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Research Article

Observer-Based Stabilization of Stochastic Systems with Limited Communication

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This paper studies the problem of observer-based stabilization of stochastic nonlinear systems with limited communication. A communication channel exists between the output of the plant and the input of the dynamic controller, which is considered network-induced delays, data packet dropouts, and measurement quantization. A new stability criterion is derived for the stochastic nonlinear system by using the Lyapunov functional approach. Based on this, the design procedure of observer-based controller is presented, which ensures asymptotic stability in the meansquare of the closed-loop system. Finally, an illustrative example is given to illustrate the effectiveness of the proposed design techniques.

1. Introduction

Stochastic variables frequently exist in practical systems such as aircraft systems, biology systems, and electronic circuits. Without taking them into account in the system design, the stochastic variables can bring negative effects on the performance of control systems and even make the systems unstable. According to the way stochastic variable occurs, stochastic system mode can be classified as Itô stochastic differential equation [1, 2], Markov switched systems [3–5], and other systems with stochastic variables [6–9]. Since the introduction of the concept of stochastic differential equation by Itô [10] in 1951, Itô stochastic system model has been used successfully in numerous applications, such as the analysis of stock systems and prediction for ecosystem. In automatic control of stochastic systems, a great number of important results have been reported in the literature [11, 12].

In the past two decades, network-based control technology has been developed to combine a communication network with conventional control systems to form the Network

Control Systems (NCSs), which have wide applications due to their advantages, such as reduced weight, power requirements, low installation cost, and easy maintenance [13]. Since the capacity of the communication channel is limited [14–16], signal transmission delay and data packet dropout are two fundamental problems in NCSs. To deal with these issues, considerable research results on this topic have been reported, see for example [17–20] and the references therein. In [21], the robust H_∞ control problem was considered for a class of networked systems with random communication packet losses.

Among the reported results, most NCSs are mainly based on deterministic physical plant. However, stochastic systems models also have wide applications in the dynamical systems. This has motivated the researches on networked control for stochastic systems and many results have been reported in the literature. In [22], the problem of network-based control for stochastic plants was studied, and a new model of stochastic time-delay systems was presented including both network-induced delays and packet dropouts. In [23], the problem of sampled-data control for networked control systems was considered. In recent years, much attention is paid to the problem of the observer-based controller design for NCSs [24–27]. In [28], the problem of the NCS design for continuous-time systems with random measurement was investigated, where the measurement channel is assumed to be subjected to random sensor delay. To the authors' knowledge, the problem of observer-based controller design for stochastic nonlinear systems with limited communication has not been fully investigated and still remains challenging, which motivates us for the present study.

In this paper, we investigate the problem of observer-based stabilization of stochastic nonlinear systems with limited communication. A new model is proposed to describe the stochastic nonlinear systems with a communication channel, which exists between the output of the stochastic plant and the input of the observer-based controller. Based on this, the design procedure of observer-based controller is proposed, which ensures the asymptotic stability of the resulting closed-loop system. Finally, a mechanical system example consisted of two cars, a spring and a damper, is given to illustrate the effectiveness of the proposed controller design method.

Notation. The notation used throughout the paper is fairly standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and the notation $P > 0 \quad (\geq 0)$ means that P is real symmetric and positive definite (semidefinite). In symmetric block matrices or complex matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry and $\mathrm{diag}\{\cdots\}$ standing for a block-diagonal matrix. $\mathrm{sym}(A)$ is defined as $A + A^T$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. $\mathbb{E}\{x\}$ means the expectation of x. The space of square-integrable vector functions over $[0, \infty)$ is denoted by $L_2[0, \infty)$, and for $w = \{w(t)\} \in L_2[0, \infty)$, its norm is given by: $\|w\|_2 = \sqrt{\int_{t=0}^{\infty} |w(t)|^2 dt}$.

2. Problem Formulation

Consider the following stochastic nonlinear system:

$$\begin{aligned} dx(t) &= \left[Ax(t) + Bu(t) + g(x(t))\right]dt + Ex(t)d\omega(t), \\ y(t) &= Cx(t), \\ x(t) &= \phi(t), \quad t \in [-2\kappa, 0], \end{aligned} \tag{2.1}$$

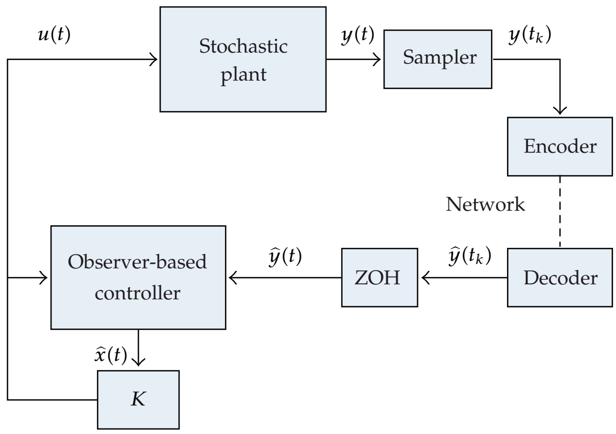

Figure 1: The stochastic systems with limited communication.

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the control input; $y(t) \in \mathbb{R}^p$ is the control output; $g(\cdot) : \mathbb{R}^n \to \mathbb{R}^{n_f}$ is unknown nonlinear function; C and E are constant matrices with an appropriate dimension; κ is the maximum delay; $\omega(t)$ is a zero-mean real scalar Wiener process, which satisfies $\mathbb{E}\{d\omega(t)\} = 0$ and $\mathbb{E}\{d\omega(t)^2\} = dt$.

For system (2.1), it is assumed that the states are not fully measured. Thus, we consider the following observer-based controller:

$$\begin{aligned} d\hat{x}(t) &= \left[A\hat{x}(t) + Bu(t) + g(\hat{x}(t)) + L\left(\hat{y}(t) - C\hat{x}(t)\right)\right]dt, \\ u(t) &= K\hat{x}(t), \end{aligned} \tag{2.2}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimation of the state vector $x(t)$; $\hat{y}(t) \in \mathbb{R}^p$ denotes the output of the zero-order hold (ZOH); K and L are the controller and observer gains.

Under control law (2.2), the closed-loop system in (2.1) is given by

$$dx(t) = \left[Ax(t) + BK\hat{x}(t) + g(x(t))\right]dt + Ex(t)d\omega(t). \tag{2.3}$$

The structure of the stochastic systems with limited communication is shown in Figure 1. In this system, for convenience of analysis, it is assumed that communication delay occurs only in the sampler-to-controller side. The stochastic plant continuously sends the output signal $y(t)$ to the controller by a network. $y(t)$ is firstly sampled by the sampler, which is assumed to be clock-driven. Then, $y(t_k)$, where t_k denotes the sampling instant for $k = 0, 1, 2, \ldots$, is encoded and decoded by the quantizer and sent to ZOH, which are assumed to be event-driven. $\hat{y}(t)$ and $u(t)$ are the input of the observer-based controller and $\hat{x}(t)$ is the output of the observer-based controller.

In this paper, the quantizer is chosen as the logarithmic quantizer. The set of quantized levels is described by:

$$\mathcal{U}_i = \left\{\pm u_i^{(j)},\ u_i^{(j)} = \rho_j^i u_0^{(j)},\ i = \pm 1, \pm 2, \ldots\right\} \cup \left\{\pm u_0^{(j)}\right\} \cup \{0\}, \quad 0 < \rho_j < 1,\ u_0^{(j)} > 0. \tag{2.4}$$

Each of the quantization level $u_i^{(j)}$ corresponds to a segment such that the quantizer maps the whole segment to this quantization level. In addition, these segments form a partition of $\mathbb{R}$, that is, they are disjoint and their union for i equals to $\mathbb{R}$. For the logarithmic quantizer, the associated quantizer $f_i(\cdot)$ is defined as

$$f_i(v) = \begin{cases} u_i^{(j)} & \text{if } \dfrac{1}{1+\sigma_j}u_i^{(j)} < v \le \dfrac{1}{1-\sigma_j}u_i^{(j)},\ v > 0, \\ 0 & \text{if } v = 0, \\ -f_j(-v) & \text{if } v < 0, \end{cases} \tag{2.5}$$

where $\sigma_j = (1-\rho_j)/(1+\rho_j\ )$.

When taking into account signal transmission delays η_k from sampler to ZOH, the quantized output signal takes the following form:

$$\hat{y}(t_k) = f(y(t_k - \eta_k)) = \left[f_1(y_1(t_k - \eta_k))\ \ f_2(y_2(t_k - \eta_k))\ \ \cdots\ \ f_n(y_n(t_k - \eta_k))\right]^T. \tag{2.6}$$

Considering the behavior of the ZOH, we have

$$\hat{y}(t) = f(y(t_k - \eta_k)), \quad t_k \le t < t_{k+1}, \tag{2.7}$$

with t_{k+1} being the next updating instant of the ZOH after t_k.

A natural assumption on the network induced delays η_k can be made as

$$0 \le \eta_k \le \overline{\eta}, \tag{2.8}$$

where $\overline{\eta}$ denotes the maximum delay. In addition, at the updating instant t_{k+1} the number of accumulated data packet dropouts since the last updating instant t_k is denoted as δ_{k+1}. We assume that the maximum number of data packet dropouts is $\overline{\delta}$, that is,

$$\delta_{k+1} \le \overline{\delta}. \tag{2.9}$$

Then, it can be seen from (2.8) and (2.9) that

$$t_{k+1} - t_k = (\delta_{k+1} + 1)h + \eta_{k+1} - \eta_k, \tag{2.10}$$

where h denotes the sampling period.

As the time sequence t_k depends on both the network-induced delays and data packet dropouts, the period $t_{k+1} - t_k$ for the sampled-data system in (2.3) is variable and uncertain. Now let us represent $t_k - \eta_k$ in (2.7) as

$$t_k - \eta_k = t - \eta(t), \tag{2.11}$$

where

$$\eta(t) = t - t_k + \eta_k. \tag{2.12}$$

Then, from (2.10) we have

$$0 \leq \eta(t) \leq \kappa, \tag{2.13}$$

where

$$\kappa = \overline{\eta} + \left(\overline{\delta} + 1\right)h. \tag{2.14}$$

Considering the quantization shown in (2.5) and by substituting (2.11) into (2.7), (2.2) can be expressed as

$$\begin{gathered} d\hat{x}(t) = \left[A\hat{x}(t) + Bu(t) + g(\hat{x}(t)) + L\left((I + \Lambda(t))y\left(t - \eta(t)\right) - C\hat{x}(t)\right)\right]dt, \\ u(t) = K\hat{x}(t), \end{gathered} \tag{2.15}$$

where

$$\Lambda(t) = \operatorname{diag}\{\Lambda_1(t), \Lambda_2(t), \ldots, \Lambda_n(t)\}, \tag{2.16}$$

with

$$\Lambda_j(t) \in \left[-\sigma_j, \sigma_j\right], \quad j = 1, \ldots, n. \tag{2.17}$$

Defining the estimation error $e(t) = x(t) - \hat{x}(t)$, we obtain

$$\begin{aligned} dx(t) &= \left[(A + BK)x(t) - BKe(t) + g(x(t))\right]dt + Ex(t)d\omega(t), \\ de(t) &= \left[LCx(t) + (A - LC)e(t) + g(x(t)) - g(x(t) - e(t))\right. \\ &\quad \left. - L(I + \Lambda(t))Cx\left(t - \eta(t)\right)\right]dt + Ex(t)d\omega(t). \end{aligned} \tag{2.18}$$

Before proceeding further, we introduce the following assumption and lemma, which will be used in subsequent developments.

Assumption 2.1. For a stochastic system mode, there exists known real constant matrices $G \in \mathbb{R}^{n\times n}$, such that the unknown nonlinear vector function $g(\cdot)$ satisfies the following boundedness condition:

$$\left|g(x(t))\right| \leq |Gx(t)|, \quad \forall x(t) \in \mathbb{R}^n. \tag{2.19}$$

Lemma 2.2 (see [29]). *Given appropriately dimensioned matrices Σ_1, Σ_2 and Σ_3, with $\Sigma_1^T = \Sigma_1$, then,*

$$\Sigma_1 + \Sigma_3 H(t)\Sigma_2 + \Sigma_2^T H^T(t)\Sigma_3^T < 0 \tag{2.20}$$

holds for all $H(t)$ satisfying $H^T(t)H(t) \leq I$ if and only if for some $\varepsilon > 0$,

$$\Sigma_1 + \varepsilon^{-1}\Sigma_3\Sigma_3^T + \varepsilon\Sigma_2^T\Sigma_2 < 0. \tag{2.21}$$

3. Main Results

In this section, the problem of asymptotical stabilization of stochastic system with limited communication is studied. We are first concerned with the asymptotical stability analysis problem. The following theorem develops a sufficient condition for system (2.18) to be asymptotically stable in the meansquare.

Theorem 3.1. *The nominal stochastic system* (2.18) *is asymptotically stable in the mean square if there exist scalars $\varepsilon_i > 0$, ($i = 1, 2, 3$) and matrices $P_j > 0$, $R_j > 0$, S_j, U_j, ($j = 1, 2$) satisfying*

$$\begin{bmatrix} \Pi_1 + \varepsilon_3\Pi_4^T\Pi_4 & \sqrt{\kappa+1}V & \Pi_2^T & \Pi_3^T & \Pi_5^T \\ * & \Pi_6 & 0 & 0 & 0 \\ * & * & -R_1^{-1} & 0 & 0 \\ * & * & * & -R_2^{-1} & -L \\ * & * & * & * & -\varepsilon_3 I \end{bmatrix} < 0, \tag{3.1}$$

where

$$\begin{gathered} \Pi_1 = \operatorname{sym}\left(W_x^T P_1 W_{r_1} + W_e^T P_2 W_{r_2} + VW_v - W_x^T\left(\varepsilon_2 G^T G\right)W_e\right) + W_g^T\Psi_1 W_g \\ + W_x^T E^T(P_1 + \kappa R_1 + P_2 + \kappa R_2)W_x, \\ W_x = \begin{bmatrix} I_n & 0_{n,5n} \end{bmatrix}, \qquad W_e = \begin{bmatrix} 0_{n,2n} & I_n & 0_{n,3n} \end{bmatrix}, \qquad V = \begin{bmatrix} \widetilde{S} & \widetilde{U} \end{bmatrix}, \\ \Psi_1 = \operatorname{diag}\left\{(\varepsilon_1 + \varepsilon_2)G^T G, \varepsilon_2 G^T G, -\varepsilon_1 I, -\varepsilon_2 I\right\}, \\ \widetilde{S} = \begin{bmatrix} S_1^T & S_2^T & 0_n & 0_n & 0_n & 0_n \end{bmatrix}^T, \qquad \widetilde{U} = \begin{bmatrix} 0_n & 0_n & U_1^T & U_2^T & 0_n & 0_n \end{bmatrix}^T, \\ W_v = \left[\begin{array}{c} \begin{matrix} I_n & -I_n & 0_{n,4n} \end{matrix} \\ \hline \begin{matrix} 0_{n,2n} & I_n & -I_n & 0_{n,2n} \end{matrix} \end{array}\right], \qquad W_g = \left[\begin{array}{c} \begin{matrix} I_n & 0_{n,5n} \end{matrix} \\ \hline \begin{matrix} 0_{n,2n} & I_n & 0_{n,3n} \end{matrix} \\ \hline \begin{matrix} 0_{n,4n} & I_n & 0_n \end{matrix} \\ \hline \begin{matrix} 0_{n,5n} & I_n \end{matrix} \end{array}\right], \\ \Pi_2 = \sqrt{\kappa}W_{r_1}, \qquad W_{r_1} = \begin{bmatrix} A + BK & 0_n & -BK & 0_n & I_n & 0_n \end{bmatrix}, \\ \Pi_3 = \sqrt{\kappa}W_{r_2}, \qquad W_{r_2} = \begin{bmatrix} LC & -LC & A - LC & 0_n & I_n & -I_n \end{bmatrix}, \\ \Pi_4 = \begin{bmatrix} 0_n & \Lambda C & 0_n & 0_n & 0_n & 0_n \end{bmatrix}, \qquad \Lambda = \operatorname{diag}\{\Lambda_1, \Lambda_2, \ldots, \Lambda_n\}, \\ \Pi_5 = \begin{bmatrix} 0_{n,p} & 0_{n,p} & -L^T P_2 & 0_{n,p} & 0_{n,p} & 0_{n,p} \end{bmatrix}, \qquad \Pi_6 = \operatorname{diag}\{-R_1, -R_2\}. \end{gathered} \tag{3.2}$$

Proof. For technical convenience, we rewrite (2.18) as

$$\begin{aligned} dx(t) &= r_1(t)dt + Ex(t)d\omega(t), \\ de(t) &= r_2(t)dt + Ex(t)d\omega(t), \end{aligned} \tag{3.3}$$

where

$$\begin{aligned} r_1(t) &= (A + BK)x(t) - BKe(t) + g(x(t)), \\ r_2(t) &= LCx(t) + (A - LC)e(t) + g(x(t)) \\ &\quad - g(x(t) - e(t)) - L(I + \Lambda(t))Cx\big(t - \eta(t)\big). \end{aligned} \tag{3.4}$$

Now, choose the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t) = x^T(t)P_1x(t) &+ \int_{t-\kappa}^{t}\int_{s}^{t} r_1^T(\theta)R_1r_1(\theta)d\theta ds + \int_{t-\kappa}^{t}\int_{s}^{t} x^T(\theta)E^TR_1Ex(\theta)d\theta ds \\ &+ e^T(t)P_2e(t) + \int_{t-\kappa}^{t}\int_{s}^{t} r_2^T(\theta)R_2r_2(\theta)d\theta ds + \int_{t-\kappa}^{t}\int_{s}^{t} x^T(\theta)E^TR_2Ex(\theta)d\theta ds, \end{aligned} \tag{3.5}$$

where $P_j > 0$, $R_j > 0$, $(j = 1, 2)$ are matrices to be determined. Then, by Itô's formula and from (3.5), we obtain the stochastic differential as

$$dV(t) = \mathcal{L}V(t)dt + 2\Big(x^T(t)P_1Ex(t) + e^T(t)P_2Ex(t)\Big)d\omega(t) \tag{3.6}$$

and

$$\begin{aligned} \mathcal{L}V(t) = {} & 2x^T(t)P_1r_1(t) + r_1^T(t)\kappa R_1r_1(t) \\ & - \int_{t-\kappa}^{t} r_1^T(s)R_1r_1(s)ds + x^T(t)E^T(P_1 + \kappa R_1)Ex(t) \\ & - \int_{t-\kappa}^{t} x^T(s)E^TR_1Ex(s)ds + 2e^T(t)P_2r_2(t) + r_2^T(t)\kappa R_2r_2(t) - \int_{t-\kappa}^{t} r_2^T(s)R_2r_2(s)ds \\ & + x(t)^TE^T(P_2 + \kappa R_2)Ex(t) - \int_{t-\kappa}^{t} x^T(s)E^TR_2Ex(s)ds \\ \le {} & 2x^T(t)P_1r_1(t) + r_1^T(t)\kappa R_1r_1(t) + x(t)^TE^T(P_1 + \kappa R_1 + P_2 + \kappa R_2)Ex(t) \\ & - \int_{t-\eta(t)}^{t} r_1^T(s)R_1r_1(s)ds - \int_{t-\eta(t)}^{t} x^T(s)E^TR_1Ex(s)ds \\ & + 2e^T(t)P_2r_2(t) + r_2^T(t)\kappa R_2r_2(t) \\ & - \int_{t-\eta(t)}^{t} r_2^T(s)R_2r_2(s)ds - \int_{t-\eta(t)}^{t} x^T(s)E^TR_2Ex(s)ds + 2X_1(t) + 2X_2(t), \end{aligned} \tag{3.7}$$

where

$$X_1(t)=\xi_1^T(t)S\left(x(t)-x(t-\eta(t))-\int_{t-\eta(t)}^{t}r_1(s)ds-\int_{t-\eta(t)}^{t}Ex(s)d\omega(s)\right)=0,$$
$$X_2(t)=\xi_2^T(t)U\left(e(t)-e(t-\eta(t))-\int_{t-\eta(t)}^{t}r_2(s)ds-\int_{t-\eta(t)}^{t}Ex(s)d\omega(s)\right)=0,$$
$$\xi_1^T(t)=\left[x^T(t)\ \ x^T(t-\eta(t))\right],\qquad S=\left[S_1^T\ \ S_2^T\right]^T,$$
$$\xi_2^T(t)=\left[e^T(t)\ \ e^T(t-\eta(t))\right],\qquad U=\left[U_1^T\ \ U_2^T\right]^T. \tag{3.8}$$

From (2.19), we obtain

$$\Upsilon_1(t)=\varepsilon_1x^T(t)G^TGx(t)-\varepsilon_1g^T(x(t))g(x(t))\geq 0,$$
$$\Upsilon_2(t)=\varepsilon_2(x(t)-e(t))^TG^TG(x(t)-e(t))$$
$$-\varepsilon_2g^T(x(t)-e(t))g(x(t)-e(t))\geq 0, \tag{3.9}$$

where ε_1 and ε_2 are positive constants. Then, taking expectation on both sides of (3.7), we have

$$\mathbb{E}\{\mathcal{L}V(t)\}+\Upsilon_1(t)+\Upsilon_2(t)\leq\mathbb{E}\left\{\xi^T(t)\left[\overline{\Pi}_6+\Sigma_4+\Sigma_5\right]\xi(t)\right\}+\Sigma_6+\Sigma_7, \tag{3.10}$$

where

$$\overline{\Pi}_1=\operatorname{sym}\left(W_x^TP_1W_{r_1}+W_e^TP_2W_{\tilde{r}_2}+VW_v\right)+W_x^TE^T(P_1+\kappa R_1+P_2+\kappa R_2)W_x+W_g^T\Psi_1W_g,$$
$$\Sigma_4=\kappa W_{r_1}^TR_1W_{r_1}+\kappa W_{\tilde{r}_2}^TR_2W_{\tilde{r}_2},\qquad \Sigma_5=(\kappa+1)\widetilde{S}R_1^{-1}\widetilde{S}^T+(\kappa+1)\widetilde{U}R_2^{-1}\widetilde{U}^T,$$
$$W_{\tilde{r}_2}=\left[LC\ \ -L(I+\Lambda(t))C\ \ A-LC\ \ 0\ \ I\ \ -I\right],$$
$$\Sigma_6=-\int_{t-\eta(t)}^{t}\left[\xi_1^T(t)S+r_1(s)R_1\right]R_1^{-1}\left[S^T\xi_1(t)+R_1r_1(s)\right]ds,$$
$$\Sigma_7=-\int_{t-\eta(t)}^{t}\left[\xi_2^T(t)U+r_2(s)R_2\right]R_2^{-1}\left[U^T\xi_2(t)+R_2r_2(s)\right]ds,$$
$$\xi^T(t)=\left[\xi_1^T(t)\ \ \xi_2^T(t)\ \ g^T(x(t))\ \ g^T(x(t)-e(t))\right]. \tag{3.11}$$

Note that $R_1>0$ and $R_2>0$, thus Σ_6 and Σ_7 are nonpositive. Therefore, from (3.10) we know that $\mathbb{E}\{\mathcal{L}V(t)\}+\Upsilon_1(t)+\Upsilon_2(t)<0$ if

$$\overline{\Pi}_1+\Sigma_4+\Sigma_5<0, \tag{3.12}$$

which by Schur complements, is equivalent to

$$\begin{bmatrix} \overline{\Pi}_1 & \sqrt{\kappa+1}V & \Pi_2^T & \overline{\Pi}_7^T \\ * & \Pi_6 & 0 & 0 \\ * & * & -R_1^{-1} & 0 \\ * & * & * & -R_2^{-1} \end{bmatrix} < 0, \tag{3.13}$$

where $\overline{\Pi}_7 = \sqrt{\kappa} W_{\tilde{r}_2}$. Now, rewrite (3.13) in the form (2.20) with

$$\Sigma_1 = \begin{bmatrix} \Pi_1 & \sqrt{\kappa+1}V & \Pi_2^T & \Pi_3^T \\ * & \Pi_6 & 0 & 0 \\ * & * & -R_1^{-1} & 0 \\ * & * & * & -R_2^{-1} \end{bmatrix},$$
$$\Sigma_2 = \begin{bmatrix} \Pi_4 & 0 & 0 & 0 \end{bmatrix}, \qquad \Sigma_3 = \begin{bmatrix} \Pi_5 & 0 & 0 & -L^T \end{bmatrix}^T, \qquad H(t) = \Lambda(t)\Lambda^{-1}. \tag{3.14}$$

By Lemma 2.2 together with a Schur complement operation, (3.13) holds if for some $\varepsilon > 0$, (3.1) holds. Thus, we have

$$\mathbb{E}\{\mathcal{L}V(t)\} < 0, \tag{3.15}$$

which ensures that the closed-loop system in (2.18) is asymptotically stable by [30]. Theorem 3.1 is proved. □

Since our main objective is to design K and L to stabilize the system (2.18), (3.1) is actually a nonlinear matrix inequality. We will transform them into tractable conditions to solve the control synthesis problem.

Theorem 3.2. *There exists an observer-based controller such that the closed-loop system in* (2.18) *is asymptotically stable in the mean square if there exist scalars $\varepsilon_i > 0$ ($i = 1, 2, 3$) and matrices $\overline{P}_1 > 0$, $P_2 \geq 0$, $R_1 > 0$, $R_2 > 0$, $Z_i > 0$, $Q_i > 0$ and S, U, $\overline{K}$, $\overline{L}$, satisfying*

$$\begin{bmatrix} \Xi_1 & \Xi_2 \\ * & \Xi_3 \end{bmatrix} < 0, \tag{3.16}$$

$$\begin{bmatrix} \Phi_1 & \Phi_2 \\ * & \Phi_3 \end{bmatrix} < 0, \tag{3.17}$$

$$\begin{bmatrix} Z_1 & I \\ * & Q_1 \end{bmatrix} > 0, \qquad \begin{bmatrix} Z_3 & I \\ * & Q_2 \end{bmatrix} > 0, \qquad \begin{bmatrix} R_1 & I \\ * & Q_3 \end{bmatrix} > 0, \tag{3.18}$$

where

$$\Xi_1 = \operatorname{sym}\left(W_{\bar{e}}^T P_2 W_{\bar{r}_2} + \overline{V} W_{\bar{v}} - W_{\hat{x}}^T\left(\varepsilon_2 G^T G\right) W_{\bar{e}}\right) + W_{\bar{g}}^T \Psi_1 W_{\bar{g}} + W_z^T Z W_z,$$

$$\Xi_2 = \left[\sqrt{\kappa}\Upsilon_1^T \quad \sqrt{\kappa+1}\,\overline{V} \quad \Upsilon_2^T \quad \Upsilon_3^T\right], \qquad Z = \operatorname{diag}\{-Z_1, Z_2, Z_3\},$$

$$\Xi_3 = \operatorname{diag}\{R_2 - 2P_2, -R_1, -R_2, -P_2, -R_2, -\varepsilon_3 I\},$$

$$\Phi_1 = \operatorname{sym}\left(W_{\bar{x}}^T W_{\bar{r}_1}\right) - W_y^T \overline{Z} W_y, \qquad \overline{Z} = \operatorname{diag}\left\{Z_2, 2\overline{P}_1 - Q_2\right\},$$

$$\Phi_2 = \begin{bmatrix} \sqrt{\kappa}\left(\overline{P}_1 A^T + \overline{K}^T B^T\right) & \overline{P}_1 E^T & \sqrt{\kappa}\,\overline{P}_1 E^T & \overline{P}_1 \\ \sqrt{\kappa} I & 0 & 0 & 0 \\ -\sqrt{\kappa}\,\overline{K}^T B^T & 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_3 = \operatorname{diag}\left\{-Q_3, -\overline{P}_1, -Q_3, -Q_1\right\}, \qquad \overline{V} = \left[\overline{S} \quad \overline{U}\right], \qquad W_{\hat{x}} = \left[0_{n,3n} \quad I_n \quad 0_{n,2n}\right],$$

$$\overline{S} = \left[0_n \quad S_2^T \quad 0_n \quad S_1^T \quad 0_n \quad 0_n\right]^T, \qquad \overline{U} = \left[U_2^T \quad 0_{n,4n} \quad U_1^T\right]^T,$$

$$W_{\bar{x}} = \left[I_n \quad 0_{n,2n}\right], \qquad W_{\bar{r}_1} = \left[A\overline{P} + B\overline{K} \quad I_n \quad -B\overline{K}\right],$$

$$W_{\bar{e}} = \left[0_{n,5n} \quad I_n\right], \qquad W_{\bar{r}_2} = \left[0_n \quad -\overline{L}C \quad -I_n \quad \overline{L}C \quad I_n \quad A - \overline{L}C\right],$$

$$\Upsilon_1 = \left[0_n \quad -\overline{L}C \quad -P_2 \quad \overline{L}C \quad P_2 \quad P_2 A - \overline{L}C\right],$$

$$\Upsilon_2 = \begin{bmatrix} 0 & 0 & 0 & P_2 E & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\kappa} R_2 E & 0 & 0 \end{bmatrix},$$

$$\Upsilon_3 = \left[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -\overline{L}^T\right],$$

$$W_{\bar{v}} = \left[\begin{array}{ccccc} 0_n & -I_n & 0_n & I_n & 0_{n,2n} \\ \hline -I_n & -0_{n,4n} & & & I_n \end{array}\right], \qquad W_y = \left[\begin{array}{ccc} 0_n & I_n & 0_n \\ \hline 0_{n,2n} & I_n & \end{array}\right],$$

$$W_{\bar{g}} = \left[\begin{array}{ccc} 0_{n,3n} & I_n & 0_{n,2n} \\ \hline 0_{n,5n} & I_n & \\ \hline 0_{n,4n} & I_n & 0_n \\ \hline 0_{n,2n} & I_n & 0_{n,3n} \end{array}\right], \qquad W_z = \left[\begin{array}{ccc} 0_{n,3n} & I_n & 0_{n,2n} \\ \hline 0_{n,4n} & I_n & 0_n \\ \hline 0_{n,5n} & I_n & \end{array}\right]. \tag{3.19}$$

Moreover, if the above conditions are satisfied, a desired controller gain and observer gain are given as follows:

$$K = \overline{K}\,\overline{P}_1^{-1}, \qquad L = \overline{P}_2^{-1}\overline{L}. \tag{3.20}$$

Proof. Define the following matrix:

$$W = \begin{bmatrix} 0_{n,3n} & I_n & 0_{n,2n} \\ \hline 0_n & I_n & 0_{n,4n} \\ \hline 0_{n,5n} & I_n & \\ \hline I_n & 0_{n,5n} & \\ \hline 0_{n,4n} & I_n & 0_n \\ \hline 0_{n,2n} & I_n & 0_{n,3n} \end{bmatrix}. \tag{3.21}$$

Perform a congruence transformation to (3.1) by $W_1 = \mathrm{diag}\{W, I, I, I, I, I\}$, which are to exchange the first row and the forth row with the third row and the sixth row, then exchange the first column and the forth column with the third column and the sixth column.

Then, by using Lemma 1 in [25] and Theorem 3.2, we have

$$\begin{bmatrix} \overline{\Xi}_1 & \overline{\Xi}_2 \\ * & \overline{\Xi}_3 \end{bmatrix} < 0, \tag{3.22}$$

$$\begin{bmatrix} \overline{\Phi}_1 & \overline{\Phi}_2 \\ * & \overline{\Phi}_3 \end{bmatrix} < 0, \tag{3.23}$$

where

$$\begin{gathered}
\overline{\Xi}_1 = \mathrm{sym}\left(W_{\overline{e}}^T P_2 W_{\overline{r}_2} + \overline{V} W_{\overline{v}} - W_{\hat{x}}^T\left(\varepsilon_2 G^T G\right) W_{\overline{e}}\right) + W_{\overline{g}}^T \Psi_1 W_{\overline{g}} + W_z^T Z W_z, \\
\overline{\Xi}_2 = \left[\overline{\Upsilon}_1^T \quad \sqrt{\kappa + 1}\,\overline{V} \quad \overline{\Upsilon}_2^T \quad \overline{\Upsilon}_3^T\right], \\
\overline{\Xi}_3 = \mathrm{diag}\left\{-R_2^{-1}, -R_1, -R_2, -P_2^{-1}, -R_2^{-1}, -\varepsilon_3 I\right\}, \\
\overline{\Upsilon}_1 = \sqrt{\kappa} W_{\overline{r}_2}, \qquad W_{\overline{r}_2} = \begin{bmatrix} 0_n & -LC & -I_n & LC & I_n & A - LC \end{bmatrix}, \\
\overline{\Upsilon}_2 = \begin{bmatrix} 0 & 0 & 0 & E & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\kappa} E & 0 & 0 \end{bmatrix}, \qquad \overline{\Upsilon}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -L^T P_2 \end{bmatrix}, \\
\overline{\Phi}_1 = \mathrm{sym}\left(W_{\overline{x}}^T P_1 W_{\overline{r}_1}\right) - W_{\overline{y}}^T Z W_{\overline{y}}, \qquad W_{\overline{r}_1} = \begin{bmatrix} A + BK & I_n & -BK \end{bmatrix}, \\
\overline{\Phi}_2 = \begin{bmatrix} \sqrt{\kappa}\left(A^T + K^T B^T\right) & E^T & \sqrt{\kappa} E^T \\ \sqrt{\kappa} I & 0 & 0 \\ -\sqrt{\kappa} K^T B^T & 0 & 0 \end{bmatrix}, \qquad W_{\overline{y}} = \begin{bmatrix} I_n & 0_{n,2n} & \\ \hline 0_n & I_n & 0_n \\ \hline 0_{n,2n} & I_n & \end{bmatrix}, \\
\overline{\Phi}_3 = \mathrm{diag}\left\{-R_1^{-1}, -P_1^{-1}, -R_1^{-1}\right\}.
\end{gathered} \tag{3.24}$$

Perform a congruence transformation to (3.22) by $J_2 = \mathrm{diag}\{I_{6n}, J_1\}$ with $J_1 = \mathrm{diag}\{P_2, I_{2n}, P_2, R_2, I_n\}$. Defining $\overline{L} = P_2 L$, we have (3.16). Performing a congruence transformation to (3.23) by $J_4 = \mathrm{diag}\{J_3, I_{3n}\}$ with $J_3 = \mathrm{diag}\{P_1^{-1}, I, P_1^{-1}\}$ and defining $\overline{P}_1 = P_1^{-1}$, $\overline{K} = KP_1^{-1}$, $Q_1 = Z_1^{-1}$, $Q_2 = Z_3^{-1}$, $Q_3 = R_1^{-1}$, $-P_1^{-1} Z_3 P_1^{-1} \le Z_3^{-1} - 2P_1^{-1}$ and $-P_2 R_2^{-1} P_2 \le R_2 - 2P_2$ we have (3.17). We can solve the inequalities (3.18) by using of the cone complementarity linearization (CCL) algorithm in [31]. The proof is completed. □

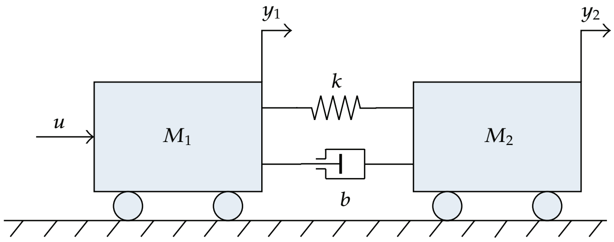

Figure 2: Mechanical system.

4. Illustrative Example

In this section, we use a mechanical example to illustrate the applicability of the theoretical results developed in this paper.

The controlled plant is a mechanical system consisted of two cars, a spring, and a damper, as shown in Figure 2. The objective is to design controllers such that the system will maintain the zero position ($y_1 = 0$ and $y_2 = 0$) when the disturbance disappears. M_1 and M_2 denote the two car mass, respectively; k is the elastic coefficient of the spring; b is the viscous damping coefficient of the damper; u denotes control input; y_1 and y_2 are the displacements of the two cars, respectively. The right is the positive direction of the force and the displacement. When $u = 0$, the balance positions are the zero place of the two cars y_1 and y_2.

Choose the following set of state variables:

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} = \begin{bmatrix} y_1 & y_2 & \dot{y}_1 & \dot{y}_2 \end{bmatrix}. \tag{4.1}$$

The equations of the mechanical system are in the following:

$$\begin{aligned} dx_1 &= x_3 dt, \\ dx_2 &= x_4 dt, \\ dx_3 &= \left(-\frac{k}{m_1}(x_1 - x_2) - \frac{b}{m_1}(x_3 - x_4) + u(t) + 0.001 \sin(0.5t) \right) dt \\ &\quad + 0.01 x_1 d\omega(t), \\ dx_4 &= \left(\frac{k}{m_2}(x_1 - x_2) + \frac{b}{m_2}(x_3 - x_4) + 0.001 \sin(0.2t) \right) dt. \end{aligned} \tag{4.2}$$

The parameters of the mechanical system are $m_1 = 1\,\text{kg}$, $m_2 = 2\,\text{kg}$, $k = 36\,\text{N/m}$, and $b = 0.06\,\text{Ns/m}$. Then the state-space matrices are given by

$$\begin{gathered} A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -36 & 36 & -0.6 & 0.6 \\ 18 & -18 & 0.3 & -0.3 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \\ E = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.01 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad G = \begin{bmatrix} 0.05 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0 \\ 0 & 0 & 0.05 & 0 \\ 0 & 0 & 0 & 0.05 \end{bmatrix}. \end{gathered} \tag{4.3}$$

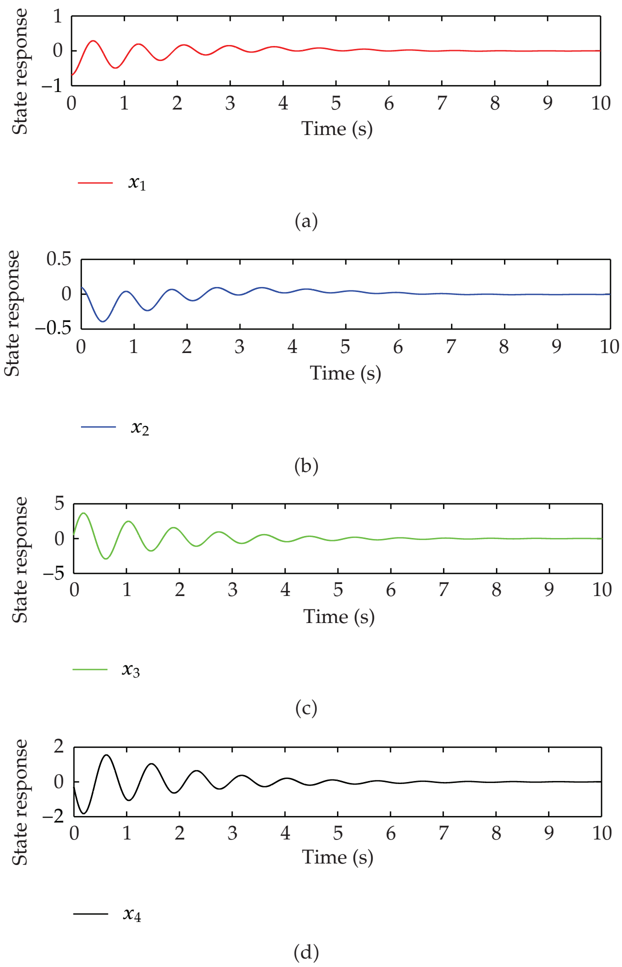

Figure 3: State responses of closed-loop system.

The eigenvalues of A are $-0.4500 \pm 7.3347i, 0, 0$, and thus this system is unstable. Our objective is to design an observer-based controller in the form of (2.2) such that the closed-loop system (2.1) is asymptotically stable in mean square. The network-related parameters are assumed: the sampling period $h = 2\,\text{ms}$, the maximum delay $\overline{\eta} = 4\,\text{ms}$, the maximum number of data packet dropouts $\overline{\delta} = 1$, the quantizer parameters $\rho = 0.9$, and $u_0 = 2$. By Theorem 3.2, we obtain the following matrices (other associated matrices are omitted here):

$$\overline{P}_1 = \begin{bmatrix} 0.5130 & 0.4367 & -0.1801 & -0.1547 \\ 0.4367 & 0.4903 & -0.1504 & -0.1654 \\ -0.1801 & -0.1504 & 3.4095 & -1.2803 \\ -0.1547 & -0.1654 & -1.2803 & 1.0595 \end{bmatrix}, \qquad \overline{K}^T = \begin{bmatrix} -0.4605 \\ -0.4650 \\ -1.6411 \\ 0.0173 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 2.7987 & -0.4600 & -0.7901 & -1.4199 \\ -0.4600 & 5.3953 & -1.2876 & -2.5287 \\ -0.7901 & -1.2876 & 0.7402 & 1.3478 \\ -1.4199 & -2.5287 & 1.3478 & 2.7867 \end{bmatrix}, \qquad \overline{L} = \begin{bmatrix} 9.2859 & -4.2200 \\ -6.3928 & 7.7879 \\ 0.5443 & 0.8766 \\ 1.2269 & 1.3828 \end{bmatrix}. \tag{4.4}$$

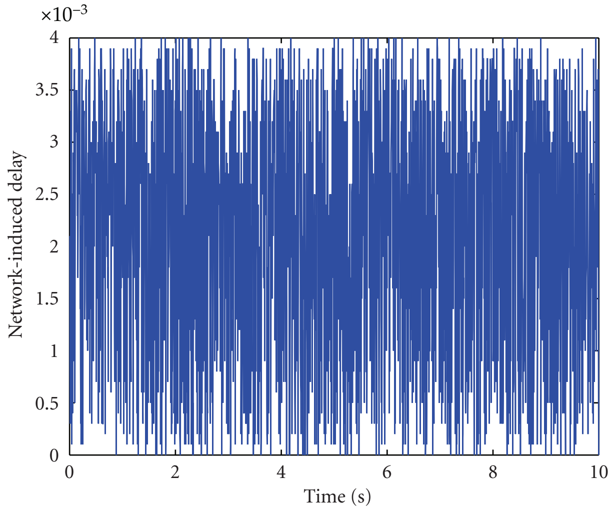

Figure 4: Network-induced delays.

According to (3.20), the gain matrices for the observer-based controller is given by:

$$K^T = \begin{bmatrix} -0.9531 \\ -1.1033 \\ -1.2654 \\ -1.8243 \end{bmatrix}, \qquad L = \begin{bmatrix} 7.7216 & 2.5661 \\ 2.8715 & 4.5438 \\ 10.5690 & 8.6714 \\ 1.8687 & 1.7328 \end{bmatrix}. \tag{4.5}$$

In the following, we provide simulation results. The initial condition is assumed to be $[-0.3, 0.7, 0.1, -0.5]$. The state responses are depicted in Figure 3, from which we can see that all the four state components of the closed-loop system converge to zero. In the simulation, the network-induced delays and the data packet dropouts are generated randomly (uniformly distributed within their ranges) according to the above assumptions, and shown in Figures 4 and 5. The output signals $y(t)$ and the successfully transmitted signal arriving at the ZOH $\hat{y}(t)$ (denotes as y_{ZOH} in figure) are shown in Figure 6, where we can see the discontinuous behavior of the transmitted measurements.

5. Conclusion

In this paper, the problem of observer-based stabilization of the stochastic nonlinear systems with limited communication has been studied. A new model has been proposed to describe the stochastic nonlinear systems with a communication channel, which exists between the output of the physical plant and the input of the dynamic controller. Based on this, the design procedure of observer-based controller has been proposed, which guarantees the asymptotic stability of the closed-loop systems. Finally, a mechanical system example is given to show the effectiveness of the proposed controller design method.

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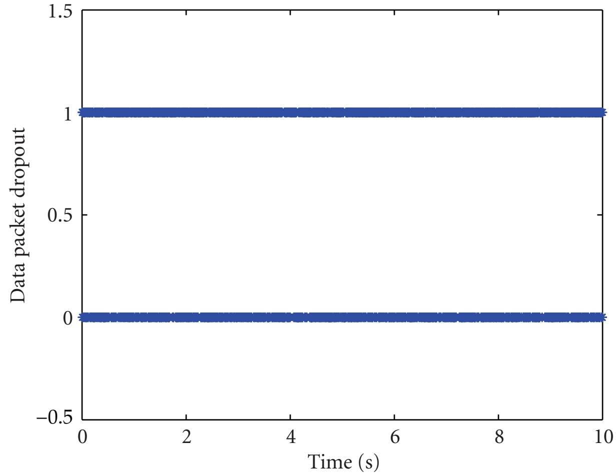

Figure 5: Data packet dropouts.

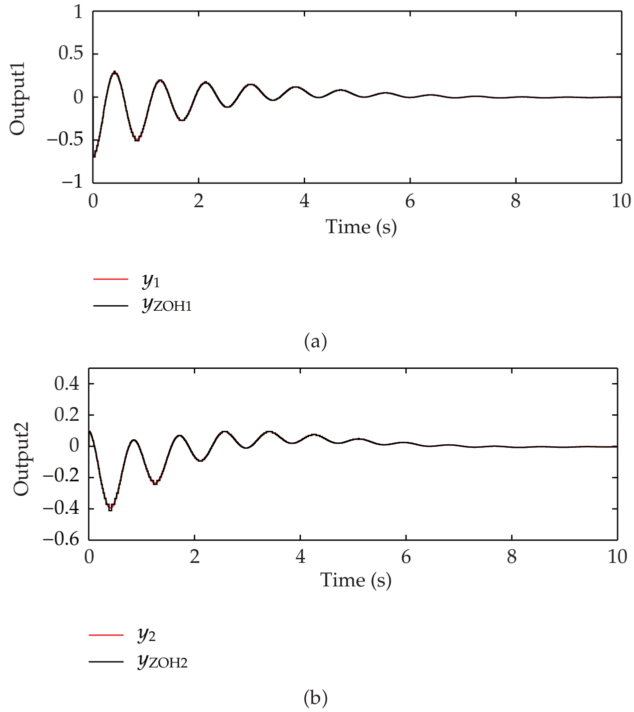

Figure 6: Measurements and transmitted signals.

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