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Declaration

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ABSTRACT

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Di Yuan

During the past few decades, continuous time diffusion models have become an integral part of financial economics. Especially, in certain core areas in finance, such as interest rate, asset pricing, option pricing, portfolio selection and volatility modelling, continuous time diffusion models have proved to be a very attractive way to conduct research and gain economic intuition. This thesis makes three main contributions to the field of continuous time diffusion models.

First, we propose regime-switching Heston, GARCH, and CEV stochastic volatility models where all parameters are allowed to vary depending on the state of the economy. Then we apply these models to describe the dynamics of S&P 500 and VIX. We find strong evidence of regime shifts for all models. The CEV model is statistically preferred to other two nested models in explaining dynamics of data.

Second, because the true transition density functions of regime-switching stochastic volatility models are unknown, the standard maximum likelihood estimation cannot be conducted. We first conduct the maximum likelihood estimation with closed-form likelihood expansions for regime-switching continuous time stochastic volatility models.

Third, to approximate a continuous time diffusion process, researchers often use the Euler approximation in the literature. Theoretically, the smaller the discretization interval is, the more accurate the Euler approximation is expected to be. However, even when the discretization interval is too small, the accuracy of the Euler approximation can get worse because of the roundoff error and random number generator bias. A variety of univariate and multivariate diffusion models from the literature are considered. We use the solution of a diffusion process when it is available and

usable as a benchmark. The Milstein approximation is also adopted to compare the accuracy of the Euler approximation. Depending on the problem of interest, different criteria are used to measure accuracy of approximation. The percentage error and strong convergence can be examined when a good approximation of sample path of a diffusion model is required. The weak convergence is preferred for the cases where approximation of moments of the process matters. In our Monte Carlo simulation studies of diverse diffusion models, we measure accuracy of the Euler approximation not only by using those criteria but also by looking at end point of the approximation. The simulation results show that an appropriate discretization interval must be picked for different diffusion models when applying the Euler approximation.

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