



# DYNAMIC MODELLING OF INDUCTION MACHINES

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## ABSTRACT

A great deal of attention has been paid to the modelling of induction machines under both steady state and dynamic operating conditions. Several different approaches adopted in formulating models include *field modelling*, based on the evaluation of the magnetic field within the machine space, and the *circuit modelling*, which uses lumped parameters. This thesis compares various different modelling approaches, and postulates that judiciously formed circuit models can be used in predicting, with acceptable accuracy, the performance of an induction machine under different operating conditions. Two approaches are developed in detail, both of which share a common starting point, namely the airgap magnetomotive force set up by a single energised coil. In the first approach, the magnetomotive force distribution is evaluated in terms of space harmonics, leading to the formation of high order harmonic circuit models. The inclusion in the models of such phenomena as saturation, current displacement and core losses dramatically improves the prediction accuracy, which justifies their being labelled as *high fidelity harmonic circuit models*. They are particularly suitable in evaluating any parasitic



behaviour under quasi dynamic operating conditions. The effects on the machine behaviour of a non-sinusoidal supply are accounted for by means of an aggregate time-harmonic supply model.

In contrast, the second approach is based on discrete circuit modelling, treating each coil within the machine as an individual circuit. The evaluation is carried out entirely in the time-domain, making the approach well suited to transient analysis. The generality of the model allows the global effects of local phenomena to be ascertained accurately. The validity of both modelling approaches is demonstrated by means of case studies.