

# **Analytical and Numerical Analyses for Rock Slope Stability Using the Generalized Hoek-Brown Criterion**

Jiayi Shen

A thesis submitted for the degree of  
Doctor of Philosophy



School of Civil, Environmental and Mining Engineering  
The University of Adelaide  
Australia

July 2013



*To my parents*

*Meihua and Binying*



## **Statement of Originality**

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

I give consent to this copy of my thesis when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968.

The author acknowledges that copyright of published works contained within this thesis resides with the copyright holder(s) of those works.

I also give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library catalogue and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

Name: Jiayi Shen

Signature:

Date: 17<sup>th</sup> July 2013



## **Acknowledgments**

I gratefully acknowledge my supervisors Dr. Murat Karakus, Associate Prof. Chaoshui Xu and Prof. Stephen Priest, for their invaluable guidance, encouragement and constructive criticism during my candidature period. Without their great contribution this thesis would not be possible. I am particularly grateful to Dr. Murat Karakus for not only gives research supports but also offers encouragement and emotional supports in the past three years.

I would like to sincerely thank the China Scholarship Council and the University of Adelaide for providing the joint PhD scholarship.

I would like to express my gratitude to Mrs Barbara Brougham for editing the submitted and published journal papers which are composed of the thesis.

Many thanks go to Associate Prof. Rafael Jimenez for supervision and cooperation on the rock failure criterion research topic when I was a visiting scholar in the E.T.S.I Caminos, Canales y Puertos at the Universidad Politécnica de Madrid, Spain.

Many thanks go to Prof. Jian Zhao for hosting me as a visiting scholar at Laboratory for Rock Mechanics at the Swiss Federal Institute of Technology Lausanne, Switzerland.

Many thanks go to all the staff of the School of Civil, Environmental and Mining Engineering for their individual help and support. I am especially grateful to Dr. Stephen Carr for helping to install research softwares and for providing IT supports.

Finally, I would also like to thank my parents, for supporting me emotionally during my PhD research.





## Introduction

Design of rock slope is one of the major challenges at every stage of open pit mining operations. Providing an optimal excavation design based on a robust analysis in terms of safety, ore recovery and profit is the ultimate goal of any slope design. The rock slope stability is predominantly controlled by the strength and deformation of the rock mass which characteristically consists of intact rock materials and discontinuities. Initially, movement of the slope occurs due to stress relaxation as a result of removal of rocks which used to provide confinement. This behavior of slope can be attributed to linear elastic deformation. In addition to this, sliding along discontinuity surfaces and dilation in consequence of formation of cracks can occur. Ultimately all these instabilities lead to failure of the slopes. Therefore, formulation of slope designs plays critical role in the process of slope stability. In conventional approaches for assessing the stability of a homogeneous slope, such as the limit equilibrium method (LEM) and shear strength reduction (SSR) method, rock mass strength is usually expressed by the linear Mohr-Coulomb (MC) criterion. However, rock mass strength is a non-linear stress function. Therefore, the linear MC criterion generally do not agree with the rock mass failure envelope, especially for slope stability problems where the rock mass is in a state of low confining stresses that make the nonlinearity more dominant.

With the aim of better understanding the fundamental rock slope failure mechanisms and improving the accuracy of the rock slope stability results, this research focuses on the application of the Hoek-Brown (HB) criterion, which can ideally represent the non-linear behavior of a rock mass, on the rock slope stability analysis.

There, three major sections are available in the thesis. The first section, from Chapters 1 to 4, proposes new methods for estimating the intact rock and rock mass properties, which will be

used for slope stability analysis. In the second section studied in Chapter 5, a new non-linear shear strength reduction technique is proposed for the analysis of three-dimensional (3D) slope modeling. In section three (Chapter 6), novel stability charts are proposed, which have the merit of estimating factor of safety (FOS) for a given slope directly from the HB parameters and rock mass properties. These charts can provide a quick and reliable assessment of rock slope stability.

The major research contributions and outcomes of the overall researches are presented in six journal publications which are forming the thesis. The titles of Chapters 1 through 6 reflect the titles of the journal papers.

**In Chapter 1**, laboratory tests conducted on Hawkesbury sandstone obtained from New South Wales are carried out to investigate the relationship between the HB constant  $m_i$  and uniaxial compressive strength (UCS) of intact rock. Based on the analysis of the laboratory tests and the existing database, a new method that can estimate the HB constant  $m_i$  values from UCS and rock types is proposed. The proposed method can reliably be used in the HB criterion for intact rock strength estimation when the triaxial tests are not available.

**In Chapter 2**, an analytical solution for estimating the instantaneous MC shear strength from the HB failure criterion for highly fractured rock mass is presented. The proposed solution is based on the assumption that the HB parameter,  $s$  is equal to zero. The proposed solution has the merit of producing very accurate shear strength for highly fractured rock mass where the Geological Strength Index (GSI) is less than 40.

**In Chapter 3**, an analytical solution, which can calculate the shear strength of rock masses accurately for the whole range GSI values, is proposed as an extension to the work in Chapter 2. The proposed approach is based on a symbolic regression analysis performed by genetic programming (GP). The proposed solution not only can be implemented into the LEM to

calculate the instantaneous shear strength of each slice of a failure surface under a specified normal stress, but also can be implemented into finite element method performed by SSR approach to calculate the instantaneous shear strength of each element under different stress state of a slope.

**In Chapter 4**, as a part of estimating rock mass strength and elastic properties in the first section, the most widely used empirical equations for the estimation of deformation modulus of rock masses ( $E_m$ ) are reviewed. Two simplified empirical equations for estimating of  $E_m$  are also presented. The proposed empirical equations use the Rock Mass Rating classification system and the deformation modulus of intact rock ( $E_i$ ) as input parameters. These equations can be used in the numerical modelling for slope stability analysis, which is conducted in Chapter 5.

**In Chapter 5**, a new non-linear shear strength reduction technique is proposed to analysis the stability of 3D rock slopes satisfying the HB failure criterion. The method for estimating the instantaneous MC shear strength from the HB criterion described in Chapters 2 and 3 are used to estimate shear strength of elements in FLAC<sup>3D</sup> model. The proposed 3D slope model is used to analyse the influence of boundary condition on the calculation of FOS using 21 real open pit cases where the values of  $m_i$  and  $E_m$  values are calculated from the methods introduced in Chapters 1 and 4, respectively. Results show that the values of FOS for a given slope will be significantly influenced by the boundary condition, especially the case where the slope angle is less than 50°.

**In Chapter 6**, extensive slope stability analyses using LEM are carried out. The calculation of FOS is based on estimating the instantaneous MC shear strength of slices of a slip surface from the HB criterion. Based on the analysis results, novel stability charts are proposed. The proposed charts are able to estimate the FOS for a given slope directly from the HB parameters,

slope geometry and rock mass properties. It is suggested that the proposed charts can be used as useful tools for the preliminary rock slope stability assessment.

## List of contents

Statement of Originality.....	VI
Acknowledgments.....	VIII
Introduction.....	X
List of Tables .....	XVI
List of Figures .....	XVIII
List of Symbols .....	XXIV
Chapter 1 A New Method for Estimating the Hoek-Brown Constant for Intact Rocks .....	3
Chapter 2 Determination of Mohr-Coulomb Shear Strength Parameters from Generalized Hoek-Brown Criterion for Slope Stability Analysis .....	29
Chapter 3 Direct Expressions for Linearization of Shear Strength Envelopes Given by the Generalized Hoek-Brown Criterion Using Genetic Programming.....	51
Chapter 4 A Comparative Study for Empirical Equations in Estimating Deformation Modulus of Rock Masses .....	81
Chapter 5 Three-Dimensional Numerical Analysis for Rock Slope Stability Using Non-linear Shear Strength Reduction Method .....	107
Chapter 6 Chart-Based Slope Stability Assessment Using the Generalized Hoek-Brown Criterion .....	137
Chapter 7 Conclusions and Recommendations for Further Work.....	175



## List of Tables

Table A (Table 1 in Chapter 1) Estimated $m_i$ values by regression analysis using triaxial test data at different confining stresses.....	8
Table B (Table 2 in Chapter 1) Best fit $m_c$ and $m_d$ constants to estimate $m_{in}$ using $\sigma_{ci}$ for specific rock types .....	15
Table C (Table 3 in Chapter 1) Comparison of the prediction performance of different methods using the sandstone laboratory test data .....	19
Table D (Table 1 in Chapter 2) Shear stresses obtained from Priest and Bray solutions over a range of GSI.....	35
Table E (Table 2 in Chapter 2) Range of input parameters.....	38
Table F (Table 3 in Chapter 2) Data for validation of the proposed approximate solution.....	39
Table G (Table 4 in Chapter 2) Comparison results of shear strength parameters with different methods.....	43
Table H (Table 1 in Chapter 3) Parameters used in GP analysis.....	63
Table I (Table 2 in Chapter 3) Range of input parameters.....	65
Table J (Table 3 in Chapter 3) Data of HB criterion for GP analysis.....	66
Table K (Table 4 in Chapter 3) Shear stresses obtained from numerical and GP analytical solutions over a range of normal stresses.....	71
Table L (Table 1 in Chapter 4) Empirical equations using RMR and GSI for predicting $E_m$ .....	85
Table M (Table 1 in Chapter 5) 3D slope stability analysis using different methods.....	109
Table N (Table 2 in Chapter 5) Comparison of failure surfaces corresponding to FOS values using different convergence criteria .....	119
Table O (Table 3 in Chapter 5) Input parameters of a slope case.....	121

Table P (Table 4 in Chapter 5) Comparison of failure surfaces, contours of $c$ and $\phi$ and FOS of a slope model under various boundary conditions.....	122
Table Q (Table 5 in Chapter 5) The results of FOS and $f_B$ of the slope with different slope angle.....	124
Table R (Table 6 in Chapter 5) The results of FOS and $f_B$ of real slope cases.....	126
Table S (Table 1 in Chapter 6) Comparison of the factor of safety estimated from different stability charts .....	145
Table T (Table 2 in Chapter 6) Slope modeling setting in <i>Slide 6.0</i> .....	146
Table U (Table 3 in Chapter 6) Comparison of the FOS of a given slope with the same value of SR.....	150
Table V (Table 4 in Chapter 6) Comparison of the FOS of a given rock slope with various Hoek-Brown parameters.....	151
Table W (Table 5 in Chapter 6) Three slope examples analyzed using the proposed stability charts.....	169



## List of Figures

Figure A (Fig. 1 in Chapter 1) The Hoek-Brown failure envelopes using different $m_i$ values.....	8
Figure B (Fig. 2 in Chapter 1) Comparison of sensitivities to the confining stress range employed for $m_i$ fitting, as indicated by the $T$ parameter .....	9
Figure C (Fig. 3 in Chapter 1) Distribution of $m_i$ values for sandstone.....	11
Figure D (Fig. 4 in Chapter 1) Correlation between $R$ and $m_i$ , after Read and Richards (2011) ..	12
Figure E (Fig. 5 in Chapter 1) Correlation between $m_{in}$ and $\sigma_{ci}$ for 28 rock types.....	13
Figure F (Fig. 6 in Chapter 1) Rock strength prediction performance using Eq. 7 for general rock types.....	14
Figure G (Fig. 7 in Chapter 1) Correlation between $m_{in}$ and $\sigma_{ci}$ for specific rock types corresponding to their rock strength prediction performances.....	16
Figure H (Fig. 8 in Chapter 1) Comparison of experimental rock strength with predicted rock strength using different methods.....	18
Figure I (Fig. 9 in Chapter 1) Cumulative distribution function (CDF) of prediction errors (AAREP) of different methods using five rock types in Table 2.....	21
Figure J (Fig. 1 in Chapter 2) (a) Major and minor principal stresses for the HB criterion, (b) Normal and shear stresses for the HB criterion.....	32
Figure K (Fig. 2 in Chapter 2) Shear stress versus GSI.....	34
Figure L (Fig. 3 in Chapter 2) Priest numerical versus proposed approximate analytical value of $\phi$ .....	40
Figure M (Fig. 4 in Chapter 2) Priest numerical versus proposed approximate analytical value of $\tau/\sigma_{ci}$ .....	41

Figure N (Fig. 5 in Chapter 2) Priest numerical versus proposed approximate analytical value of $c/\sigma_{ci}$ .....	42
Figure O (Fig. 6 in Chapter 2) Comparison of angle of friction $\phi$ results .....	44
Figure P (Fig. 7 in Chapter 2) Comparison of cohesion $c$ results.....	45
Figure Q (Fig. 8 in Chapter 2) Comparison of shear stress $\tau$ results.....	46
Figure R (Fig. 1 in Chapter 3) (a) The basic of method of slices, (b) Forces acting on a given slice.....	52
Figure S (Fig. 2 in Chapter 3) The MC criterion showing shear strength defined by angle of friction $\phi$ and cohesion $c$ .....	53
Figure T (Fig. 3 in Chapter 3) (a) Maximum and minimum principal stresses for the GHB criterion, ( b) Normal and shear stresses for the GHB criterion [27] .....	56
Figure U (Fig. 4 in Chapter 3) A typical tree structure of the function of $x*y-\sin(z)$ .....	60
Figure V (Fig. 5 in Chapter 3) A basic flow chart for GP.....	60
Figure W (Fig. 6 in Chapter 3) Crossover operation in genetic programming.....	62
Figure X (Fig. 7 in Chapter 3) Mutation operation in genetic programming.....	62
Figure Y (Fig. 8 in Chapter 3) Numerical versus GP value of $\tau/\sigma_{ci}$ .....	68
Figure Z (Fig. 9 in Chapter 3) Discrepancy analysis of the proposed analytical solution.....	69
Figure AA (Fig. 10 in Chapter 3) Discrepancy analysis of the analytical solution which has the lowest value of AAREP.....	70
Figure BB (Fig. 11 in Chapter 3) Hoek-Brown shear strength envelope in shear stress/normal stress space.....	72
Figure CC (Fig. 12 in Chapter 3) Comparison of shear stress $\tau$ results.....	73

Figure DD (Fig. 1 in Chapter 4) Empirical equations in Group 1 for estimating $E_m$ compared with <i>in-situ</i> data.....	88
Figure EE (Fig. 2 in Chapter 4) Empirical equations in Group 2 for estimating $E_m / E_i$ compared with <i>in-situ</i> data.....	89
Figure FF (Fig. 3 in Chapter 4) Empirical equations in Group 3 for estimating $E_m$ compared with <i>in-situ</i> data.....	91
Figure GG (Fig. 4 in Chapter 4) Empirical equations in Group 4 for estimating $E_m / E_i$ compared with <i>in-situ</i> data.....	92
Figure HH (Fig. 5 in Chapter 4) Empirical equations in Group 5 for estimating $E_m$ compared with <i>in-situ</i> data, $\sigma_{ci}=80\text{MPa}$ .....	93
Figure II (Fig. 6 in Chapter 4) Plot the Eq. 4 for the <i>in-situ</i> data.....	95
Figure JJ (Fig. 7 in Chapter 4) Estimated $E_m$ values from Eq. 4 versus <i>in-situ</i> data.....	96
Figure KK (Fig. 8 in Chapter 4) Plot the Eq. 5 for the <i>in-situ</i> data.....	97
Figure LL (Fig. 9 in Chapter 4) Estimated $E_m / E_i$ values from Eq. 5 versus <i>in-situ</i> data.....	98
Figure MM (Fig. 10 in Chapter 4) $E_m$ values estimated from Eq. 4 compared with Hoek and Diederichs (2006) <i>in-situ</i> data.....	99
Figure NN (Fig. 11 in Chapter 4) $E_m / E_i$ values estimated from Eq. 5 compared with Hoek and Diederichs (2006) <i>in-situ</i> data.....	100
Figure OO (Fig. 1 in Chapter 5) Instantaneous MC envelope of the HB criterion in the normal and shear stress plane.....	112
Figure PP (Fig. 2 in Chapter 5) The correlations between MC parameters and $\sigma_3$ .....	114
Figure QQ (Fig. 3 in Chapter 5) Flow chart of the application of HB criterion into FLAC <sup>3D</sup> using non-linear SSR technique.....	116

Figure RR (Fig. 4 in Chapter 5) Boundary conditions for a slope model.....	118
Figure SS (Fig. 5 in Chapter 5) Plot of FOS values versus mesh elements.....	120
Figure TT (Fig. 6 in Chapter 5) The correlations between $f_B$ and $\beta$ under different boundary conditions for a slope case.....	124
Figure UU (Fig. 7 in Chapter 5) The correlations between $f_B$ and $\beta$ under different boundary conditions for open pit cases.....	127
Figure VV (Fig. 8 in Chapter 5) The correlations between $f_{B,xy}$ and $H, \sigma_{ci}, GSI, m_i$ .....	128
Figure WW (Fig. 1 in Chapter 6) Relationship between HB and equivalent MC envelopes ....	141
Figure XX (Fig. 2 in Chapter 6) Slope stability chart ( $\beta=45^\circ, a=0.5$ ) [12] .....	142
Figure YY (Fig. 3 in Chapter 6) (a) Slope stability chart with $D=0$ [13], (b) Slope stability chart with $D=0.7$ [14], (c) Slope stability chart with $D=1.0$ [14] .....	143
Figure ZZ (Fig. 4 in Chapter 6) (a) The basic of method of slices, (b) Stresses acting on a given slice.....	148
Figure AAA (Fig. 5 in Chapter 6) Proposed stability charts for rock mass slope, $\beta=45^\circ, D=0$ ( $5 \leq m_i \leq 35$ ) .....	155
Figure BBB (Fig. 6 in Chapter 6) Proposed stability charts for rock mass slope, $\beta=45^\circ, D=0$ (SR=0.1, 1, 10, 40) .....	156
Figure CCC (Fig. 7 in Chapter 6) (a) Relationship between $c/\sigma_{ci}$ and GSI for different $m_i$ values [29], (b) Relationship between $\phi$ and GSI for different $m_i$ values [29] .....	157
Figure DDD (Fig. 8 in Chapter 6) Alternative form of Fig. 6b using the stability number $N$ ...	158
Figure EEE (Fig. 9 in Chapter 6) Chart for estimating disturbance weighting factor $f_D$ , SR=10, $\beta=45^\circ$ .....	160

Figure FFF (Fig. 10 in Chapter 6) Chart for estimating disturbance weighting factor, $f_D$ ( $m_i=5, 15, 25, 35$ ) .....	164
Figure GGG (Fig. 11 in Chapter 6) Slope angle weighting factor chart.....	165
Figure HHH (Fig. 12 in Chapter 6) Discrepancy analysis of the proposed rock slope stability charts.....	166



## List of Symbols

$a$	Hoek-Brown input parameter for the rock mass
$c$	Cohesion
$D$	Disturbance factor
$E_i$	Deformation modulus of the intact rock
$E_m$	Deformation modulus of the rock mass
$f_B$	Boundary weighting factor
$f_D$	Disturbance weighting factor
$f_\beta$	Slope angle weighting factor
$H$	Slope height
$m_b$	Hoek-Brown input parameter for the rock mass
$m_i$	Hoek-Brown constant for the intact rock
$m_{in}$	Normalized $m_i$ for the Hoek-Brown criterion
$m_c$	Constant for calculating $m_{in}$
$m_d$	Constant for calculating $m_{in}$
$s$	Hoek-Brown input parameter for the rock mass
$\beta$	Slope angle
$\phi$	Angle of friction
$\gamma$	Unit weight of the rock mass
$\nu$	Poisson's ratio
$\sigma_1$	Major principal stress
$\sigma_3$	Minor principal stress
$\sigma_{ci}$	Uniaxial compressive strength of the intact rock

$\sigma_n$  Normal stress  
 $\sigma_t$  Tensile strength of the intact rock  
 $\tau$  Shear stress