T = 0.9 T_c$. The line denotes the string model, Eq. (4.3.17), fit of R_0 to the data as described in the text.	93

7.1	(a) The temporal link U_4 is updated based on the neighboring links. The shaded area represents the 3D spatial smeared lattice. The heat bath starts from links of a low action configuration. The overrelaxation or reflection steps starts updates the time links based on action minimal	99
7.2	(a) The quark–antiquark potential measured at each depicted smearing level for 3 dimensional smearing (above) and 4 dimensional smearing (below). The lines correspond to the string picture predictions of Eq. (4.3.8). The standard value of the string tension is used.	100
7.3	Plot of the density distribution $\mathcal{C}(\vec{\rho})$ in the middle plane for source separation $R = 9a$, $T = 0.9T_C$	102
7.4	The mean square width of the flux tube $W_n^2(R/2)$ in the middle plane between the quarks. The lattice data, corresponding to the action density minimization, approach the string model predictions at short distances. At large distances the predicted linear divergence of the flux tube width is manifest in lattice data.	103
7.5	The normalized width of the flux tube $W_n^2(x_i)$ versus $q\bar{q}$ separations measured in the planes (a) $x = 2$, (b) $x = 3$, (c) $x = 4$. $\beta = 6$, $T = 0.9T_C$. The coordinates x_i are lattice coordinates (lattice units) and are measured from the quark position $x = 0$. The line denotes the one parameter string model, Eq. (7.4.1), fit to lattice data for $R \geq 1$ fm. The numbers in the legend denote the number of smearing sweeps.	104
7.6	The normalized width of the flux tube $W_n^2(x_i)$ versus $q\bar{q}$ separations measured in the plane $x = 3$. The temporal links have not been smeared in the evaluation of Polyakov loops, rather the temporal links have been integrated out. This time, the Polyakov loops are taken from the unsmeared configurations and correlated with the smeared action density. The legend denotes the number of smearing sweeps of the QCD vacuum.	105
7.7	The change of the tube’s width $\delta W^2 = W^2(x_i) - W^2(x_0)$ measured from the central plane for the depicted $q\bar{q}$ separations. The smearing level of the lattice data is illustrated. $\beta = 6$, $T = 0.9T_C$. The line denotes the width difference δW^2 as predicted by the string model Eq. (7.4.1).	106
7.8	The measured change in the ultraviolet cutoff R_0 along the flux tube normalized by a factor $(\pi\sigma)^{-1}$	107
8.1	Schematic representation of link-blocking	112
8.2	Schematic representation of smearing the spatial links in the Wilson loop. Each spatial link $U_\mu(x)$, $\mu \neq 4$ is smeared with the operators $(S^n)U$. The temporal links are left unsmeared. While not indicated in the figure, the links U^\dagger at the top of the figure are also smeared.	113
8.3	Schematic diagram of the smearing profile. h is the minimum number of smearing sweeps applied at the last link giving rise to smearing radius L_1 . n is the maximum number of smearing sweeps in the middle resulting in the radius L_2	113

8.4	The overlap with the ground state C_0 , the distance between the quark anti-quark source is $R = 10 a$, $\beta = 6$	115
8.5	The overlap with the ground state C_0 , for $R = 1$ fm. The lines connect the states corresponding to variation of the ellipse semi-major axis for each rectangular base corresponding to sweeps $h = 15$ and $h = 18$. . .	116
8.6	Comparison between non-uniformly smeared profiles $n \neq h$ and flat states $n = h$ represented by the smooth line. The quark source separation distance $R = 1.0$ fm.	117
8.7	The flux tube operator Eq. (8.3.8). Each operator consists of a family of five Gaussians. The operators correspond to the states $(h = 13, n = 34)$, $(h = 15, n = 32)$, $(h = 20, n = 34)$ and $(h = 24, n = 33)$. These states maximize the value of the overlap with the ground state. The source separation distance $R = 1.0$ fm.	117
8.8	Same as Fig. 8.6, for quark-antiquark separation distance $R = 1.2$ fm.	118
8.9	The overlap with the ground state C_0 versus the physical width of the flux-tube operator in the middle L_2 of the tube. Each line corresponds to a fixed ratio between the width of the tube in the middle and at the last link L_1 . The quark-antiquark separation distance of $R = 1.2$ fm is considered here.	118
9.1	Schematic diagram for the isosceles configuration of the 3Q system. The large spheres represent the motion of the diffused field of characteristic smearing radius of R_s centred at the quarks (small spheres).	126
9.2	The force for the isosceles 3Q configurations with base lengths (a) $A = 0.6$ fm, (b) $A = 0.8$ fm and (c) $A = 1.0$ fm, respectively. The x -axis denotes the position R of the third quark. Smearing effects are manifest for $R < 0.95$ fm, $R < 0.85$ fm and $R < 0.75$ fm for $A = 0.6$ fm, $A = 0.8$ fm, and $A = 1.0$ fm. Only subtle smearing effects remain beyond these distance scales.	127
9.3	Surface plot (inverted) of the flux distribution $\mathcal{C}(\vec{\rho})$ of Eq. (9.3.1) evaluated in the plane of the (3Q) system $\vec{\rho}(x, y, 0)$, for isosceles configuration of base length $A = 0.4$ fm and separation distances (a) $R = 0.6$ fm, (b) $R = 0.8$ fm and (c) $R = 1.0$ fm, at $T = 0.8 T_C$. The spheres refer to the positions of the quarks.	130
9.4	The flux action iso-surface at the quark positions, plotted together with a surface plot for the density distribution $\mathcal{C}(\vec{\rho})$, in the 3Q plane at temperature $T = 0.9 T_c$, for equilateral triangular configuration $R = 1.1$ fm and $A = 1.0$ fm.	131
9.5	Surface plot in the plane of the 3Q system $\vec{\rho}(x, y, z = 0)$ and iso-surface of the flux distribution $\mathcal{C}(\vec{\rho}; \vec{r}_1, \vec{r}_2, \vec{r}_3)$ for the isosceles configuration with $A = 1$ fm and the third quark separation distance R as indicated. $T = 0.8 T_C$	132
9.6	Comparison of the flux contour lines of the density distribution in the 3Q plane \mathcal{C} for triangular base $A = 1.0$ fm and third quark separation $R = 0.9$ fm at (a) $T = 0.8 T_c$ and (b) $T = 0.9 T_c$, in the $z = 0$ plane. As illustrated in Fig. 9.3, the maximal effect is near the Fermat point.	133

- 9.7 The density distribution $\mathcal{C}'(\vec{\rho})$ for the isosceles configuration with the base, $A = 1.0$ fm, and height $R = 0.8$ fm at $T/T_c = 0.8$ ($n_{\text{sw}} = 60$ sweeps). Data are plotted for the transverse planes $x = 1$ to $x = 6$. The lines correspond to the Gaussian fits to the density in each plane $\bar{\rho}(x_i, y, 0)$. The highest amplitude lies close to the Fermat point plane $x = 2.88$ of this 3Q configuration. 134
- 9.8 The radius profile of the flux-tube measured in the plane of the quarks for each isosceles configuration with base $A = 6a$, $A = 8a$ and $A = 10a$ ($a = 0.1$ fm), at two temperatures $T/T_c = 0.8$ (above) and $T/T_c = 0.9$ (below). The legend (in the upper right corner graph) signifies the third quark's position. 139
- 9.9 The radius profile of the flux-tube displaying a Y-shape like profile for quark configurations of base $A = 1.0$ fm and the third quark position $R = 1.3$ fm at temperature $T/T_c = 0.8$. In the background are the corresponding flux action-density contours. 141
- 9.10 Same as Fig. 9.8 for isosceles configuration bases of $A = 0.6$ fm, and $A = 1.0$ fm. The upper and lower figures compare the measured radius profile for two levels of smearing, 60 sweeps and 80 sweeps, respectively. The radius is in lattice units. 141
- 9.11 The profile of the action density amplitude, $H_y(x_i)$ (scaled by a factor of 10^1) for each isosceles configuration with base $A = 0.6$ fm, $A = 0.8$ fm and $A = 1.0$ fm, for the two temperatures $T/T_c = 0.8$ (upper), $T/T_c = 0.9$ (lower). The legend signifies the third quark position. . . 143
- 9.12 The squared flux distribution width at the depicted planes, $x_i = 2, 3$ and 7 , are compared for two smearing levels. The isosceles configuration base length is $A = 1.0$ fm at temperature $T/T_c = 0.9$. Smearing merely shifts the profile by a constant. The broadening pattern is not affected. 145
- 9.13 The squared flux-tube width at the depicted planes for the isosceles configuration $A = 0.8$ fm compared at two temperatures $T/T_c = 0.8$ (top) and $T/T_c = 0.9$ (bottom). The plane coordinates are indicated in the legend. 146
- 9.14 Same as Fig. 9.13 for a larger isosceles base length of $A = 1.0$ fm. . . 146
- 9.15 Comparison of the mean square width of the flux distribution at three distinct planes, $x = 2, 3$, and 7 , for two isosceles bases $A = 0.6$ fm and $A = 0.8$ fm. The upper graphs show the comparison at $T/T_c = 0.9$ whereas the lower are at $T/T_c = 0.8$ 147
- 9.16 Comparison of the ratio, α (Eq. (9.6.7)), of the mean squared width of the flux distribution parallel and perpendicular to the quark plane for three isosceles bases $A = 0.6$ fm, $A = 0.8$ fm and $A = 1.0$ fm. The upper graphs show comparison at $T/T_c = 0.8$ whereas the lower are at $T/T_c = 0.9$ 148

- 9.17 Surface plot of the flux density surface in the quark plane, $\rho(x, y, z = 0)$, together with contour lines. The contour lines are projected onto the surface plot. The density of the contour lines increases near the edges in accord with the gradient of the density scalar field along the x -axis. The flux contours of the maximum value are the inner most lines inside the triangle. As the density plot illustrates, the flux-tube configuration is a filled Δ -shape with maximum action expulsion inside the triangle near the Fermat point of the configuration. These measurements are taken for an isosceles quark geometry of base $A = 0.8$, height $R = 1.2$ fm and temperature $T = 0.9 T_c$ 151
- 9.18 Schematic diagram for the isosceles configuration of the 3Q system. L_1 , L_2 and L_3 are distances from the Fermat point to the quark positions. 152

Statement of originality

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Ahmed Saad El Bakry Mahmoud

Dedication

To my family.

Abstract

The distribution of the gluon action density in mesonic systems is investigated at finite temperature. The simulations are performed in pure SU(3) Yang-Mills gauge theory for two temperatures below the deconfinement phase. The action-density isosurfaces display a prolate-spheroid-like shape. The curved width profile of the flux tube is found to be consistent with the prediction of the free bosonic string model at large distances.

In the intermediate source separation distance, where the free string picture poorly describes the flux tube width profile, we find the topological characteristics of the flux tube converge and compare favourably with the predictions of the free bosonic string upon reducing the vacuum action towards the classical instanton vacuum. As a byproduct of these calculations, we find the broadening of the QCD flux tube to be independent of the UV filtering at large distances. Our results exhibit a linearly divergent pattern in agreement with the string picture predictions.

We investigate the overlap of the ground state meson potential with sets of mesonic-trial wave functions. We construct trial states with non-uniform smearing profiles in the Wilson loop operator at $T = 0$. The non-uniformly UV-regulated flux-tube operators are found to optimize the overlap with the ground state.

The gluon flux distribution of a static three quark system has been revealed at temperatures near the end of the QCD plateau, $T/T_c \approx 0.8$, and another just before the deconfinement point, $T/T_c \approx 0.9$. The flux distributions at short distance separations between the quarks display an action-density profile consistent with a rounded filled Δ shape iso-surface. However the Δ shape action iso-surface distributions are found to persist even at large inter-quark separations. The action density distribution in the quark plane exhibits a nonuniform pattern for all quark separations considered. We systematically measure and compare the main aspects of the profile of the flux distribution at the two considered temperature scales for three sets of isosceles triangle quark configurations. The radii, amplitudes and rate of change of the width of the flux distribution are found to reverse their behavior as the temperature increases from the end of the QCD plateau towards the deconfinement point. Remarkably, we find the mean square width of the flux distribution shrinks and localizes for quark separations larger than 1.0 fm at $T/T_c \approx 0.8$ which results in an identifiable Y-shaped radius profile. Near the deconfinement point, the action-density delocalizes and the width broadens linearly with the quark separation at large quark separations.

We present a method to include the thermal effects into the junction width of the baryonic string model. The profile of the baryonic gluonic distribution is compared with the width of the string picture's junction fluctuations. The comparison reveals that the best fits to the junction fluctuations of the baryonic string are near the Fermat point of the triangle made up by the quarks. This result supports the underlying picture of Y-shaped string-like flux tubes connected at a junction.