

# Quantum Computation and a Universal Quantum Computer



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# Glossary

<b>1WQC</b>	One-Way Quantum Computing
<b>CPU</b>	Central Processing Unit
<b>EPR</b>	Einstein Podolsky Rosen
<b>GRK</b>	Grover Radhakrishnan
<b>GSQC</b>	Ground State Quantum Computation
<b>HQC</b>	Holonomic Quantum Computing
<b>I/O</b>	Input Output
<b>QAC</b>	Quantum Adiabatic Computing
<b>QFT</b>	Quantum Fourier Transform
<b>QGA</b>	Quantum Gate Array
<b>QTM</b>	Quantum Turing Machine
<b>UQC</b>	Universal Quantum Computer
<b>UQTM</b>	Universal Quantum Turing Machine
<b>UTM</b>	Universal Turing Machine

## **GLOSSARY**

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## Abstract

This thesis covers two main topics in quantum computing: universal quantum computation and quantum search. We first demonstrate how a quantum harmonic oscillator can be used to implement the universal set of quantum gates and thereby serve as one possible building block for a universal quantum computer. We then address the core and primary focus of this thesis, the theoretical construction of a machine that can compute every computable function, that is, a universal (i.e. *programmable*) quantum computer. We thereby settle the questions that have been raised over the years regarding the validity of the UQTM proposed by Deutsch in 1985. We then demonstrate how to interface the universal quantum computer to external quantum devices by developing programs that implement well-known oracle based algorithms, including the well-known Grover search algorithm, using networked quantum oracle devices. Finally, we develop a partial search oracle and explore symmetry based partial search algorithms utilizing this oracle.

## Declaration

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution to myself and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give consent to this copy of my thesis when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968. The author acknowledges that copyright of the published work contained in Appendix B of this thesis resides with the copyright holder(s) of the publishing journal. I also give permission for the digital version of my thesis to be made available on the web, via the Universitys digital research repository, the Library catalogue, the Australasian Digital Theses Program (ADTP) and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

The research work presented in this thesis was conducted from February 2007 to March 2010 under the supervision of Associate Professor Max Lohe, Dr. Lorenz von Smekal, and Professor Anthony Williams.

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