The University of Adelaide

### Semiparametric Models with Endogeneity and their Application to an Empirical Demand Analysis

by

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#### Abstract

During the past few decades, nonparametric models have been extensively applied to empirical studies in various fields of economics due to its flexibility for depicting any type of relationship among key economic variables. However, one of the most well-known shortfalls of the model is the curse of dimensionality. It can be conveniently overcome with semiparametric modelling such as partially linear (PL) models and/or single-index (SI) models. Nonetheless, the practicality of these models in the empirical studies has been hampered by the lack of appropriate estimation procedures and a method to address endogeneity. Hence the ultimate goal of this thesis is to establish a novel econometric method for estimating semiparametrics, specifically a PL model and an extended generalised partially linear single-index (EGPLSI) model, with the presence of endogeneity. Furthermore, semiparametric analysis is an important tool for analysing empirical Engel curves, which often involve endogeneity in total expenditure. We show that, our newly developed estimation procedures and methods are able to address the endogeneity problem in the semiparametric analysis of empirical Engel curves. These goals can be broken down into a few research objectives.

- (1) Firstly, this thesis aims to construct a comprehensive and systematic treatment of endogeneity in semiparametrics, given the complexity of the models containing both parametric and nonparametric components.
- (2) Secondly, it aims to develop novel estimation procedures and methods to address endogeneity in a PL model and an EGPLSI model.
- (3) Lastly, it aims to analyse the empirical demand function semiparametrically by applying the estimation procedures and methods in this thesis.

#### Publications arising from the thesis

- Kim, N. and Saart, P. W. (2013). Estimation in Partially Linear Semiparametric Models with parametric and/or nonparametric endogeneity. Under review at the Econometrics Journal.
- (2) Kim, N., Saart, P. W. and Gao, J. (2013). Semi-parametric Analysis of Shape-Invariant Engel Curves with Control Function Approach. Under review at the Journal of Econometrics.

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