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A decomposition and multistage optimization approach applied to the optimization of water distribution systems with multiple supply sources

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[1] The aim of this paper is to present a decomposition and multistage approach for optimizing the design of water distribution systems with multiple supply sources (WDS-MSS). An algorithm is first proposed to identify the optimal source partitioning cut-set for a WDS-MSS. A WDS with K supply sources is therefore decomposed to K disconnected subnetworks by the removal of the determined cut-set. Then, a total of K separate differential evolution (DE) algorithms are used to optimize the designs for the K subnetworks, respectively. This is the first optimization stage. The optimal solutions for the K subnetworks plus the optimal cut-set being the minimum allowable pipe sizes are used to create a tailored seeding table. This table is used to initialize a second-stage DE algorithm to optimize the whole of the original WDS, which is the second stage of the optimization process. Four WDS-MSS case studies are used to demonstrate the effectiveness of the proposed method. A standard DE algorithm seeded by the total choice table rather than the tailored seeding table is applied to the entire network for each case study, and the results are compared with those of the proposed method in terms of efficiency and solution quality. The comparison demonstrates that the proposed method (i.e., decomposition followed by multistage optimization) shows better performance than results from a whole of network optimization. In addition, the proposed method also exhibits significantly improved performance compared with the optimization techniques that have been previously used to optimize these case studies.

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1. Introduction

[2] Over the last four decades, significant research has been undertaken to develop techniques to optimize the design of water distribution systems (WDSs). Various optimization techniques including traditional optimization methods and evolutionary algorithms (EAs) have been applied to WDS optimization, and these are summarized in Table 1 (it should be noted that only the first significant paper for each optimization technique applied to WDS optimization is provided in Table 1). Traditional optimization techniques such as linear programming (LP) and nonlinear programming (NLP) often converge at local optimal solutions due to the nonsmoothness properties of the WDS optimization problem [*Eiger et al.,* 1994]. EAs given in Table 1 have been demonstrated to be able to find better quality solutions than traditional optimization methods based on testing on a number of WDS case studies. One major drawback with using EAs, however, is that they require a large number of network evaluations to find optimal solutions, resulting in an expensive computational overhead, especially for relatively large case studies. Thus, it is difficult for these EAs to find good quality optimal solutions for the real-world sized WDSs, as these systems are generally complex, with large numbers of decision variables.

[3] Much research has been done in an attempt to improve the efficiency of EAs applied to large WDS optimization problems [*Bolognesi et al.,* 2010]. Decomposing the original WDS using graph theory to facilitate the optimization process is one of these research lines.

2. Decomposition of WDSs

[4] Normally, decomposition of a water network is used to carry out an analysis of network connectivity, reliability, and management strategies. *Ostfeld* [2005] employed graph theory to undertake a connectivity analysis for WDSs. *Deuerlein* [2008] decomposed complex water networks into forests, blocks, and bridges using graph theory. Based on the decomposition algorithm proposed by *Deuerlein* [2008], the original whole network can be simplified to several parts that are able to improve the understanding of the interaction among different network components, thereby enabling a network vulnerability analysis and improved

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Table 1. Types of Previously Used Optimization Techniques Applied to WDS Optimization

Algorithm[a]	First Reference
Linear programming (LP)	*Alperovits and Shamir* [1977]
Nonlinear programming (NLP)	*Fujiwara and Khang* [1990]
Standard genetic algorithm (SGA)	*Simpson et al.* [1994]
Modified genetic algorithm (MGA)	*Dandy et al.* [1996]
Simulated annealing (SA)	*Loganathan et al.* [1995]
Tabu search (TS)	*Lippai et al.* [1999]
Harmony search (HS)	*Geem et al.* [2002]
Shuffled frog leaping algorithm (SFLA)	*Eusuff and Lansey* [2003]
Ant colony optimization (ACO)	*Maier et al.* [2003]
ANN metamodels	*Broad et al.* [2005]
Particle swarm optimization (PSO)	*Suribabu and Neelakantan* [2006]
Scatter search (SS)	*Lin et al.* [2007]
Cross-entropy algorithm (CE)	*Perelman and Ostfeld* [2007]
Hybrid discrete dynamically dimensioned search (HD-DDS) algorithm	*Tolson et al.* [2009]
Differential evolution (DE)	*Suribabu* [2010]
Honey-Bee Mating Optimization (HB)	*Mohan and Babu* [2010]
Genetic Heritage Evolution by Stochastic Transmission (GHEST)	*Bolognesi et al.* [2010]

[a]Only the first significant paper for each optimization technique applied to WDS optimization is provided.

management of the network. *Yazdani and Jeffrey* [2010] used graph theory and complex network principles to conduct a robustness analysis for WDSs; *Di Nardo and Di Natle* [2010] proposed a design support method for district metering of WDSs using graph decomposition.

[5] Few attempts have been made to utilize graph decomposition to facilitate WDS design optimization. *Krapivka and Ostfeld* [2009] proposed a network decomposition based genetic algorithm (GA-LP) scheme for the least cost pipe sizing of WDSs. In their work, the looped water network was first decomposed into a number of spanning trees and chords. Then, an LP was utilized to optimize each spanning tree, allowing the identification of the least cost spanning tree. Finally, a GA was used to alter the flows for the least cost spanning tree (referred to the "outer" problem), and the LP was employed to optimize the tree network with the updated flows (the "inner" problem).

[6] *Cisty* [2010] proposed another network decomposition-based GA-LP model for solving WDS design problems. In this proposed GA-LP method, a GA was used to generate various trees for a complex looped network, and LP was used to optimize each tree network. *Haghighi et al.* [2011] developed a hybrid model incorporating a GA and integer linear programming (GA-ILP) to optimize the design of WDSs. As for the GA-LP method proposed by *Cisty* [2010], the GA in the GA-ILP model proposed by *Haghighi et al.* [2011] randomly generated tree networks for the original looped WDS and the ILP was utilized to optimize each tree network.

[7] *Zheng et al.* [2011a] proposed a combined NLP-differential evolution (NLP-DE) method for optimizing WDS design. In the proposed NLP-DE approach, the original WDS was decomposed into a shortest-distance tree and chords. Then, an NLP was employed to arrive at an approximate optimal solution for the decomposed WDS. The approximate optimal solution obtained from the NLP was then used to seed a DE to generate improved quality solutions for the original full WDS.

3. Proposed Decomposition and Multistage Optimization Method

[8] The above analysis indicates that graph theory is normally used to find various trees for the looped WDSs in previously proposed decomposition-based optimization methods. This is motivated by the fact that optimal solutions for trees can be obtained by deterministic optimization methods such as LP, NLP, or ILP with great efficiency. In contrast, in this paper, a novel decomposition method is proposed to alternatively decompose the original complex WDS into subnetworks rather than into trees to facilitate network design optimization.

[9] For a real-world WDS, multiple sources of supply (i.e., multiple tanks) are normally incorporated into the system in addition to having loops to improve the reliability of supply. For such a complex WDS with multiple supply sources (WDS-MSS), existing optimization algorithms normally tackle the system as a whole to find optimal design solutions. Normally, design of a large-scale water network with multiple sources is computationally very rigorous. This is due to the size of the search space as well as the time for hydraulic simulation of the network. The method proposed here has (i) developed a graph decomposition method to partition the larger optimization problems into smaller ones that in turn reduces the computational overhead for optimizing the design of the WDS-MSS and (ii) developed a multistage DE method to optimize the design of the subnetworks obtained by decomposing the WDS-MSS and then the original whole network. The outcome is a significantly more efficient and effective method for the optimization of the design of water networks with multiple sources.

[10] In the proposed decomposition and multistage optimization method, an algorithm is developed to identify the optimal source partitioning cut-set for a WDS with K supply sources. By removing the optimal source partitioning cut-set, the whole original WDS is decomposed to K subnetworks. For each subnetwork, one and only one supply source is assigned. Each subnetwork is then optimized by a DE algorithm independently, which is the first stage of optimization.

[11] The optimal solutions for all subnetworks are then combined to provide an approximate optimal solution for the whole original network. However, this approximate optimal solution needs to be further improved because the pipes within the optimal source partitioning cut-set were not included during the first stage of the subnetwork optimization. A second-phase DE is therefore used to explore the search space around the obtained approximate optimal solution, and better quality solutions for the whole WDS are expected to be found with significantly reduced computational effort. This is the second stage of the optimization process.

[12] The concept of multistage optimization is based on the decomposition of large-scale and complex systems into independent subsystems (although these subsystems are actually interconnected and are not truly independent of one another). Each subsystem is optimized independently,

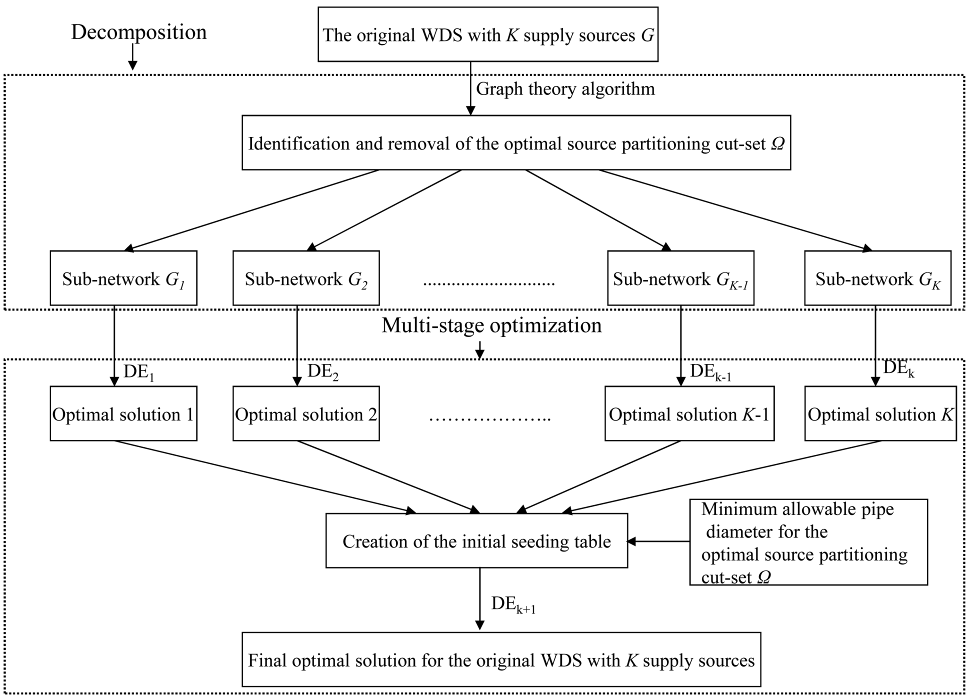

Figure 1. Flowchart of the proposed optimization approach.

and the optimal solutions for each subsystem are then combined together to derive the optimal solution for the whole system. Although a multistage optimization approach has been used to control the pollution of water resource systems [*Hass*, 1970, *Haimes*, 1971], optimize urban water management [*Zhu et al.*, 2005], and deal with the reservoir operation problem [*Canon et al.,* 2009], the method proposed here is the first time that multistage optimization has been used to optimize the design of a WDS.

[13] Although the DE algorithm is used in this study, other EAs such as a GA could also be implemented in the proposed optimization framework. However, the performance comparison of the DE algorithm with other optimization algorithms has not been carried out in this study. The methodology of the proposed decomposition and multistage method are given later.

4. Formulation of the WDS-MSS Optimization Problem

[14] Typically, single-objective optimization of a WDS is the minimization of system total life cycle costs (pipes, tanks, and other components) while satisfying head constraints at each node. In this paper, the proposed decomposition and multistage optimization method is verified using WDS-MSS case studies with pipes only for a single demand load case. Thus, the formulation of the WDS-MSS optimization problem can be given by

$$\text{Minimize} \qquad F = a\sum_{i=1}^{np} D_i^b L_i. \tag{1}$$

Subject to:

$$H_k^{\min} \le H_k \le H_k^{\max} \quad k = 1, 2, \ldots, nj, \tag{2}$$

$$G(H_k, D) = 0, \tag{3}$$

$$D_i \in \{A\}, \tag{4}$$

where F = network cost that is to be minimized [*Simpson et al.,* 1994]; D_i = diameter of the pipe i; L_i = length of the pipe i; a, b = specified coefficients for the cost function; np = total number of pipes in the network; nj = total number of nodes in the network; $G(H_k, D)$ = nodal mass balance and loop (path) energy balance equations for the whole network, which is solved by a hydraulic simulation package (EPANET2.0 in this study); H_k = head at the node $k = 1, 2, \ldots, nj$; $H_k^{\min}$ and $H_k^{\max}$ are the minimum and maximum allowable head limits at the nodes, respectively; and A = a set of commercially available pipe diameters.

5. Methodology of the Proposed Method

[15] The flowchart in Figure 1 outlines the features of each step of the proposed decomposition and multistage optimization approach.

5.1. Decomposition of the WDS-MSS

5.1.1. Source Partitioning Cut-Set of the WDS-MSS

[16] In a connected graph $G(V, E)$, a cut-set is a set of edges whose removal from G results in G being disconnected [*Deo*, 1974], where V is a set of vertices and E is a set of edges. In this paper, a source partitioning cut-set (C) for a WDS-MSS is a set of pipes whose removal from the system results in the WDS-MSS being separated in such a way that each subnetwork is attached to one and only one unique supply source. That is, the original WDS with K supply sources is decomposed into K disconnected subnetworks after removal of the source partitioning cut-set. For a WDS-MSS with two supply sources (reservoirs)

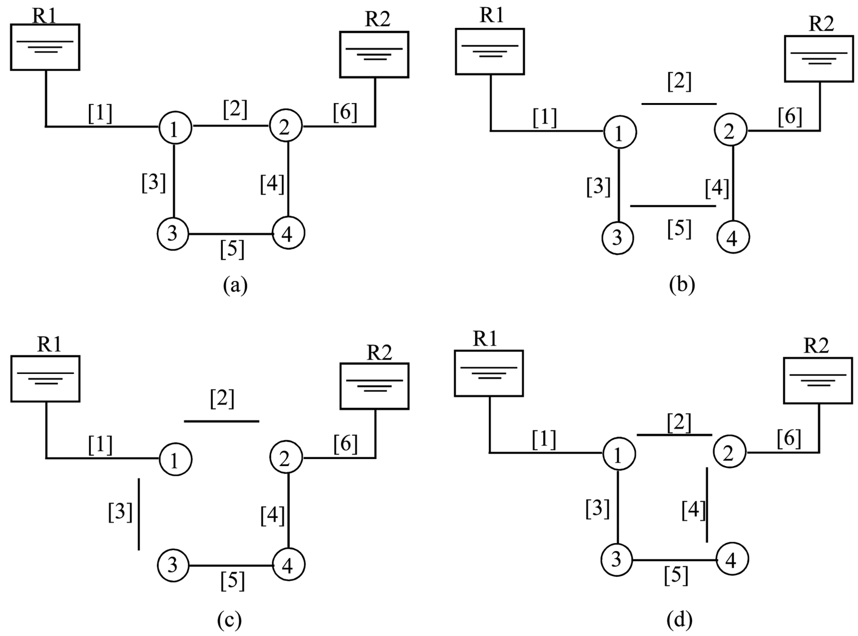

Figure 2. An example of cut-sets and the subnetworks for a two reservoir WDS: (a) the two-reservoir water network; (b) source partitioning cut-set (pipes 2 and 5) and subnetworks; (c) optimal source partitioning cut-set (pipes 2 and 3) and subnetworks; and (d) source partitioning cut-set (pipes 2 and 4) and subnetworks.

(see Figure 2a), all source partitioning cut-sets (C) and their corresponding two subnetworks after removal of the cut-set are given in Figures 2b–2d).

[17] As shown in Figure 2a, the original WDS $G(V, E)$, where $V = \{R1, R2, 1, 2, 3, 4\}$ and $E = \{1, 2, 3, 4, 5, 6\}$, has two reservoirs ($R1$ and $R2$), six links and four nodes. An arbitrarily selected source partitioning cut-set $C = \{2, 5\}$ is shown in Figure 2b. The original two-reservoir WDS is decomposed to two subnetworks $G_1(V_1, E_1)$, $G_2(V_2, E_2)$ after removal of the cut-set $C = \{2, 5\}$, where $V_1 = \{R1, 1, 3\}$, $E_1 = \{1, 3\}$, $V_2 = \{R2, 2, 4\}$, $E_2 = \{4, 6\}$. It can be observed that a total of three cut-sets exist in this two-reservoir WDS, which enable the network disconnection. In the proposed decomposition and multistage optimization method, an optimal source partitioning cut-set Ω is proposed to decompose the WDS-MSS. The definition of the Ω and the algorithm that has been developed in this study to identify the Ω for a WDS-MSS are outlined in the next section.

5.1.2. Identification of the Optimal Source Partitioning Cut-Set Ω of the WDS-MSS

[18] For a WDS-MSS with K supply sources, each node i in the water network has K different potential water supply sources and a number of potential supply paths from each supply source. For a given supply source k and the demand i, there exists a finite set of independent paths joining these two nodes, symbolized here as P_{ki}. For each supply path $\lambda \in P_{ki}$, the available friction slope is calculated as

$$S_{ki}(\lambda) = \frac{H_k - H_i^{\min}}{\sum_{l \in \lambda} L_l}, \qquad (5)$$

where $S_{ki}(\lambda)$ is the available friction slope from source k to node i based on the supply path $\lambda \in P_{ki}$, H_k is the head of the source k, and $H_i^{\min}$ is the minimum allowable head requirement at node i; L_l is the length of link l ($l \in \lambda$). Among the different paths $\lambda \in P_{ki}$, the path that has the *largest* available friction slope (λ_{ki}^*) is considered to be the most economic supply path for this node i from source k [*Zheng et al.*, 2011a], which is given as

$$\lambda_{ki}^* = \arg\max_{\lambda \in P_{ki}} S_{ki}(\lambda). \qquad (6)$$

[19] Then, for a given node i, the available friction slope for the economic paths from each source can be constructed to form the set $\xi_i = \{S_{1i}(\lambda_{1i}^*), \quad S_{2i}(\lambda_{2i}^*), \ldots, S_{Ki}(\lambda_{Ki}^*)\}$. Given this, the source k with the greatest available friction slope λ_{ki}^* for node i is taken to be the supply source for node i. This is based on heuristic reasoning that it is economical overall for a demand node to receive flows from a supply source having a relatively high available head and/or a relatively short distance to this demand node. As such, each node i is assumed to receive flows from one and only one supply source in the proposed method according to the heuristic approximation.

[20] It is noted that the largest available friction slope (λ_{ki}^*) is determined by the distance and the allowable heads but without the inclusion of consideration of the nodal demands. This is because that it is impossible to consider the flows in each path to determine the most effective path for each node to receive demands from sources as the flows in the network are unknown before the determination of the network configuration (pipe diameters in the network). However, it is acknowledged that the demands at nodes may influence the final decomposition results and hence the path determined by the largest available friction slope is an

Table 2. Network Data of the WDS With Two Reservoirs

Nodes	Elevation (m)	Pressure Head Requirement (m)	Water Demands (L/s)	Links	Length (m)
R1	54			5	550
R2	56			6	400
1	27	20	50	1	800
2	29	20	60	2	800
3	31	20	75	3	650
4	33	20	90	4	700

approximation to the truly optimal path for delivery of flows.

[21] By assigning demand nodes to different supply source nodes, a demand node set N_k can be constructed for each supply source node k, which consists of all nodes for which k is the supply source. Then links that have different supply sources for two nodes on each side are obtained, which is defined as:

$$\Omega = \{(i,j) : i \in N_k, j \in N_m, k \neq m, k, m = 1, \ldots, K\}, \quad (7)$$

where (i, j) is the link having node i and j on each side. This set of links is defined as the optimal source partitioning cut-set Ω for the WDS-MSS and the removal of the optimal cut-set leaves the original WDS-MSS decomposed into several subnetworks. Each subnetwork is composed of one and only one supply source and a particular number of nodes and pipes. Each supply source only provides water to specific nodes established when the optimal source partitioning cut-set is removed. Thus, the optimal source partitioning cut-set is actually the estimated optimal supply boundary of different supply sources in a WDS.

[22] The two-reservoir WDS presented in Figure 2a is used to explain the proposed Ω to decompose the network. The network data are given in Table 2. Each supply path for each node (λ) and the available friction slope for each path ($S(\lambda)$) are provided in Table 3. The path having the largest available friction slope has been highlighted for each node in Table 3. $N_{R1} = \{1\}$ as $\lambda_{\text{R1-1}}$ is the most economical path that has the largest available friction slope for node 1. $N_{R2} = \{2, 3, 4\}$ as these nodes have the largest available friction slopes from $R2$. Thus, the optimal source partitioning cut set is given as $\Omega = \{2, 3\}$ as the nodes on each side of these two links are assigned to different reservoirs. The optimal partitioning cut-set Ω and the subnetworks after removal of the Ω are given in Figure 2c.

[23] For a relatively small WDS-MSS (the water network in Figure 2a), the Ω can be determined using complete enumeration. However, it is impossible to enumerate all the paths for a relatively large WDS-MSS. An algorithm that is used to efficiently identify the optimal source partitioning cut-set Ω for a large WDS-MSS has been developed in this research. The proposed approach is motivated by the fact that the shortest-distance path P^*_{ki} of all the available paths from the same supply source to a particular node always has the largest available friction slope λ^*_{ki}. This is reflected in equation (5), which shows that the available head for a particular node to a particular supply source is constant. Therefore, the shortest path between a node and a particular supply source has the largest available friction slope, i.e., $P^*_{ki} = \lambda^*_{ki}$. The Dijkstra algorithm [*Deo*, 1974] is employed in this study to find the shortest-distance path for each node to different supply sources. The details of Dijkstra algorithm [*Deo*, 1974] are given as follows.

[24] In the Dijkstra algorithm, either a permanent label or temporary label is assigned to each node. A permanent label is given to a node once the shortest path from this node to the source node has been determined. The value of the permanent label is made equal to the sum of lengths of the shortest path. In contrast, a temporary label is given to a node for which the shortest path has not yet been identified. The value of this temporary label is set to be equal to the sum of lengths of the shortest path in the current iteration, and this value will be updated in later iterations.

[25] The Dijkstra algorithm begins by assigning a permanent label 0 to the starting node (supply source node) and a temporary label ∞ (this is replaced by a large number in the computer algorithm) to the remaining nodes (demand nodes in a WDS-MSS). In the search procedure, at each iteration, another node gets a permanent label according to the following rules [*Deo*, 1974]:

[26] **Rule 1.** Every node j that has not yet permanently been labeled is updated with a new temporary label whose value is given by min [old label j, old label $i + d_{ij}$], where i is the latest node permanently labeled in the previous iteration. dij is the direct length from node i to node j. If nodes i and j are not directly connected, then $d_{ij} = \infty$.

[27] **Rule 2.** At each iteration, the smallest value amongst all temporary labels is found and the corresponding node is permanently labeled with this value. Thus, a new permanently labeled node is produced in this iteration. If more than one temporary label has the same value, then any one of the candidates for permanent labeling is selected.

[28] Rules 1 and 2 are repeated until all the nodes are permanently labeled. An example illustration of the Dijkstra algorithm performed for the source node $R1$ to other demand nodes in the looped water network of Figure 2a is given in Table 4. The shortest-distance path for source node $R1$ to other demand nodes is presented in the last column of Table 4.

Table 3. Supplying Paths and the Available Friction Slope for Each Node

Nodes (i)	Pipes in Path (λ)[a]	Length (m)	Available Head ($H_k - H_i^{\min}$) (m)	Available Friction Slope ($S(\lambda)$)
1	***R1-1***	***800***	***7***	***0.0088***
	R2-6-2	1200	9	0.0075
	R2-6-4-5-3	2300	9	0.0039
2	R1-1-2	1600	5	0.0031
	R1-1-3-5-4	2700	5	0.0019
	R2-6	***400***	***7***	***0.0175***
3	R1-1-3	1450	3	0.0021
	R2-6-2-3	1850	5	0.0027
	R2-6-4-5	***1650***	***5***	***0.0030***
4	R1-1-2-4	2000	1	0.0005
	R1-1-3-5	2300	1	0.0004
	R2-6-4	***1100***	***3***	***0.0027***

[a]The paths in bold have the largest available frictions slope for each demand node.

Table 4. The Dijkstra Algorithm for Identifying the Shortest-Distance Tree

	Length to Node[a]						
Iteration	R1	1	2	3	4	Description	Shortest Path P^*_{ki}
1	**0**	∞	∞	∞	∞	Starting at the source node *R1*. It is labeled 0 and all the other nodes are labeled ∞.	*R1*-R1
2	0	**800**	∞	∞	∞	All successors of *R1* are labeled using Rule 1. The smallest label (node 1) is permanently labeled (Rule 2).	1-*R1*
3	0	800	1600	**1450**	∞	All successors of 1 are labeled using Rule 1. The smallest label (node 3) is permanently labeled (Rule 2).	3-1-*R1*
4	0	800	**1600**	1450	2000	All successors of 3 are labeled using Rule 1. The smallest label (node 2) is permanently labeled (Rule 2).	2-1-*R1*
5	0	800	1600	1450	**2000**	All successors of 2 are labeled using Rule 1. The smallest label (node 4) is permanently labeled (Rule 2).	4-3-1-*R1*

[a]The bold values are the succession of assignment of permanent labels. ∞ would be designated as a large number in a computer implementation.

[29] The details of the proposed algorithm to identify the optimal source partitioning cut-set Ω for a WDS with K supply sources are given in Figure 3. As can be seen from Figure 3, three steps are involved in this proposed algorithm to identify the optimal source partitioning cut-set Ω. In step 1, the Djikstra algorithm is performed to identify the shortest-distance path $P^*_{ki} = \lambda^*_{ki}$ for each supply source node k to each node i within the WDS. Then, the available friction slope for the shortest distance path $S_{ki}\left(\lambda^*_{ki}\right)$ is computed using equation (5). As such, a total of K different $S_{ki}\left(\lambda^*_{ki}\right)$ values are obtained for each node i. In step 2, node $i = 1, \ldots, n$ is assigned to the set N_k if $S_{ki}\left(\lambda^*_{ki}\right)$ is the largest value from the K total available friction slope values, indicating that k is the supply source node for node i. In step 3, all the links (i, j) that have the nodes on each side assigned to different supply source nodes are identified and form the optimal source partitioning cut-set Ω.

[30] It is observed from Figure 3 that the Djikstra algorithm is performed K times to determine the optimal source partitioning cut-set for a WDS with K supply sources. The computational time required to identify the optimal source partitioning cut-set for each WDS-MSS case study is analyzed in later discussion. The subnetworks are obtained after removal of the optimal source partitioning cut-set. These subnetworks are independent and can be optimized separately.

```
For a WDS with graph G(V, E), with K supply sources, n nodes and nl links.
Step 1:
  FOR k=1,…..K
    Perform the Dijkstra algorithm for the supply source k to identify the shortest-
    distance path P*_ki = λ*_ki to each node i =1,…..,n as illustrated in Table 4.
    Compute the S_ki(λ*_ki) for each node i using Equation (5).
  END FOR
Step 2:
  FOR i=1,…..n
    Select k such that S_ki(λ*_ki) > S_ji(λ*_ji) for all j=1, …, K, j≠k using Equation (6)
    Node i is assigned to set N_k for which k is the supply source for node i.
  END FOR
Step 3:
  FOR all (i, j) ∈ E
    IF i ∈ N_k, j ∈ N_m, k ≠ m, k, m = 1,⋯⋯, K , i.e., the link with nodes at either end
    assigned to different sources.
      Link (i, j) is assigned to the optimal source partitioning cut-set Ω (Equation (7))
    END IF
  END FOR
```

Figure 3. Optimal source partitioning cut-set identification algorithm.

5.1.3. Summary of the Proposed Decomposed Method for WDS-MSS

[31] The proposed decomposition method partitions the whole water distribution system with K supply sources into K subnetworks. This differs significantly to the majority of the previously used decomposition approaches. These previous approaches identified a tree network as an approximation for the original full network [*Krapivka and Ostfeld*, 2009; *Kadu et al.*, 2008; *Zheng et al.*, 2011a]. In the proposed decomposition method, the shortest-distance path only is used to assign the nodes to different supply sources, and each node may receive flows via various paths from the assigned supply source (not only the shortest-distance path). This is due to the fact that loops are retained within each subnetwork obtained by the proposed decomposition method. However, in *Krapivka and Ostfeld* [2009], *Kadu et al.* [2008], and *Zheng et al.* [2011a], each node has one and only one path to receive flows to meet the demands from the source node.

[32] The available friction slope for each node is used in the proposed decomposition method to determine the optimal source partitioning cut-set Ω for a WDS-MSS, and the magnitude of the demands at each node is not considered during the decomposition. It is assumed to be cost effective overall for a demand node to receive the flows to meet the demands from a source having a relatively large available head and/or the shortest distance to this node. Thus, an approximate supply boundary is produced using the proposed decomposition method since each demand node receives the flows from one and only one supply source. However, it should be acknowledged that the supply boundary obtained by the proposed decomposition is an approximation to that of the real supply system as some nodes (especially nodes at the supply boundary) in the real WDS may receive the flows to supply demands from multiple supply sources.

[33] The available friction slope concept has also been used by *Kadu et al.* [2008] to identify a tree for a looped WDS. Thus, it is necessary to clarify the differences between the method used by *Kadu et al.* [2008] and the approach proposed here in terms of decomposing the WDS. The proposed decomposition method aims to specify a particular supply source for each demand node, for which this supply source has the *largest* available friction slope to this

demand node, while *Kadu et al*. [2008] used the *smallest* available friction slope to identify the critical path for the original WDS. In addition, disconnected subnetworks are obtained using the proposed decomposition method, within which loops are involved, while a tree network is finally obtained using the method proposed by *Kadu et al*. [2008].

[34] It is also useful to highlight the difference between the proposed decomposition method and the network aggregation method proposed by *Perelman and Ostfeld* [2007]. The main differences include: (i) in the new method presented here, the whole network is decomposed into several disconnected subnetworks, while the aggregation method keeps the general topology of the original system and only removes some nodes and links from the original system; (ii) in the proposed decomposed method, the decomposition results for a WDS are based on the number of different supply sources, while the aggregation result is dependent on the connectivity properties of the original system (such as the location of the monitor stations); and (iii) the demand distribution and link properties (such as link length and conductance) are not varied in the proposed decomposition approach, while they are changed in the aggregation network of *Perelman and Ostfeld* [2007] to resemble the hydraulics and water quality performance of the original system.

5.2. Multistage Optimization for the WDS-MSS

5.2.1. DE Algorithm Applied to Each Subnetwork (First-Stage Optimization)

[35] The DE algorithm, introduced by *Storn and Price* [1995], has performed well when used to find optimal solutions in a number of numerical optimization case studies [*Vesterstrom and Thomsen*, 2004]. *Vasan and Simonovic* [2010] and *Suribabu* [2010] first applied DE to the optimization of WDSs and concluded that the performance of the algorithms was at least as good as, if not better, than other EAs such as GAs and ant colony optimization. More recently, *Zheng et al*. [2011a, 2011b] further investigated the performance of DE and reported that DE was effective in finding optimal solutions for WDS. A total of three operators including mutation, crossover, and selection operators are involved in the application of DE in an optimization problem. Three parameters need to be prespecified: the population size (N), mutation weighting factor (F), and the crossover rate (CR). The general ranges of these three parameters are $1D \leq N \leq 10D$ (where D is the number of decision variables) $0.1 \leq F \leq 1.0$ and $0.1 \leq \mathrm{CR} \leq 1.0$ [*Storn and Price*, 1995].

[36] The basic DE algorithm is a continuous global optimization search algorithm [*Storn and Price*, 1995] and requires modification when used to solve discrete WDS optimization problems. In this study, the modification made to the DE algorithm was based on the approach used in *Suribabu* [2010]. To handle the head constraints, constraint tournament selection [*Deb*, 2000] was used in the DE algorithm. The pseudocode for the DE algorithm applied to WDS optimization is given in Figure 4. Assume the WDS to be optimized has D decision variables (pipes), and a total of TD available pipe diameters can be used for each decision variable.

[37] During the first stage of the optimization process, each subnetwork is optimized by a separate DE. In this paper, only the pipes in the subnetwork are considered for each separate subnetwork optimization. The subnetwork optimization problem formulation is similar to that for the original whole network (equations (1)–(4)). Because the dimensionality of each subnetwork is significantly reduced compared with the original network, the DE algorithm is expected to be able to more efficiently find optimal solutions for each subnetwork than for the whole network.

[38] For the water network given in Figure 2, 14 pipe diameters including $\{150, 200, 250, 300, 350, 400, 450, 500, 600, 700, 750, 800, 900, 1000\}$ mm can be selected for each pipe, and all the pipes are assigned to have an identical Hazen-Williams coefficient (HW) of 130. The unit costs for each pipe diameter are given by *Kadu et al*. [2008]. Two separate DEs were employed to optimize the two subnetworks ($S_1 = \{R1, 1, [1]\}$, $S_2 = \{R2, 2, 3, 4, [4], [5], [6]\}$) as shown in Figure 2c obtained by removing the optimal source partitioning cut-set $\Omega = \{2, 3\}$. The DE optimal solutions for S_1 and S_2 were \$37,910 and \$166,896, respectively, and the pipe diameters for the optimal solutions are [1] = 250 mm, [4] = 450 mm, [5] = 300 mm, and [6] = 500 mm. It is noted that the optimal cut-set $\Omega = \{2, 3\}$ was not included in the first stage of the proposed multistage optimization method.

5.2.2. Creation of the Seeding Table

[39] In the proposed method, the optimal solutions for K subnetworks are obtained after the first-stage optimization, and an optimal pipe diameter is assigned for each link in all subnetworks. As the optimal source partitioning cut-set Ω of the original complete network is not included during the first-stage optimization, the minimum allowable pipe diameters are therefore assigned to all the links in the Ω in this study. Each link of the complete network is given a pipe diameter by combining the optimal solutions of the subnetworks and assigning the minimum allowable pipe diameters for the Ω. This, therefore, creates an approximate optimal solution (or a near optimal in a topological sense) for the complete network. For the example given in Figure 2, the approximate optimal solutions were \$240,374 and the corresponding network configuration is [1] = 250 mm, [2] = 150 mm, [3] = 150 mm, [4] = 450 mm, [5] = 300 mm, and [6] = 500 mm (note 150 mm is the minimum allowable pipe diameter).

[40] The approximate optimal solution is now used to create a tailored seeding table to enable the second stage of optimization. For each link in this seeding table, three pipe diameters are included, namely (i) the pipe diameter from the approximate optimal solution of the whole network, (ii) and the pipe diameters that are immediately smaller, and (iii) the pipe diameters that are immediately larger than the diameter provided by the approximate optimal solution. For a pipe that is already the minimum or maximum allowable diameters, the three adjacent smallest or largest pipe diameters are assigned to the seeding table for this pipe.

[41] Table 5 is used to illustrate the process of the creation of the seeding table based on the approximate optimal solution of the water network given in Figure 2. The pipe diameters of the approximate optimal solution obtained after the first-stage optimization are given in column 2 of Table 5. As shown in Table 5, for links 1, 4, 5, and 6, three adjacent pipe diameters are included in the seeding table, and the middle one is the pipe diameter for the approximate

```
Step 0. Specify the following inputs of the differential evolution (DE): the population size (N), the
crossover rate (CR), the mutation weighting factor (F), and the maximum allowable number of generations
(MG).
Step 1: Randomly generate N initial solutions X_{i,G} ={ x^1_{i,G}, x^2_{i,G},……… x^D_{i,G} }, i=1,…..,N, G=0.
Step 2: Evaluate the objective function of the N initial solutions f(X_{i,G}).

Count=1

REPEAT

UNTIL Count ≥ MG
Step 3: Perform the DE mutation operator to generate N mutant solutions V_{i,G} ={ v^1_{i,G}, v^2_{i,G},……… v^D_{i,G} }
  FOR i=1,…..,N
     V_{i,G} = X_{r1,G} + F(X_{r2,G} − X_{r3,G}), where r1 ≠ r2 ≠ r3 and they are randomly generated for each i.
  END FOR
Step 4: Perform the DE crossover operator to generate trial solutions U_{i,G} ={ u^1_{i,G}, u^2_{i,G},……… u^D_{i,G} }
  FOR i=1,…..,N
     FOR j=1,…..,D
        IF Rand_j(0,1) ≤ CR, Rand_j(0,1) is a uniformly distributed random number between 0 and 1.
           u^j_{i,G} = v^j_{i,G}
        ELSE
           u^j_{i,G} = x^j_{i,G}
        END IF
     END FOR
  END FOR
Step 5: Alter the continuous pipe diameter solution to the nearest discrete diameter for each decision
        variable and then evaluate N trial solutions f(U_{i,G}).
Step 6: Select the next generation X_{i,G+1} ={ x^1_{i,G+1}, x^2_{i,G+1},……… x^D_{i,G+1} } i=1,….,N
  FOR i=1,…..,N
     IF f(X_{i,G}) ≤ f(U_{i,G})
           X_{i,G+1} = X_{i,G}
     ELSE
           X_{i,G+1} = U_{i,G}
     END IF
  END FOR

Count=Count+1
```

Figure 4. Pseudocode for the DE algorithm.

optimal solution (column 2 of Table 5). For links 2 and 3, three adjacent smallest pipe diameters are assigned to the seeding table as the diameter of links 2 and 3 given in column 2 of Table 5 are already the minimum allowable diameter (150 mm). This proposed method for the creation of the seeding table is applied to each case study in this paper.

5.2.3. Final Optimal Solution for the Original WDS-MSS (Second-Stage Optimization)

[42] In the proposed decomposition and multistage optimization method, another DE algorithm (denoted the final DE algorithm) is used in the second stage of optimization to find the optimal solutions for the original WDS with multiple supply sources. It is noted that the first-stage optimization does not include the pipes in the optimal source partitioning cut-set Ω. In the proposed approach, an approximate optimal solution was generated by combining the subnetwork optimal solutions and setting the pipes in the Ω to be the minimum allowable pipe diameters. However, this approximate optimal solution is not acceptable for the original whole network. This is because (i) the network reliability will be reduced by simply assigning the pipes in the Ω to be the minimum allowable diameter size as these pipes are the connections between subnetworks; and (ii) the approximate optimal solution produced in the first-stage optimization may be infeasible for the original whole network with the inclusion of the minimum diameter pipes in the Ω. Thus, the approximate optimal solution obtained in the first-stage optimization need to be further polished. This is achieved by applying the DE at the second-stage optimization of the proposed method.

[43] During the second-stage optimization phase (the formulation is given by equations (1) to (4)), the final DE algorithm is seeded by a tailored seeding table (column 4 of Table 5) rather than the total choice table (14 pipe diameter options). Thus, the initial solutions of the final DE algorithm are randomly located in the search space specified by the tailored seeding table rather than the whole search space. The final DE algorithm therefore focuses on exploring promising regions specified by the tailored seeding table and hence avoids wasting computational effort

Table 5. Process for Creating the Seeding Table (Applies to Any-Sized Network)

Links	Diameters for the Approximate Optimal Solutions (mm)	Link Membership	Pipe Diameters in the Seeding Table (mm)
1	250	Belongs to S1	200, 250, 300
2	150	Cut-set	150, 200, 250
3	150	Cut-set	150, 200, 250
4	450	Belongs to S2	400, 450, 500
5	300	Belongs to S2	250, 300, 350
6	500	Belongs to S2	450, 500, 600

Total pipe diameters choice table = {150, 200, 250, 300, 350, 400, 450, 500, 600, 700, 750, 800, 900, 1000} mm.

investigating infeasible or unnecessarily high cost regions within the search space. It is expected therefore that the final DE algorithm is able to locate better quality solutions for the original WDS-MSS with great efficiency and reliability as it has been seeded with good initial estimates [*Grefenstette*, 1987; *Harik and Goldberg*, 2000].

[44] The second-stage DE was applied to the original full water network as shown in Figure 2a, but it is initialized by the seeding table in the column 4 of Table 5. A further better optimal solution with a cost of \$239,034 was obtained after the second-stage optimization, and this optimal solution was feasible when determined by EPANET2.0.

6. Case Studies

[45] The algorithms for identifying the optimal source partitioning cut-set, creating the seeding table and the DE algorithm were all coded in C++ using MinGW Developer Studio 2.05. The program EPANET2.0 [*Rossman*, 2000] was used as a network solver in this study. Four case studies have been used to verify the effectiveness of the proposed decomposition and multistage optimization approach: two artificial double-reservoir WDSs; a real-world three-reservoir WDS; and a realistic four-reservoir WDS. It should be noted that the water network layout for each case study is drawn at different scales. In addition, the cost for each diameter used for each case study is the sum of the pipe material cost and the pipe construction cost.

6.1. Case Study 1: Two-Reservoir WDS

[46] The layout of the two-reservoir WDS is given in Figure 2, and the network data are included in Table 2. The global optimal solution for this small network was \$239,034 by using the full enumeration approach. To investigate the impact of the different decomposition strategies on the final solution, this water network decomposed by all cut-sets obtained by the full enumeration were optimized by the proposed multistage DE method. A standard DE (SDE) algorithm seeded by the total choice table (14 pipe options) was also applied to this network to enable the performance comparison with the proposed approach. Table 6 presents the statistical results of different algorithms. It is noted that the parameters of the DE (N = 30, F = CR = 0.5) were fine tuned. A maximum number of allowable evaluations was set to be 6000 for this case study.

[47] As shown in Table 6, for this small network, all the algorithms are able to find the global optimal solution with a cost of \$239,034. The proposed multistage DE method with $\Omega = \{2, 3\}$ (denoted as CS1) significantly outperformed the proposed multistage DE but with the cut-sets $C_1 = \{2, 4\}$ (CS2) and $C_2 = \{2, 5\}$ (CS3) in terms of the solution quality and the efficiency. This is proven by the fact that CS1 found the global optimal solution with a success rate of 100%, which is significantly higher than CS2 (54%) and *CS*3 (14%). In addition, the proposed multistage DE method with $\Omega = \{2, 3\}$ performed slightly better than the SDE in terms of the percent with the best solution found.

[48] The computational overhead for a hydraulic evaluation of one subnetwork with EPANET 2.0 is different from the computational effort required to evaluate the original whole network because of the smaller size of the subnetwork. To enable a fair comparison, the computational overhead for the evaluation of each subnetwork has been converted to the equivalent number of evaluations for the whole network. Each subnetwork and the full network were run 1000 times with randomly selected pipe configurations using the code developed for this proposed method. Then, the average computational time for one subnetwork simulation was converted to the equivalent number of corresponding full network simulations. This approach has been used for each case study investigated in this paper. The code was developed in C++ (linked to EPANET2.0 through the Tookit) and run on a Pentium PC (Inter R) at 3.0 GHz.

[49] In terms of comparing the efficiency, CS1 performed the best as it only required an average of 376 equivalent full network evaluations to find the optimal solutions. This is only 24%, 57%, and 47% of those required by CS2, CS3, and SDE respectively.

6.2. Case Study 2: Double-Reservoir WDS

[50] The double-reservoir network (DRN) was first presented by *Kadu et al*. [2008]. The DRN consists of 24 demand nodes, 34 pipes, and 9 loops and is fed by

Table 6. Algorithm Performance for the Two-Reservoir WDS (F = CR = 0.5)

	Methods	Number of Trial Runs	Best Solution Found (\$)	Percentage of Trials With Best Solution Found (%)	Average Number of Equivalent Full Two-Reservoir WDS Evaluations to Find Best Solution
CS1	Proposed cut-set based on friction slope method with $\Omega = \{2, 3\}$	100	239,034	100	376
CS2	Alternative 1 cut-set with C_1={2, 4}	100	239,034	54	1568
CS3	Alternative cut-set 2 with C_2={2, 5}	100	239,034	14	658
-	SDE	100	239,034	98	792

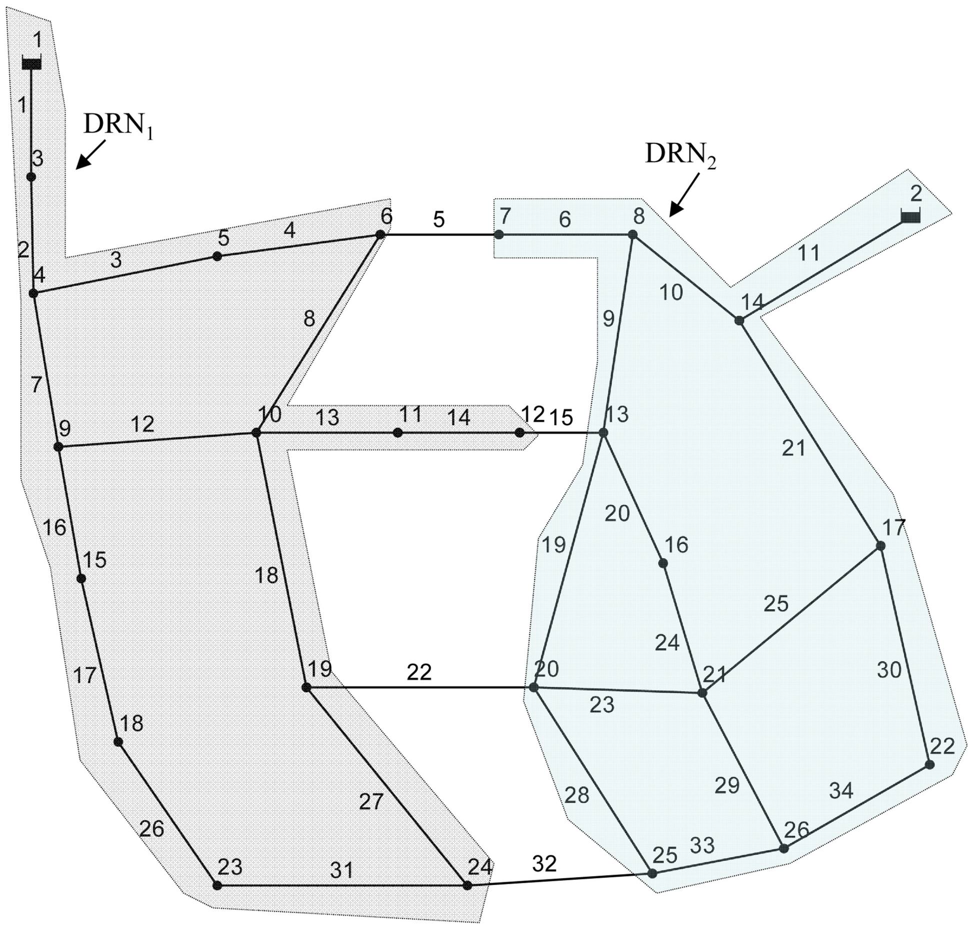

Figure 5. Layout, the optimal source partitioning cut-set (Ω) and the subnetworks (DRN$_1$ and DRN$_2$) of the two-reservoir network (DRN).

2 reservoirs with 100 and 95 m of fixed head, respectively. The layout of the DRN is given in Figure 4. A total of 14 pipe diameters are available in the DRN case study and hence the total choice table includes 14 pipe diameters for each pipe. The search space is therefore $14^{34} \approx 9.2972 \times 10^{38}$. Details of this network and the cost of the pipes are given by *Kadu et al*. [2008].

[51] The optimal source partitioning cut-set for the DRN identified through the developed graph decomposition approach (Figure 5) included pipes 5, 15, 22, and 32 ($\Omega = \{5, 15, 22, 32\}$). The original DRN was therefore partitioned into two subnetworks (as shown in Figure 5): subnetwork one (DRN$_1$) and subnetwork two (DRN$_2$). DRN$_1$ included reservoir 1, 13 nodes, and 15 pipes on the left side of the optimal source partitioning cut-set. DRN$_2$ was composed of reservoir 2, with 11 nodes and 15 pipes on the right side of the optimal source partitioning cut-set.

[52] To enable a performance comparison, the runs of the SDE algorithm seeded by the total choice table (14 pipe diameters) with different starting random number seeds were also conducted for the DRN case study. Table 7 provides the parameter values used for the DE algorithm applied to the DRN case study. As shown in Table 7, a population size (N) of 50 and a maximum number of allowable evaluations of 30,000 were used for the DE applied to subnetworks DRN$_1$ and DRN$_2$ (the first-stage optimization of the proposed method). For the DE algorithm used in the second-stage optimization phase and the SDE applied to the original whole DRN, a population size of 100 and a maximum number of allowable evaluations of 400,000 were used. Values of $F = 0.6$ and CR $= 0.5$ were utilized for all DE used in the proposed method and the SDE applied to the DRN case study. These values were selected based on trials of a number of different parameter values.

[53] A total of 100 runs of the proposed method with different starting random number seeds were performed for the DRN case study. A typical run of the proposed method is illustrated in Table 8.

[54] As shown in Table 8, DRN$_1$ and DRN$_2$ were optimized by DE algorithm during the first optimization stage of the proposed method and hence optimal solutions with costs of \$1.405 million and \$1.191 million were obtained for DRN$_1$ and DRN$_2$, respectively (see columns 2 and 3 of Table 8). By assigning the optimal source partitioning cut-set with the minimum allowable pipe diameters (150 mm for the DRN case study), an approximate optimal solution was produced for the original full DRN with a cost of \$2.752 million, which is given in the column 4 of Table 8. A seeding table was constituted based on the obtained approximate optimal solution (column 5 of Table 8), and this

Table 7. The DE Algorithm Parameter Values Applied to Different Subnetworks and the Whole DRN ($F = 0.6$, $CR = 0.5$)

Network	Number of Decision Variables (Pipes)	Population Size (N)	Maximum Number of Allowable Evaluations
DRN1	15	50	30,000
DRN2	15	50	30,000
DRN (the second-phase DE algorithm)	34	100	400,000
DRN (the SDE)	34	100	400,000

seeding table was used to initialize the DE for the second-stage optimization of the proposed method.

[55] The final solution yielded by the proposed method after the second-phase optimization was $2.750 million (column 6 of Table 8), which is lower than the approximate optimal solution obtained after the first optimization stage. It should be highlighted here that the approximate optimal solution with a cost of $2.752 million was slightly infeasible as determined by EPANET2.0 with the maximum head deficit of 0.5 m. This is because that (i) the water flow distribution was slightly changed after combining the subnetworks; and (ii) the optimal source partitioning cut-set was simply assigned the minimum allowable pipe diameters. However, this slightly infeasible solution was located at the vicinity of the final optimal solution. This is reflected by the fact that 28 of a total of 34 pipes had the same diameters for the approximate optimal solution and the final optimal solution (as shown in Table 8). In addition, the pipe diameters for each link of the final optimal solution are located in the seeding table that was created based on the approximate optimal solution.

[56] The statistical results of the proposed method, the SDE, and other previously reported feasible solutions (determined by EPANET2.0) for the DRN case study are given in Table 9.

[57] In this study, a new best solution (feasible when verified by EPANET2.0) was produced at a cost of $2.750 million. *Kadu et al*. [2008] and *Haghighi et al*. [2011]

Table 8. Typical Run of the Proposed Method for DRN Case Study

Links	Subnetwork Optimization Results (the First-Stage Optimization) (mm)		Approximately Optimal Solution (mm)[a]	Creation of Choice Table	Final Optimization Results (the Second-Stage Optimization) (mm)[a]
Networks	DRN_1	DRN_2	DRN_1+DRN_2+cut-set pipes		DRN
1	1000		1000	800, 900, 1000	900
2	900		900	800, 900, 1000	900
3	350		350	300, 350,400	350
4	300		300	250, 300, 350	300
5[b]			150	150, 200, 250	150
6		250	250	200, 250, 300	250
7	800		800	750, 800,900	800
8	150		150	150, 200, 250	150
9		450	450	400, 450, 500	450
10		500	500	450, 500, 600	500
11		800	800	750, 800, 900	750
12	700		700	600, 700, 750	700
13	500		500	450, 500, 600	500
14	450		450	400, 450, 500	500
15[b]			150	150, 200, 250	150
16	450		450	400, 450, 500	500
17	350		350	300, 350,400	350
18	400		400	350, 400, 450	400
19		150	150	150, 200, 250	150
20		150	150	150, 200, 250	150
21		700	700	600, 700, 750	700
22[b]			150	150, 200, 250	150
23		450	450	400, 450, 500	450
24		350	350	300, 350,400	350
25		700	700	600, 700, 750	700
26	200		200	150, 200, 250	250
27	300		300	250, 300, 350	250
28		300	300	250, 300, 350	300
29		200	200	150, 200, 250	200
30		300	300	250, 300, 350	300
31	150		150	150, 200, 250	150
32[b]			150	150, 200, 250	150
33		150	150	150, 200, 250	150
34		150	150	150, 200, 250	150
Cost ($ million)	1.405	1.191	2.752[c]		2.750
Minimum pressure surplus (m) and its corresponding node	0.08 (Node 23)	0.42 (Node 20)	−0.50 (Node 23)		0.15 (Node 12)

[a]The cost of the solution is the sum of the unit cost for each selected pipe multiplied by the length of this pipe.
[b]Optimal source partitioning cut-set pipes for the DRN.
[c]Infeasible solution.

Table 9. Algorithm Performance for the DRN Case Study

Row	Algorithm		Number of Trial Runs	Best Solution Found ($M)	Percentage of Trials With Best Solution Found (%)	Average Cost Solution ($M)	Average Number of Original Evaluations to Find Best Solution	Average N umber of Equivalent Full DRN Evaluations to Find Best Solution
1	Proposed method using Ω	DRN_1	100	1.405	85	1.410	10,765	2702
2	(This study)	DRN_2	100	1.191	80	1.206	7955	2991
3		DRN_1+ DRN_2+ cut-set pipes[a]	100	2.752[b]	80	2.772	18,720	5693
4		DRN	100	2.750[c]	75	2.755	66,740	66,740
5		Total	100					72,433[d]
6	SDE (This study)		100	2.750	32	2.762	201,457	201,457
7	GA [*Kadu et al.*, 2008]		10	2.847	0	NA	NA	NA
8	GA-ILP [*Haghighi et al.*, 2011]		NA	2.839	0	NA	NA	NA
9	Proposed method using C_1[e]		100	2.898	0	2.901		78,965
10	Proposed method using C_2[f]		100	2.755	0	2.783		156,620

[a]The cost of the cut-set pipes is $0.156 million by assigning them with the minimum pipe diameters (150 mm).
[b]Infeasible solution determined by EPANET2.0 with the maximum head deficit of 0.5 m.
[c]The best solution based on the new method proposed in this paper.
[d]The total computational overhead required by the proposed method has been converted to the equivalent number of the whole network evaluations (DRN_1+DRN_2+DRN_3+cut-set+DRN).
[e]The proposed method applied to the DRN decomposed by cut-set $C_1 = \{4, 12, 31\}$.
[f]The proposed method applied to the DRN decomposed by cut-set $C_2 = \{6, 15, 19, 23, 33\}$.

found the previous best solutions for this case study with costs of $2.847 and $2.839 million, respectively. The new best known solution with a cost of $2.750 million was found with a success rate of 75% by the proposed method, whereas the SDE only returned a success rate of 32%.

[58] As shown in Table 9, the current best solutions for DRN_1 and DRN_2 found by the first-stage optimization of the proposed method were $1.405 and $1.191 million, respectively. These two optimal solutions for DRN_1 and DRN_2 were found with success rates of 85% and 80% respectively. The approximate optimal solutions for the original whole DRN were obtained by combining the optimal solutions for both subnetwork and assigning the minimum pipe diameters for the optimal source partitioning cut-set. As can be seen from Table 9, the best approximate optimal solution provided after the first optimization stage was $2.752 million and this solution was found with a success rate of 80%.

[59] The average computational time of one evaluation for the DRN_1 and DRN_2 was equivalent to 0.251 and 0.376 evaluations for the whole DRN network, respectively. Since the original average number of evaluations for DRN_1 and DRN_2 during the first-stage optimization were 10,756 and 7955 (column 7 of Table 9), the equivalent number of full DRN evaluations was, therefore, 2702 and 2991, respectively (column 8 of Table 9).

[60] The computational time required to find the optimal source partitioning cut-set was also converted to the equivalent number of whole network evaluations. For the DRN case study, the computational time required to find the optimal source partitioning cut-set was equivalent to 19 evaluations of the whole DRN network.

[61] As shown in Table 9, the total equivalent average number of evaluations required to find the optimal solutions using the proposed approach was 72,433, which is only 36% of the number of evaluations required by the SDE algorithm. This shows that the proposed method significantly outperforms the SDE algorithm in terms of efficiency. It was observed that the first optimization stage found the approximate optimal solutions that are extremely close to the final best solution ($2.750 million) using only 5693 equivalent full DRN evaluations.

[62] A convergence comparison between a DE algorithm seeded with the initial seeding table (the proposed method) and a SDE algorithm is given in Figure 6. It is evident that that the proposed algorithm converges significantly faster than the SDE algorithm. To further investigate the impact of the different decomposition strategies on the final solution, the proposed method was also applied to the DRN case study decomposed by $C_1 = \{4, 12, 31\}$ and $C_2 = \{6, 15, 19, 23, 33\}$, respectively ($C$ is the source partitioning cut-set), and the results are included in Table 9. As shown in Table 9, the best solutions found by the proposed method with decomposition cut-sets $C_1 = \{4, 12, 31\}$ and $C_2 = \{6, 15, 19, 23, 33\}$ were $2.898 and $2.755 million, respectively, which are both larger than the current best known solution of the DRN case study.

[63] In contrast, the proposed method using the optimal source partitioning cut-set $\Omega = \{5, 15, 22, 32\}$ was able to find the current best known solution with a success rate of 75% (see row 4 of Table 9). In addition, the proposed method with $\Omega = \{5, 15, 22, 32\}$ used fewer average equivalent full DRN evaluations (72,433 in row 5 of Table 9) to find optimal solutions than the proposed method with $C_1 = \{4, 12, 31\}$ (78,965 in row 9 of Table 9) and $C_2 = \{6, 15, 19, 23, 33\}$(156,620 in row 10 of Table 9*)*.

[64] Based on the results of case study 1 (Table 6) and case study 2 (Table 9), it can be concluded that (i) the search performance of the proposed method in terms of both solution quality and efficiency is significantly affected by the decomposition strategy used and (ii) the proposed optimal source partitioning cut-set Ω, as developed in this paper, is effective in terms of decomposing the water network for design optimization.

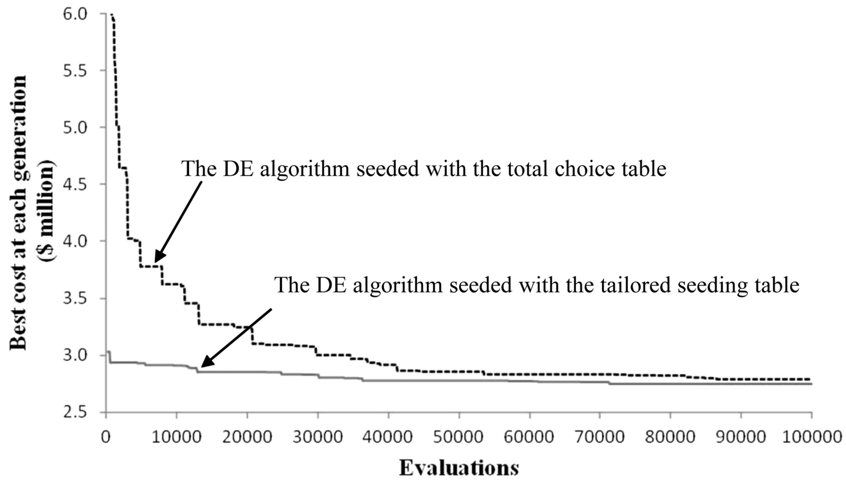

Figure 6. A convergence comparison between DE algorithm seeded with tailored seeding table and the DE algorithm seeded with total choice table.

6.3. Case Study 3: Three-Reservoir WDS

[65] The three-reservoir network (TRN) is an actual water network supplied by three reservoirs located in an eastern province of China. This case study is the first time that it has been investigated. The three reservoirs are denoted as R_1, R2, and R_3 as shown in Figure 7, and have fixed heads of 44, 45, and 47 m, respectively. The TRN has 287 pipes, 199 demand nodes, and 86 primary loops. At each demand node, a minimum pressure of 20 m is required. All the pipes are assigned to have an identical HW of 130. The objective of this case study is to determine the least cost design of this water network, while satisfying the pressure constraints. A total of 14 commercially available pipe diameters ranging from 150 mm up to 1000 mm are available for selection for each pipe (as in case study 1). Thus, the total search space is $14^{287} \approx 8.6845 \times 10^{328}$.

[66] Utilizing the proposed algorithm, 14 links were identified to form the optimal source partitioning cut-set for the TRN case study. Hence, the original TRN was disassembled into three subnetworks, denoted TRN_1, TRN_2, and TRN_3 as shown in Figure 7. Reservoir 1 (R_1), with 73 demand nodes and 91 pipes, was assigned to TRN_1. Reservoir 2 (R_2), with 65 demand nodes and 98 pipes, was assigned to TRN_2. The remaining reservoir (R_3), with 61 demand nodes and 84 pipes was given to TRN_3. These three subnetworks are shown in Figure 7 in different shades of gray.

[67] The computational time required to identify the optimal source partitioning cut-set for the TRN case study was the equivalent of 15 evaluations of the original TRN (using EPANET 2.0). As for the same method used for the DRN case study, the evaluations of TRN_1, TRN_2, and TRN_3 were found to be the equivalent of 0.11, 0.10, and 0.091, respectively, of the whole TRN evaluation in terms of average computational time based on 1000 runs with randomly selected pipe configuration.

[68] For the TRN case study, 10 runs of the proposed method and 10 SDE algorithm runs with different starting number seeds were performed to compare the performance of the two methods. Table 10 provides the parameter values used for the DE algorithm applied to the TRN case study.

[69] As displayed in Table 10, for subnetwork optimization, the population size (N) of the DE algorithms was 150 and the maximum number of allowable evaluations used was 150,000. A population size of $N = 200$ was used for the DE algorithm in the second phase of the proposed method and two population sizes of $N = 200$ and 500 were used for the SDE algorithm. The maximum number of allowable evaluations for DE algorithms applied to optimize the complete TRN (including the SDE and the DE used in the second phase optimization of the proposed method) was 2.5 million. Values of $F = 0.3$ and $CR = 0.5$ were selected for all DE algorithm runs for this case study based on a parameter sensitivity analysis.

[70] The solution distribution obtained by the proposed method and the SDE algorithm applied to the TRN case study is given in Figure 8. It should be noted that the number of evaluations of the proposed method shown in Figure 8 has been converted to the equivalent number of evaluations for the complete TRN using the same approach as for the DRN case study.

[71] As can be seen from Figure 8, the proposed method exhibits superior performance when compared with the SDE algorithm in term of solution quality and efficiency. The SDE algorithm with $N = 500$ was able to find better quality solutions than the SDE algorithm with $N = 200$, but at expense of significantly more evaluations. The final solutions found by the SDE algorithm trial runs with different starting random number seeds are more scattered in distribution than those found by the proposed method. This demonstrates that the performance of the proposed method is less sensitive to the randomized starting points of the search. The statistical results for this case study are shown in Table 11.

[72] As shown in Table 11, the proposed method found the current best solution for the TRN case study with a cost of \$6.822 million. The best solutions found by the SDE algorithms with $N = 500$ and $N = 200$ were \$6.874 and \$6.902 million, respectively, which are 0.73% and 1.17%

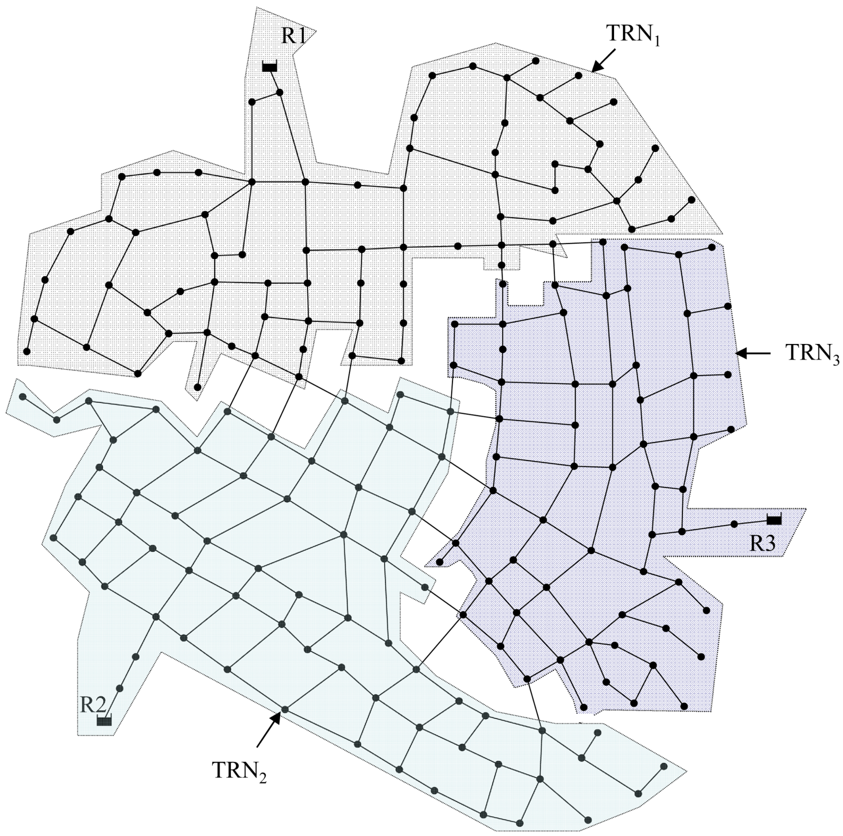

Figure 7. Layout, the optimal source partitioning cut-set, and the subnetworks (TRN_1, TRN_2, and TRN_3) of the three-reservoir network (TRN).

higher than the current best solution found by the proposed method. It was also found that the proposed method performed better than the SDE algorithm in terms of the average cost of solution quality based on 10 different runs. The most noticeable advantage of the proposed method was that it converged to the optimal solutions with significantly greater speed than the SDE algorithm. This is reflected by the fact that the proposed method required an average 270,171 total equivalent full TRN evaluations to find the optimal solutions, while the SDE algorithm with $N = 200$ and $N = 500$ used an average of 559,860 and 1,737,300 evaluations, respectively, as shown in Table 11.

Table 10. DE Algorithm Parameter Values Applied to Different Subnetworks and the Whole TRN ($F = 0.3$, $CR = 0.5$)[a]

Network	Number of Decision Variables (Pipes)	Population Size (N)	Maximum Number of Allowable Evaluations
TRN_1	91	150	150,000
TRN_2	98	150	150,000
TRN_3	84	150	150,000
TRN (the second-stage DE algorithm)	287	200	2,500,000
TRN (the SDE)	287	200/500	2,500,000

[a]The solution distribution obtained by the proposed method and the SDE algorithm

[73] The best and the average approximate optimal solution obtained by the first-stage optimization were \$6.874 and \$6.883 million, respectively, which is only 0.75% and 0.88% larger than the current best solution found by the proposed method after the second-stage optimization (\$6.823 million). In addition, these approximate optimal solutions were located extremely quickly since they only required an average number of 24,411 equivalent full TRN evaluations, as presented in Table 11.

[74] For this case study, a sensitivity analysis for variations in the nodal demands and HWs has been conducted to investigate the impact on the final solution. A nodal demand multiplier (R) was used to adjust the demands for each node. For example, $R = 0.9$ indicates the new demands of each node are 0.9 times the current demand. In this study, values of $R = 0.9$ and 1.1 were used to undertake the sensitivity analysis on the nodal demands, while maintaining a consistent HW value (130).

[75] Additionally, the values of HW of 100 and 115 were used to analyze the sensitivity of the final solution on the HW for the TRN case study. The nodal demands for each node were kept constant ($R = 1.0$). Finally, each node was randomly assigned a value of R in the range of [0.9, 1.1], and each link was assigned a value of HW in the range of [100, 130] for the TRN case study. The results of the proposed decomposition and multistage method applied to the TRN case study with the variation of demands and HW values are presented in Table 12.

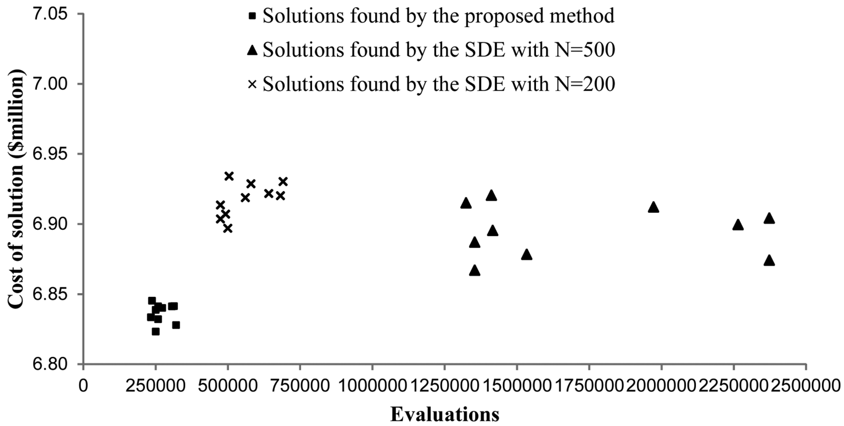

Figure 8. Solution distributions of proposed method and the SDE applied to the TRN case study.

[76] As shown in Table 12, for a HW = 130, the cost of the final optimal solutions obtained by the proposed method increases for an R value that is greater. The cost of the best solution and the average cost solution for the TRN case study with $R = 1.0$ increases by 4.3% and 4.5%, respectively, compared to those with $R = 0.9$, while it decreases by 4.0% and 3.8% compared to those with $R = 1.1$. When the nodal demand was constant ($R = 1$), the proposed method found the lower cost solutions as the value of HW increases as displayed in Table 12. This is to be expected as a larger HW value reflects a smoother pipe.

[77] The best solution obtained for the TRN with HW = 100 is $7.629 million ($R = 1$), which is 6.3% and 11.8% higher than those found for the TRN with HW = 115 and HW = 130, respectively. The best solution found by the proposed method for the TRN with randomly assigned R values (in the range of [0.9, 1.1]) for each node and randomly assigned HW values (in the range of [100, 130]) for each link is $7.176 million, which is 5.2% higher than the best solution found for the original TRN with $R = 1.0$ and HW = 130 ($6.823 million).

[78] The average number of equivalent full TRN evaluations required by the proposed method applied to each network with variations of demands and HW values are similar. This shows that the search efficiency of the proposed method is not significantly affected by network parameter variations (demands and HW values).

6.4. Case Study 4: Four-Reservoir WDS (Balerma Network)

[79] The four-reservoir network (FRN) is the Balerma network, which was first investigated by *Reca and Martínez* [2006]. It consists of 4 reservoirs, 8 loops, 454 pipes, and 443 demand nodes as shown in Figure 9. Ten PVC commercial pipes with nominal diameters from 125 to 600 mm are to be selected for this network and hence the search space is 10^{454}. All the pipes are assumed to have an absolute roughness height of $k = 0.0025$ mm, and the minimum required pressure at each node is 20 m. Pipe costs are given by Reca and *Martínez* [2006]. For this case study, the total choice table is composed of 10 pipe diameters for each pipe.

[80] The optimal source partitioning cut-set for the FRN case study was identified to be composed of five pipes using the proposed method given in Figure 3. The whole FRN was partitioned into four subnetworks after removal of the optimal source partitioning cut-set. These include FRN_1, FRN_2, FRN_3, and FRN_4 as shown in Figure 9. There were 45 demand nodes and 45 pipes in FRN_1; 130 demand nodes and 132 pipes in FRN_2; 41 demand nodes and 41

Table 11. Algorithm Performance for the TRN Case Study

Algorithm		Number of Trial Runs	Best Solution Found ($M)	Percentage of Trials With Best Solution Found (%)	Average Cost Solution ($M)	Average Number of Original Evaluations to Find Best Solution	Average Number of Equivalent Full TRN Evaluations to Find Best Solution
Proposed method (this study)	TRN_1	10	2.311	10	2.322	101,190	11,131
	TRN_2	10	2.291	10	2.294	76,535	7654
	TRN_3	10	2.050	10	2.058	61,820	5626
	$TRN_1+TRN_2+TRN_3+$ cut-set pipes[a]	10	6.874[b]	10	6.883	239,545	24,411
	TRN	10	6.823	10	6.844	245,760	245,760
	Total	10					270,171[c]
SDE ($N = 500$, this study)		10	6.874	0	6.904	1,737,300	1,737,300
SDE ($N = 200$, this study)		10	6.902	0	6.923	559,860	559,860

[a]The cost of the cut-set pipes is $0.211 million by assigning them with the minimum pipe diameters (150 mm).
[b]Infeasible solution determined by EPANET2.0 with the maximum head deficit of 0.2 m.
[c]The total computational overhead required by the proposed method has been converted to the equivalent number of the whole network evaluations ($TRN_1+TRN_2+TRN_3$+cut-set+TRN).

Table 12. Sensitivity Analysis for the TRN Case Study

Values of HW and R		Number of Trial Runs	Best Solution Found ($M)	Average Cost Solution ($M)	Average Number of Equivalent Full TRN Evaluations to Find Best Solution
HW = 130	$R = 0.9$	10	6.542	6.549	279,985
	$R = 1.0$	10	6.823	6.844	270,171
	$R = 1.1$	10	7.100	7.107	315,700
$R = 1.0$	HW = 100	10	7.629	7.637	280,720
	HW = 115	10	7.177	7.182	303,600
	HW = 130	10	6.823	6.844	270,171
$R = [0.9, 1.1]$, HW = [100,130]		10	7.176	7.186	288,520

pipes in FRN_3; and 227 demand nodes and 231 pipes in FRN_4. For the FRN case study, the computational time to identify the optimal source partitioning cut-set was equivalent to 32 whole FRN evaluations. The average computational time for one evaluation of FRN_1, FRN_2, FRN_3, and FRN_4 was equivalent to 0.031, 0.20, 0.031, and 0.52 whole FRN evaluations, respectively, based on 1000 runs using the same method as for the DRN case study. The pipe configuration for each subnetwork and the full network was randomly generated for the 1000 runs.

[81] For the FRN case study, because the size of the subnetworks varies significantly, the population size (N) and the maximum number of allowable evaluations of DE algorithms applied to different subnetwork optimizations need to be slightly tuned. Table 13 gives the parameter values used for the DE algorithms run for the optimization of each subnetwork and for the whole FRN optimization. These parameters values were selected based on a few trials. As can be seen from Table 13, the larger subnetwork was given a larger population size and the maximum number of allowable evaluations. Two SDE algorithms with population sizes of $N = 500$ and $N = 2000$ were applied to the FRN case study. A sensitivity analysis on the F and CR has been carried out for this case study and values of $F = 0.3$ and CR = 0.5 were selected for all the DE algorithms. The statistical results for these different algorithms and the published results for this case study are provided in Table 14.

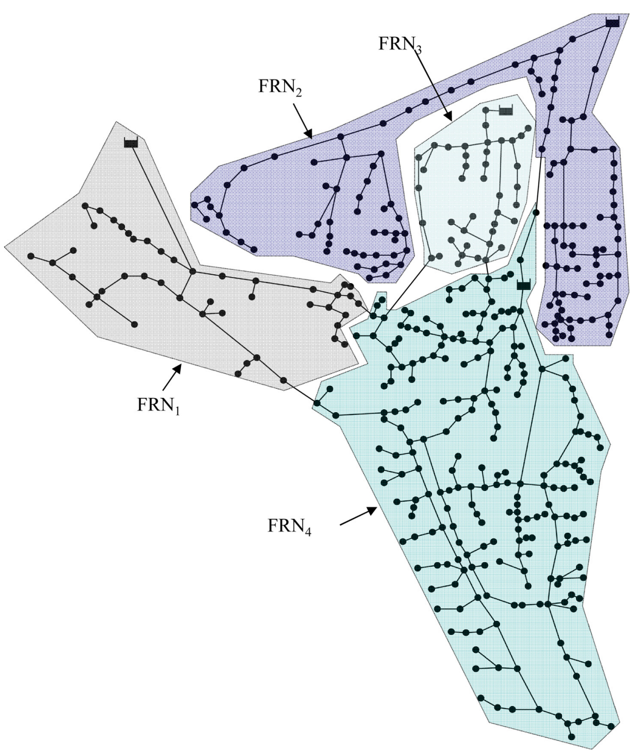

Figure 9. Layout, the optimal source partitioning cut-set and the subnetworks (FRN_1, FRN_2, FRN_3, and FRN_4) of the four-reservoir network (FRN).

[82] As displayed in Table 14, the current best known solution for the FRN case study was first reported by *Zheng et al*. [2011a] with a cost of €1.923 million using a NLP-DE method. This best solution was also found by the proposed method in this paper, however, using only an average of 639,906 total equivalent full FRN evaluations based on 10 different runs, compared to 1,427,850 evaluations required by the NLP-DE method [*Zheng et al.,* 2011a]. The best solution found by HD-DDS [*Tolson et al.,* 2009] was €1.940 million using 30 million evaluations. The SDE algorithm with $N = 500$ produced the best solution of €1.988 million after 2,042,000 evaluations and the SDE algorithm with $N = 2000$ yielded the best solution of €1.982 million with 9,230,000 evaluations.

[83] *Reca and Martínez* [2006] and *Geem* [2009] employed the GANOME GA and HS to find the best solutions of €2.302 and €2.018 million for this case study respectively, running a total of 10 million evaluations. As shown in Table 14, the worst solution found by the proposed method based on the 10 different runs is €1.935 million, which is still lower than the best solutions found by the majority of other algorithms presented in Table 14. From these results, it is concluded that the proposed method is able to find better solutions for this case study with higher reliability than the majority of other optimization techniques.

[84] In terms of efficiency (total equivalent number of evaluations), the proposed method found the best solution 1.23 times faster than the NLP-DE method; 44.8 times faster than the HD-DDS; 13.6 times faster than the SDE algorithm with population size of $N = 2000$; 1.83 times faster than the SDE algorithm with population size of $N = 500$; and 14.3 times faster than GANOME GA and HS. This implies that the proposed decomposition and multistage optimization approach is able to find optimal solutions for such a relatively large case study (454 decision variables) with substantially improved efficiency compared with all other algorithms presented in Table 14.

[85] It is interesting to note that the best approximate optimal solution generated by the first-stage optimization

Table 13. DE Algorithm Parameter Values Applied to Different Subnetworks and the Whole FRN (Balerma Network)

Network	Number of Decision Variables (Pipes)	Population Size (N)	Maximum Number of Allowable Evaluations
FRN_1	45	100	20,000
FRN_2	132	200	200,000
FRN_3	41	100	20,000
FRN_4	231	300	800,000
FRN (the second-phase DE algorithm)	454	200	10,000,000
FRN (the SDE)	454	500/2000	10,000,000

of the proposed method was €1.930 million, which is only 0.7% higher than the current best solution for the FRN case study produced by the proposed method after the second-stage optimization. The average cost of the 10 approximate optimal solutions was €1.931 million, which is also extremely close to the current best solution. In addition, these approximate optimal solutions were found with extremely good efficiency as shown in Table 14. Although these approximate optimal solutions were infeasible when determined by EPANET2.0, they were able to specify promising regions for the second-stage optimization of the proposed method, thereby allowing good quality solutions for the whole FRN to be found efficiently.

6.5. Subnetwork Optimization Analysis (First-Stage Optimization)

[86] Table 15 summarizes the number of pipes for which the diameters in the approximate optimal solutions (produced by the subnetwork optimization during the first-stage optimization) are different from the current best known solutions for each case study.

[87] As can be seen from Table 15, the number of different pipes diameters range from only 1 to 2 for the two-reservoir WDS case study (6 total pipes), 6 to 8 for the DRN case study (34 total pipes) based on 100 different runs, from only 29 to 35 for the TRN case study (287 total pipes), and from only 52 to 61 for the FRN case study (454 total pipes) based on 10 different runs. Thus, the majority of the pipes in the approximate optimal solution obtained in the first-stage optimization have the same diameters as for those in the current best known solution for each case study. This demonstrates that the proposed source partitioning approach for a WDS with multiple supply sources is effective in terms of providing good initial estimates for the whole-of-network optimization. This is proven in that the network configuration obtained by combining each subnetwork's design is extremely close to that provided by the final optimal solution as shown in Table 15.

[88] Thus, it can be concluded that during the first-stage optimization phase of the proposed decomposition and multistage optimization approach, the approximate optimal solutions for the whole network were efficiently found with very satisfactory quality in terms of both cost and network configuration compared to the current best known solution for each case study. The benefits are attributed to two factors including: (i) each DE algorithm is used to deal with a portion of the whole network and hence explore a significantly reduced search space in the proposed method. This allows good quality solutions for each subnetwork to be located with substantially improved efficiency; and (ii) the sum of computational overhead for each subnetwork's hydraulic evaluation is smaller than that of one whole network evaluation.

7. Conclusions and Future Work

[89] A novel decomposition and multistage optimization method is proposed to optimize the design of WDS with multiple supply sources. The proposed method begins by identifying an optimal source partitioning cut-set for a given water network with K supply sources based on the available friction slopes at each node. The whole

Table 14. Algorithm Performance for the FRN Case Study (Balerma Network)

Algorithm		Number of Trial Runs	Best Solution Found (€M)	Average Cost Solution (€M)	Worst Solution Found (€M)	Average Number of Original Evaluations to Find Best Solution	Average Number of Equivalent Full FRN Evaluations to Find Best Solution
Proposed method (this study)	FRN	10	0.182	0.182	0.182	14,867	461
	FRN_2	10	0.710	0.712	0.714	122,889	24,578
	FRN_3	10	0.133	0.133	0.133	15,400	477
	FRN_4	10	0.883	0.884	0.884	567,366	295,030
	$FRN_1+FRN_2+FRN_3+FRN_4$+cut-set pipes[a]	10	1.930[b]	1.931	1.931	720,522	320,546
	FRN	10	1.923	1.931	1.935	319,360	319,360
	Total	10					639,906[c]
NLP-DE [*Zheng et al*., 2011a]		10	1.923	1.927	1.934	1,427,850	1,427,850
HD-DDS [*Tolson et al*., 2009]		1	1.940			30,000,000	30,000,000
SDE ($N = 2000$,this study)		10	1.982	1.985	1.987	9,294,666	9,294,666
SDE ($N = 500$,this study)		10	1.988	2.208	2.050	1,814,700	1,814,700
HS [*Geem*, 2009]		1	2.018			10,000,000	10,000,000
GANOME GA [*Reca and Martínez,* 2006]		10	2.302	2.334	2.350	10,000,000	10,000,000

[a]The cost of the cut-set pipes is $19,674 by assigning them with the minimum pipe diameters.

[b]Infeasible solution determined by EPANET2.0 with the maximum head deficit of 2.2 m.

[c]The total computational overhead required by the proposed method has been converted to the equivalent number of the whole network evaluations ($FRN_1+FRN_2+FRN_3+FRN_4$+cut-set+FRN).

Table 15. Summary of the Number of Different Pipe Diameters for the Approximate Optimal Solutions and the Current Best Known Solutions for Each Case Study

Case Study	Number of Pipes	Number of Pipes in Optimal Source Partitioning Cut-Set (Ω)	Number of Different Runs	Number of Pipes Different in Diameters Between the Approximate Solution and the Current Best Known Solution
Two-reservoir WDS	6	2	100	1–2
DRN	34	4	100	6–8
TRN	287	14	10	29–35
FRN	454	5	10	52–61

water network is then partitioned into K disconnected subnetworks after the removal of the optimal source partitioning cut-set. A total of K independent DE algorithms are used to optimize the K subnetworks individually during the first-stage optimization. The optimal solutions for each subnetwork plus the optimal source partitioning cut-set with the minimum allowable pipe diameter are used to create a tailored seeding table. Another DE algorithm is seeded with this given seeding table to optimize the design of the original whole network during the second-stage optimization.

[90] The proposed method was applied to four case studies, and the results were compared with those of standard DE algorithms seeded with the total choice table also applied to these four case studies. It was found that the proposed method (decomposition followed by two-stage optimization) significantly outperforms the SDE algorithms in terms of solution quality and efficiency. Based on the results of the proposed method applied to the four case studies, the following observations can be made:

[91] The proposed optimization strategy (decomposition using optimal cut-set followed by the multistage optimization) has shown better performance than results from a whole of network optimization. This is proven by the fact that the proposed approach is able to find the same or better quality solutions than the SDE applied to the full network with significantly improved efficiency for each case study presented in this paper.

[92] 1. The proposed partitioning approach for a WDS with multiple supply sources based on the available friction slopes at each node is effective. This is reflected by the fact that (i) the approximate optimal solutions obtained from the subnetwork optimizations were extremely close to the current best solution for each case study in terms of both solution costs and network configurations and (ii) the good quality solutions for each case study were found efficiently by a DE seeded by the tailored seeding table obtained from subnetwork optimization.

[93] 2. The computational overhead required to find the optimal source partitioning cut-set for a given WDS with multiple supply sources is negligible compared with that required by the whole optimization process (smaller than 0.01% of the total time). This indicates that the proposed algorithm given in Figure 3 used to identify the optimal source partitioning cut-set for a WDS with multiple supply sources is extremely efficient.

[94] 3. The DE algorithm seeded with the tailored seeding table based on the approximate optimal solution efficiently produces better quality solutions than the standard DE algorithm seeded with the total choice table.

[95] 4. The proposed method found the new current best solution for the DRN with a cost of \$1.750 million and the current best known solution for the FRN case study with the best known efficiency. The proposed method produced a current best solution for the TRN case study, with a value of \$6.823 million ($R = 1$ and $\mathrm{HW} = 130$).

[96] The performance of the proposed method has been compared with other previously reported optimization techniques based on the four case studies. It was found that the newly proposed method (decomposition followed by two-stage optimization) yielded better optimal solutions than other optimization techniques such as GAs and the HD-DDS with an extremely faster convergence speed.

[97] It is important to note that the computational time for each subnetwork optimization was added to the total computational time for the whole proposed optimization process in this study. This is because subnetwork optimization is individually completed in a predetermined sequence. However, subnetwork optimization using this proposed method can actually be undertaken utilizing parallel computing technology or multiple computers. In this case, all the subnetwork optimizations could be started simultaneously, therefore further improving the efficiency of the whole optimization process. Thus, the proposed method provides an opportunity to exploit parallel computing techniques for the design optimization of a WDS with multiple supply sources.

[98] The proposed decomposition and multistage optimization method presented in this paper has been demonstrated to be effective in finding the least cost design (single objective optimization) for WDS-MSS. A further future extension to the proposed method would be to deal with multiobjective optimization problems for WDS-MSS, in which say both the network cost and reliability will be considered. For the purpose of multiobjective optimization for WDS-MSS, a multiobjective optimization technique (such as NSGA-II: *Deb et al.* [2002]; Borg MOEA: *Hadka and Reed* [2012]) could be used to deal with subnetworks separately during the first-stage optimization phase. Then, another multiobjective optimization run would be seeded by the results obtained from the first-stage optimization to generate multiobjective optimal solutions for the original whole WDS-MSS. This extension could be a focus of future work.

[99] It should be noted that, in this study, the proposed decomposition and multistage optimization method have been verified by WDS-MSS with pipes only. The proposed method will need to be modified to deal with the optimization of more complex networks with pumps and/or valves. For example, if pressure reducing valves are used to

partition a local water supply system into different zones, then application of the new method will require appropriate modification. Another future focus will be to extend the proposed method to deal with the optimization design of more complex networks, for which the pumps, valves, tanks, multiple demand loading cases, and variable source energy may be incorporated.

[100] Although the proposed decomposition and multistage optimization method was applied to find the optimal design for WDS-MSS in this paper, this concept (i.e., decomposition followed by multistage optimization) could be also transferred or extended to deal with other optimization problems, such as leakage hot spot detection [*Wu and Sage*, 2006], contaminant detection [*Weickgenannt et al.,* 2010], and the real-time optimization problems for WDSs [*Kang and Lansey*, 2010].

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