# Essays on Economics of School and Residential Choices

by

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# Abstract

This thesis studies school and residential choices when private schooling is available and attending a public school is free of charge but requires a residence in the school attendance zone. Each of the three chapters focuses on different issues.

The first chapter develops a model of competition between neighbourhood (public) schools and private schools. A model is presented in which a school's quality is determined by the average ability of the student body. Private schools set their own tuition and admission policies to attract particular types of students. All schools are equally effective in providing any given school quality. The theoretical results show that in equilibrium private schools cream skim relatively richer and higher ability students and produce higher school qualities than public schools even though neighbourhood schools also generate segregation among public school students. A policy implication is that price subsidisation to private schooling would intensify the cream-skimming problem in this environment. This is likely to worsen the welfare of students who are left in the public sector as public schools lose relatively higher ability students to the private sector.

The second chapter argues, by developing a simple multiple jurisdiction model, that price subsidisation for private education can be a Pareto improving policy if (i) school quality is measured by levels of educational services, (ii) private education is more costly per unit, (iii) the level of educational services of public schools within a jurisdiction is determined by majority voting of the residents, and (iv) the housing capacities of jurisdictions cannot accommodate perfect segregation among heterogeneous households.

The third chapter studies school and residential choices when there are frictions in housing markets and agents only value school quality during the early stages of their lives. An overlapping-generation model is developed to explain the relocation of agents across frictional housing markets due to different valuation and quality of

local amenities such as public schools. There exist steady-state equilibria in which young agents who value school quality and live in a location with a low-quality public school have a potential to move to another location with a better public school. With frictions, some of such young agents who are willing to relocate get stuck in the low-quality school location and obtain relatively low life-time utility. The equilibria exist under sufficiently high differences in public school qualities across locations. For individuals, the benefits of moving into a good school location are not only derived from school quality but also from the resale value of the house once school services are no longer valued. Equilibria exist in which increasing the quality of the low-quality school improves the total welfare but affects agents across locations differently. In addition, when relatively good quality private schools are available, the young house buyers are better off while some old house owners are worse off due to a reduction in the returns from house sales.

# Acknowledgments

# Dedication

*To Mom and Dad*

# Contents

# List of Tables

# List of Figures

# Chapter 1

# Introduction

This thesis studies school and residential choices when private schooling is available and attending a public school is free of charge but requires a residence in the school attendance zone. This residential requirement creates barriers to public school access. Thus, it allows households to segregate themselves according to their characteristics such as income and children's ability. In contrast, private schools generally charge tuition, set their own admission policy, and have no residential requirement. Availability of private alternatives breaks the tie between school quality and residency, thus increasing school and residential options for households. In the following three chapters, each develops a theoretical model to analyse different issues of school and residential choices.

The first chapter examines a cream-skimming problem: private schools attract relatively richer and higher ability students and produce higher school quality than public schools. Unlike previous studies that tend to treat all public schools as identical, this chapter incorporates both heterogeneous neighbourhood schools and private schools into the same model. A school's quality is determined by the average ability of the student body. All schools are assumed to be equally effective in providing any given school quality. Considering a single jurisdiction, the theoretical

results show that the cream-skimming problem persists even though neighbourhood schools also generate segregation among public school students. Investigating the cream-skimming issue is useful for the debate over private school subsidisation. Frustration over the quality of public schools has led to policies such as private-school vouchers. Such policies are expected to increase educational choice and school competition which would drive inefficient public schools to operate efficiently, thus improving the quality of public schools. However, this argument is challenged by some studies which show that subsidising private schools via vouchers could lower the quality of public schools because the problems of rent seeking behavior and cream-skimming nature of private schools are intensified. This chapter contributes to the debate by confirming that, under the assumptions of the model, the cream-skimming problem persists even when incorporating heterogeneous neighbourhood schools. Therefore, subsidising private schools is likely to intensify the cream-skimming problem, and thus worsening the welfare of students who are left in the public sector as public schools lose relatively higher ability students to the private sector.

The second chapter also contributes to the debate by arguing that price subsidisation for private education can be a Pareto improving policy if (i) school quality is measured by levels of educational services, (ii) private education is more costly per unit, (iii) the level of educational services of public schools within a jurisdiction is determined by majority voting of the residents, and (iv) the housing capacities of jurisdictions cannot accommodate perfect segregation among heterogeneous households. A simple two-jurisdiction and two-income type model is developed to address the point. The second chapter differs from the first chapter by abstracting from heterogeneous public schools within a jurisdiction and from peer effects. To some extent, this abstraction is arguably reasonable. Compared to levels of educational services such as per-student educational expenditure, information about

schools' peer-quality, generally measured by average test scores, may not be equally accessed among households and may be complicated to translate into meaningful information. Moreover, given that all schools tend to receive the same per-student educational expenditure within a jurisdiction, if an open enrolment policy is implemented all neighbourhood schools are expected to have the same peer-quality, otherwise students will have incentives to move to a better peer-quality public school. Another difference is unequal effectiveness in providing school quality. Assuming that education is a divisible good, the second chapter assumes that public educational providers charge a lower unit price than private providers because they are less costly to operate. When housing capacities of jurisdictions cannot accommodate perfect segregation between high and low income types, some high income households who demand high educational services inevitably have to live in the low income jurisdiction, where educational expenditure is smaller, and opt out for pricier private education. This creates a possibility of Pareto-improving price subsidisation for private education. Under the assumptions of the model, the theoretical results show that using an income tax applied to all high income households regardless of their educational sector choice can be used to finance the policy. Pareto-improving subsidisation exists and it is either full subsidisation, which eliminates the unit price difference between two sectors, or partial subsidisation, that maximises the total welfare. This last result is the main contribution of this chapter. In addition, an important observation from the first and the second chapters is that information about efficiency of private schools is crucial for the rationale of private school subsidisation.

The third chapter studies school and residential choices when there are frictions in housing markets and agents only value school quality during the early stages of their lives, such as when they have school-age children. It is natural in housing markets that frictions are present. It takes time for buyers to find suitable sellers

and vice versa. Frictional housing markets are ignored in studies of school and residential choices to allow for the focus on endogenous school or educational quality. The third chapter instead assumes exogenous public school quality of each location and focuses on how agents relocate themselves according to different valuations of such local amenities. An overlapping-generation model is developed to address the problem. The results show that there exist steady-state equilibria in which young agents who value school quality and live in a location with a low-quality public school have a potential to move to another location with a better public school. With frictions, some of such young agents who are willing to relocate get stuck in the low-quality school location and obtain relatively low life-time utility. For individuals, the benefits of moving into a good school location are not only derived from school quality but also from the resale value of the house once school services are no longer valued. Increasing the quality of low-quality schools improves the total welfare but affects agents across locations differently. In addition, when relatively good quality private schools are available, the young house buyers are better off while some old house owners are worse off due to a reduction in the returns from house sales. These results point out that frictions matter to the effects of changes in school quality on the relocation and welfare of heterogeneous agents.

# Chapter 2

# A Model of Competition between Neighbourhood Schools and Private Schools

## 2.1 Introduction

Attending a public school is generally free of charge but may require students to reside in the school attendance zone. Empirical studies have shown that housing prices are significantly higher in locations where the measure of public school quality is higher.[1] This capitalisation of school quality suggests that public school students may have to pay some premium (or implicit tuition) through housing prices when attending a high quality neighbourhood school. Students with high enough household income then can reside in the zones of high quality schools and then segregate themselves from poorer students. Private schools, on the other hand, charge tuition, set their own admission policy, and impose no residential requirement. This

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[1]See Black (1999) and Black and Machin (2011) as well as references therein. Examples of school attendance zones are illustrated in Figure I in Black (1999), for a case of Massachusetts, U.S., and Figure 2 in Davidoff and Leigh (2008), for a case of the Australian Capital Territory.

allows them to generate their preferred student body. Competition among these neighbourhood schools and private schools results in student sorting. Measuring school quality by the average standard test score of the student body, students in a high quality school tend to have parents with high socio-economic advantage. In this sense, private schools may be seen as cream-skimmers who attract relatively high income and ability students. Also, private school students outperform public school students when comparing the average test scores between sectors.[2]

Unlike previous studies that treat all public schools as identical within a jurisdiction, this chapter incorporates both heterogeneous neighbourhood schools and private schools into the same model. It shows that private schools are still superior in the ability to cream skim relatively richer and higher ability students even though neighbourhood schools also generate segregation among public school students. The theoretical results confirm higher school quality in the private sector.

This chapter provides a theoretical model that addresses how households choose their residence and children's school when neighbourhood schools and private schools coexist. The analysis also shows how neighbourhood school zones and flexible tuition and admission policies of private schools induce a sorting of students in each sector. The model is built on Epple and Romano (1998)'s single jurisdiction model by adding a housing market with identical houses and public school zones. In the model, students differ in their ability and household income. Neighbourhood schools charge zero tuition but impose residential requirement, while private schools can admit students from any locations and are free to set their tuition and admission policies conditional on income and ability. A school's quality is determined by the average ability of the student body. The housing market is frictionless and consists of identical houses with a reservation price to facilitate the analysis of how housing

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[2]See the appendix for the relevant stylised facts. For the proper test of cream-skimming, see Epple et al. (2004).

prices are affected by school quality. All schools are equally effective in providing any given school quality. The main theoretical findings are as follows. First, in the private sector, there is a hierarchy of private school qualities and a student sorting with stratification by income and ability. Second, in the public sector, there is a hierarchy of neighbourhood school qualities and student sorting with stratification by income if an ability elasticity of demand for educational quality is weakly positive and there is a positive correlation between income and ability. Also, high quality neighbourhood schools locate in high housing-price neighbourhoods. Third, private schools have better quality than neighbourhood schools. These results suggest that, compared to neighbourhood schools, private schools are still able to cream skim relatively richer and higher ability students. This has a policy implication that introducing private school voucher programs would create a decrease in quality of public schools due to the likeliness of losing relatively high ability students who now can afford private education.

The results of this chapter depend on three important assumptions. The first assumption is that the utility function satisfies a strictly single crossing in income condition ($SCI$). Intuitively, under $SCI$, a household with higher income but the same ability child is willing to pay more for a given level of school quality. With freedom to set tuition and admission policies conditional on students' types, private schools can internalise peer effects into their tuition. The student body of a private school then consists of richer but lower ability students who pay higher tuition to subsidise poorer but higher ability students. This cross-subsidisation explains student sorting with stratification by income and ability in the private sector. A hierarchy of private school qualities results from the fact that if there is an equal quality of any two private schools, either of them can increase profits by expelling some of its relatively lower income and lower ability students. Then, the school admits some of the relatively higher income and higher ability students who value

quality more from the other school by the same amount. The new student body will have higher quality and the school can charge higher tuition while the total cost remains unchanged. The profits then increase. This violates a condition that in equilibrium no private school can increase profits by changing their tuition and admission policies.

The second and the third important assumptions are a weakly single crossing in ability condition ($WSCB$) and a positive correlation between income and ability, respectively. These two assumptions are crucial for the equilibrium properties of the public sector. Intuitively, $WSCB$ ensures that a household with the same income but higher ability child is not willing to pay less for a given level of school quality. A positive correlation between income and ability combining with $WSCB$ and $SCI$ guarantees that differentiated neighbourhood schools exist. This is because high income households, which also tend to have high ability children, can segregate themselves from poorer households by living in a neighbourhood with higher rent since their neighbourhood school also have higher peer quality. Immediately, student sorting with stratification by income is implied.

The result that private schools have better quality than neighbourhood schools comes from the fact that private schools can internalise peer effects in their pricing while, under identical housing, each neighbourhood school costs their students the same rent premium regardless of their types. Therefore, if there is a neighbourhood school with a higher quality than a private school, the private school then can expel their relatively poorer and lower ability students and admit only students with higher income and higher ability, who also value quality more, from the neighbourhood school by the same amount. The new student body of the private school will have higher quality and higher tuition can be charged while the total cost remains unchanged. Again, profits increase, violating the equilibrium condition.

The chapter relates to studies on economics of public and private education. A

set of papers focuses on allocation in the public sector. For example, De Bartolome (1990) constructs a two-community model in which families differ in children ability but not income and there are peer-group externalities. He shows that, when the level of educational inputs in each community is determined by voting, the equilibrium is inefficient and land prices fail to internalise the externalities. Fernandez and Rogerson (1996) develop a multi-community model with different income households but departing from peer-group effects. They show that welfare improvement can be achieved by redistribution toward the poorest and by increasing the educational spending as well as the attractiveness of the poorest community. Arnott and Rowse (1987) focus on the social planner's allocation of students and educational expenditures across classrooms in the presence of peer-group effects, given a distribution of student abilities and a fixed budget. Epple and Romano (2003) provide an extensive analysis of neighbourhood schools and the effects of finance policies. Their model predicts a sorting of neighbourhood school qualities, that is, a higher quality school locates in a higher rent neighbourhood. The results of the public sector in this chapter are essentially the same as in Epple and Romano (2003) since they also utilise the $SCI$ and identical houses assumptions. None of these studies, however, has a private sector.

Another set of papers considers the consequences of the private sector in providing education. The existence and properties of equilibria when majority-voting, tax-financed public school expenditure coexists with private alternatives are studied in Stiglitz (1974), Glomm and Ravikumar (1992), and Epple and Romano (1996). One of the important points of these papers is that when there are private alternatives some high income households may find it optimal to opt for private education if the majority-voting educational expenditure of the public sector is too low for their preference. The opt-out behavior of the rich implies higher educational inputs or quality in the private sector. These papers, however, focus on endogenous pub-

lic educational expenditure and competition between sectors while abstract from differentiated schools. In contrast, this chapter departs from endogenous public expenditure but focuses on competition among differentiated schools. Similar results are obtained, that is, opt-out households are those who have relatively richer and higher ability children.

Some studies develop models featuring coexistence of public and private education to analyse the consequences of educational policies such as vouchers that subsidise private school households. Notable papers, among others, are Epple and Romano (1998, 2008), Nechyba (1999, 2000, 2003), and Ferreyra (2007). Nechyba's multi-district model has households with different income and student peer quality and houses with different desirability. Each community then has two main features, which are its (majority-voting) public school spending and housing quality to attract households. Modeling this way facilitates his analysis to focus on the effects of migration across and within communities induced by private school vouchers. Based on the model of Nechyba (1999), Ferreyra (2007) adds religious private schools and then estimates the model. The estimates are used to simulate two voucher programs, universal and nonsectarian private school vouchers, and show their effects on school and residential choices and school qualities. These multi-district models with endogenous public school expenditure and heterogeneous housing are complex and have to rely on numerical results. This chapter abstracts from such settings to focus on how neighbourhood schools, which is not modeled in these studies, compete with private schools in a single jurisdiction.[3] This simplification allows the model to be able to highlight properties of equilibrium without relying on numerical results.

Epple and Romano (1998) develop a model of competition between public and private schools within a district and analyse the effects of universal vouchers. Stu-

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[3]A model featured multiple jurisdictions and different effectiveness of public and private sectors will be discussed explicitly in the next chapter.

dents in the model differ in ability and household income. Moreover, private schools can set tuition conditional on ability and income while public schools charge zero tuition without residential requirement. A school's quality is determined by the peer-group quality measured by the average ability of the student body. The theoretical results show a strict hierarchy of school qualities and stratification of students by ability and income. All public schools have the same quality which is lower than any private school quality in equilibrium because private schools have the ability to cream skim relatively richer and higher ability students. Unlike Nechyba's model, equilibrium properties of Epple and Romano's theoretical model can be shown without relying heavily on numerical results.

The model in this chapter is built on the model of Epple and Romano (1998). Also, this chapter differs by adding the implementation of neighbourhood schools, which is absent in their paper. The equilibrium properties of the private sector in this chapter confirm the main finding in Epple and Romano (1998), that is, private schools are cream-skimmers who attract richer and higher ability students, thus having higher school qualities than any neighbourhood schools. The reason that these results remain unchanged is that each neighbourhood school still cannot price discriminately their students while private schools can. The cream-skimming problem persists. Thus, this implies that voucher programs without restricting the cream-skimming problem would create a decrease in quality of public schools because those who utilise the vouchers are former public school students who have relatively higher income and ability, as shown in Epple and Romano (1998, 2008). In addition, Epple et al. (2004) have tested the predictions of Epple and Romano (1998)'s model, which the model in this chapter is built on, and show that the cream-skimming problem is supported by the data. Specifically, the propensity to opt out for private schooling rises with income and student ability. Within the private sector the propensity to attend the highest-tuition private schools also

11

increases with both income and ability. Their empirical findings confirm the cream-skimming result of this chapter.

The chapter proceeds by describing the model in the next section. Section 2.3 discusses properties of equilibrium. Section 2.4 discusses the results and policy implications. The last section is a conclusion.

## 2.2   The Model

The model features households with different income and children's ability, coexistence of public and private schools, residential requirement for attending public schools, and peer-group quality as the measure of school quality. The model is built on Epple and Romano (1998)'s model. The settings are essentially equivalent. The minor difference is that school zones are applied along with identical houses in the frictionless housing market.

### 2.2.1   Households

There is a continuum of households of measure one. Each household endows with an income $y$ and one school-age child with ability $b$. Let $f(b, y)$ be the joint marginal distribution assumed to be continuous and positive on its support, $S \equiv (0, \bar{b}] \times (0, \bar{y}]$. All households have the same utility function, $U$, which is assumed to be continuous, increasing, and twice differentiable in numeraire consumption and educational achievement. Educational achievement is defined by a function $a = a(\theta, b)$, which is continuous and increasing in both arguments, where $\theta$ is the measurement of school quality determined by the mean ability of the student body in the attending school. Assume that all households strictly prefer free public education than no schooling. Let $y_t = (1 - t)y$ denote after-tax income, $p$ denote tuition ($p = 0$ if attending a public school), and $r$ denote rent of the house the household lives in.

The utility function is then given by $U = U(y_t - p - r, a(\theta, b))$. Assume that $U$ satisfies a strictly single crossing in income condition ($SCI$):

$$\partial\left(\frac{\partial U/\partial\theta}{\partial U/\partial y_t}\right)/\partial y_t > 0, \tag{2.1}$$

which implies a positive income elasticity of demand for educational quality for all qualities and for all types. In other words, preference over school quality depends on income. In addition, $U$ satisfies a weakly single crossing in ability condition ($WSCB$) if

$$\partial\left(\frac{\partial U/\partial\theta}{\partial U/\partial y_t}\right)/\partial b \geq 0. \tag{2.2}$$

Similarly, $WSCB$ corresponds to a weakly positive ability elasticity of demand for educational quality. In other words, preference over school quality may depend on ability.

The timing of a household's decision involves two stages. In the first stage, taking the income tax rate and the school choice as given, a household chooses where to live and pays the rent, and then the housing market clears. In the second stage, given the residence, the household chooses which school his child attends, either the neighbourhood school or a private school. Then, the household pays income taxes to finance public schools regardless of the school sector choice.

## 2.2.2 Schooling

All schools, both public and private, have the same cost function:

$$C(k) = V(k) + F \tag{2.3}$$

where $V$ is a variable cost function depending only on the number of students ($k$), and $F$ is a fixed cost. The marginal cost is positive and increasing, $V' > 0, V'' > 0$.

## *Public Schools*

Attending a public school is free of charge ($p = 0$) but requires students to live in the corresponding school zone. This kind of public schools will be called neighbourhood schools. The model assumes that all neighbourhood schools have the same size and each has its school zone which is not overlapped with the others. Moreover, there are no exogenous geographical neighbourhood boundaries. Rather the model assumes that a school zone is a neighbourhood and its size is equal to the cost-minimising number of students. In other words, school zones are flexible to adjust so that, given the demand for public education, all neighbourhood schools have the same number of students. Let $\hat{N}$ and $\hat{k}$ denote the cost-minimising number of neighbourhood schools and its student number, respectively. Both variables are determined by $\min_{\{\hat{N}, \hat{k}\}} \hat{N}(C(\hat{k}) + F)$, subject to $\hat{N}\hat{k} = X$, where $X$ is a given total number of students demanding public education. In this setting, there will be $\hat{N}$ neighbourhoods that each has a public school. Let $N$ denote the total number of neighbourhoods, therefore $\hat{N} + 1 = N$. One can think of the $N$-th neighbourhood as one large neighbourhood without a public school because of no demand for it. The set of neighbourhoods is then denoted by $\{1, 2, ..., \hat{N}, N\}$.

Public schools are financed by income taxes. The tax rate $t$ is set to balance the government budget, given $\hat{N}$ and $\hat{k}$:

$$t \iint\limits_{S} yf(b, y)db\,dy = \hat{N}(C(\hat{k}) + F). \tag{2.4}$$

## *Private schools*

Private schools maximise profits by choosing admission and tuition policies which can be conditional on student types assumed to be observable. This allows for price discrimination in the private sector. Private schools can admit students from any locations. Private schools are also free to enter and exit which implies a zero profit

condition.

Let a subscript $i$ denote the $i$-th private school, $i = 1, 2, ..., M$, where $M$ is the total number of private schools. The total number of schools is then $M + \hat{N}$. Let a subscript $n \in \{1, 2, ..., \hat{N}, N\}$ denote the $n$-th neighbourhood, and a subscript $0n$ denote neighbourhood $n$ with a public school (when $n \in \{1, ..., \hat{N}\}$). Given the residential choices of households and other schools' policies, the problem of a private school $i$ is to choose the school quality ($\theta_i$), the number of students ($k_i$), the tuition policy ($p_i(b, y)$) and the admission policy ($\alpha_i(b, y)$) for each type, to maximise profits:

$$\max_{\theta_i, k_i, p_i(b,y), \alpha_i(b,y)} \pi_i \equiv \iint_S [p_i(b, y)\alpha_i(b, y) \times f(b, y)db\, dy] - V(k_i) - F \qquad (2.5)$$

$$\text{subject to} \qquad \alpha_i(b, y) \in [0, 1], \forall(b, y); \qquad (2.6)$$

$$U(y_t - p_i(b, y) - r_n, a(\theta_i, b)) \geq \max U(y_t - p_j(b, y) - r_n, a(\theta_j, b)), \forall(b, y); \qquad (2.7)$$

$$j \in \{\{1, ..., M\} \cup \Gamma | j \neq i, \alpha_j(b, y) > 0$$

is in the optimal set of j,

$$\Gamma = \{0n\} \text{ if } n \in \{1, ..., \hat{N}\}, \text{ otherwise } \Gamma = \varnothing\}$$

$$k_i = \iint_S \alpha_i(b, y)f(b, y)db\, dy; \qquad (2.8)$$

$$\theta_i = \frac{1}{k_i} \iint_S b\alpha_i(b, y)f(b, y)db\, dy. \qquad (2.9)$$

Constraint (2.6) imposes non-negativity of school $i$'s admission policy, that is, $\alpha_i(b, y)$ is a non-negative fraction of type $(b, y)$ that private school $i$ would admit. Constraint (2.7) is a utility taking assumption. To admit a student, private school $i$ has to set the tuition such that the utility of the household is at least as high as the utility getting from the best alternative, given the neighbourhood the household

15

lives in. The school alternatives for a student are: all private schools, and the neighbourhood school, if any. Constraints (2.8) and (2.9) are the calculations of the school size and the mean ability (school quality), respectively.

The private-public sector equilibrium is characterised by the following conditions.

Utility maximisation (UM):

$$
\begin{aligned}
U^*(b, y) \;=\; & \max U(y_t - p_i(b, y) - r_n, a(\theta_i, b)), \; \forall (b, y), \\
& i \in \{\{1, ..., M\} \cup \Gamma \mid \alpha_i(b, y) > 0 \\
& \text{is in the optimal set of i,} \\
& \Gamma = \{0n\} \text{ if } n \in \{1, ..., \hat{N}\}, \text{ otherwise } \Gamma = \varnothing\}.
\end{aligned}
$$

Profit maximisation (ΠM):

$$
[\theta_i, k_i, p_i(b, y), \alpha_i(b, y)] \quad \text{satisfy (2.5), for } i = 1, 2, ..., M.
$$

Zero-profit condition (ZM):

$$
\pi_i = 0, \; i = 1, 2, ..., M.
$$

Public sector policy (PSP):

$$p_{0j}(b,y) \;=\; 0,\; \forall (b,y)$$

$$\alpha_{0j}(b,y) \;=\; 1,\; \forall (b,y) \text{ living in } j \text{ and } U^*(b,y) = U(y_t - r_j, a(\theta_{0j}, b))$$

$$\alpha_{0j}(b,y) \;\in\; [0,1],\; \forall (b,y) \text{ living in } j \text{ and } U^*(b,y) > U(y_t - r_j, a(\theta_{0j}, b))$$

$$\alpha_{0j}(b,y) \;=\; 0,\; \forall (b,y) \text{ living outside } j$$

$$k_{0j} \;=\; \iint_S \alpha_{0j}(b,y) f(b,y) db\, dy = \hat{k}$$

$$\theta_{0j} \;=\; \frac{1}{k_{0j}} \iint_S b\alpha_{0j}(b,y) f(b,y) db\, dy$$

where $j = 1, 2, ..., \hat{N}$.

Market clearing (MC):

$$\sum_{i=1}^{M} \alpha_i(b,y) + \sum_{j=1}^{\hat{N}} \alpha_{0j}(b,y) \;=\; 1,\; \forall (b,y).$$

Condition UM states that a household chooses the school, either the neighbourhood school or a private school, that maximises her utility, taking the residence with the rent $r_n$, admission and tuition policies, school qualities, and the tax rate as given. Condition ΠM is the profit maximisation condition for private schools. With the free entry and exit assumption, the zero profit condition (ZM) is imposed in equilibrium. Condition PSP summarises public school policies. Each neighbourhood school charges zero tuition and admits all students who choose public education and live in the school zone. The student body and the mean ability are calculated the same way as in the case of private schools. Lastly, the market clearing condition (MC) ensures no household prefers no schooling.

### 2.2.3   Housing

The model focuses on a jurisdiction (district) level. The total housing capacity is normalised to one which is equal to total number of households. Each household can occupy only one house by renting. Housing is identical. All houses are owned by landlords who live outside the model and operate in the competitive housing market. A landlord can dismiss the tenant and re-rent the house with no cost. Also, the opportunity cost of a house is $c \geq 0$, which can be interpreted as the best outside option of a house. Thus, the rent in neighbourhood $n$ has to be at least $c$, $r_n \geq c$. The rent of each house in the same neighbourhood is assumed to be equal.[4] To avoid complication, the model assumes that $c$ is low enough to allow all households to afford a house in equilibrium. Without loss of generality, the model later sets $c = 0$ to facilitate the proofs of some equilibrium properties. There are also no property taxes, and no transportation cost in the district, that is, households are perfectly mobile.[5]

## 2.3   Equilibrium

In this section, the definition of equilibrium is defined. Then, the solution of the private school problem and the equilibrium properties will be stated.

**Definition** An equilibrium is a set $\{\theta, k, p(b, y), \alpha(b, y), t, r\}$ such that

(i) given $r$ and $t$, $\{\theta, k, p(b, y), \alpha(b, y)\}$ satisfies condition UM, ΠM, ZΠ, PSP, and MC,

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[4]In the same neighbourhood, suppose there is a house cheaper than others. Households can offer the landlord of that house $r + \varepsilon$, $\varepsilon > 0$. Since all houses have the same attributes and the landlord can dismiss the tenant with no cost by assumption, the house will then belong to the new household with such higher offer. In equilibrium, it has to be the case that no household has incentive to move. Therefore, an inequality of rents in the same neighbourhood violates such condition.

[5]Adding an exogenous property tax rate ($\tau$) will not qualitatively change the properties of equilibrium. See the discussion section of this chapter.

(ii) given $\{\theta, k, p(b,y), \alpha(b,y), t\}$, $r$ clears the housing market, and

(iii) $t$ balances the government budget (2.4).

In equilibrium, no private school can increase profits by changing its tuition and admission policies. In addition, no household can increase its utility by moving within or across neighbourhoods.

## 2.3.1 Solution to the Private School Problem

Given that each household takes the tax rate and the rent of the house as given when making the school choice, the solution to private school $i$'s problem requires:[6]

$$U(y_t - p_i^*(b,y,\theta_i) - r_n, a(\theta_i, b)) = U^*(b,y), \ \forall (b,y); \qquad (2.10)$$

$$\alpha_i(b,y) = \left\{ \begin{array}{l} = 0 \\ \in [0,1] \\ = 1 \end{array} \right\} \text{ as } p_i^*(b,y,\theta_i) \left\{ \begin{array}{l} < \\ = \\ > \end{array} \right\} V'(k_i) + \eta_i(\theta_i - b); \qquad (2.11)$$

$$\eta_i = \frac{1}{k_i} \iint\limits_S \left[ \frac{\partial p_i^*(b,y,\theta_i)}{\partial \theta_i} \alpha_i(b,y) \times f(b,y) db\, dy \right]. \qquad (2.12)$$

Even though school zones are added to the model, the solution to the private school problem is essentially the same as in Epple and Romano (1998). Condition (2.7) and UM hold with equality, and the optimal tuition policy $p^*(b,y,\theta)$ is given by (2.10). For a type $(b,y)$ attending private school $i$ with quality $\theta_i$, $p_i^*(b,y,\theta_i)$ is the reservation price that makes the student just indifferent between school $i$ and her best alternative. The optimal admission policy is characterised by (2.11). Let the effective marginal cost be denoted by $EMC_i(b) = V'(k_i) + \eta_i(\theta_i - b)$. Admitting a student with type $(b,y)$ not only increases school $i$'s marginal cost $(V'(k))$, but also changes the revenue of the school due to a change in $\theta_i$ through peer effects $(\eta_i(\theta_i - b))$. In condition (2.12), the Lagrangian multiplier of constraint

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[6]See the appendix for the derivation of the solution.

(2.9), $\eta_i$, is the average revenue change due to a change in $\theta_i$. As $\eta_i$ is constant for each private school, the effective marginal cost is only a function of ability (denoted by $EMC_i(b)$). Also, $EMC_i(b)$ is decreasing in $b$. A student type $(b, y)$ with her reservation price lower than her effective marginal cost is not admitted, that is, $\alpha_i(b, y) = 0$ if $p_i^*(b, y, \theta_i) < EMC_i(b)$. The school admits any number of the type, $\alpha_i(b, y) \in [0, 1]$, if $p_i^*(b, y, \theta_i) = EMC_i(b)$, and admits all of the type with the reservation price above the effective marginal cost, that is, $\alpha_i(b, y) = 1$ if $p_i^*(b, y, \theta_i) > EMC_i(b)$. For any public school $j$, the model assumes that $EMC_{0j} = 0$, and $\eta_{0j} = 0$ since the students see zero cost of public education once they already reside and pay rent in the neighbourhood and take the tax rate as given.

## 2.3.2   Properties of Equilibrium

The model assumes the existence of an equilibrium where public and private schools coexist.[7] The model obtains essentially equivalent equilibrium properties to Epple and Romano (1993) and Epple and Romano (1998) with respect to sorting in school qualities and stratification of students by income and ability. Specifically, private schools have better quality than public schools (see Proposition 2). Sorting in housing prices according to differentiated neighbourhood school qualities (see Proposition 1) occurs just as in Epple and Romano (2003). Therefore, only new or relevant proofs will be shown in this chapter.[8] The main contribution here, perhaps trivial, is to show that, in equilibrium the ability to cream skim relatively richer and higher ability students of private schools does not change when public schools are modeled as neighbourhood schools, given that the housing market is frictionless and consists of identical houses. Now, the equilibrium properties of neighbourhood

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[7]See the appendix of Epple and Romano (1998) for a discussion on the existence problem in this kind of model (club economies). Exact equilibrium may fail to exist because the number of schools is an integer.

[8]For very details of the proofs refer to Epple and Romano (1993, 1998, 2003).

schools are considered.

**Proposition 1.** *For neighbourhood schools $1, 2, ..., \hat{N}$, if $\theta_{01} > \theta_{02} > ... > \theta_{0\hat{N}}$, then*

*(i) there is a sorting of rents, $r_1 > r_2 > ... > r_{\hat{N}} = c$,*

*(ii) equilibrium exhibits stratification by income (SBI): given the same student ability, if a household with income $y$ chooses $\theta_{0j}$ and a household with income $y'$ chooses $\theta_{0j'}$ with $\theta_{0j'} > \theta_{0j}$, then $y' > y$,*

*(iii) if condition (2.2) holds with inequality (that is, strictly single crossing in ability), stratification by ability (SBA) obtains in equilibrium: having the same income, if a household with student ability $b$ chooses $\theta_{0j}$ and a household with student ability $b'$ chooses $\theta_{0j'}$ with $\theta_{0j'} > \theta_{0j}$, then $b' > b$. If (2.2) holds with equality, residential choice is invariant to student ability,*

*(iv) student types on a boundary locus (measure zero) between two adjacent neighbourhood schools are indifferent between attending either school and strictly prefer their neighbourhood school within the boundary, and*

*(v) differentiated neighbourhood schools exist if preferences satisfy $WSCB$ and $E[b|y]$ is increasing in $y$.*

*Proof.* See the appendix. □

Essentially, Proposition 1 shows that if there is a positive correlation between ability and income along with $WSCB$, there will be a sorting in neighbourhood school qualities associated with a sorting in rental prices. This indicates capitalisation of neighbourhood school qualities into housing prices. With differentiated school qualities, rental prices have to vary positively with the corresponding neighbourhood school qualities to prevent the incentive for any public school households to move (Proposition 1(i)). Stratification by income ($SBI$) is guaranteed by $SCI$ while stratification by ability ($SBA$) arises only when condition (2.2) holds in inequality. As all houses in the same neighbourhood cost the same rent, continuity

of the utility function implies that household types on a boundary are indifferent between two adjacent neighbourhood schools. A boundary locus is a horizontal straight line in a $(b, y)$-space if $SBA$ does not arise since the willingness to pay for a given school quality varies only by income. If $SBA$ obtains, the boundary locus has negative slope because higher ability students would like to pay more for a given school quality. Before heading to equilibrium properties of the private sector, a property on housing of private school households is noted.

**Lemma 1.** *All private school households live in the lowest rent houses.*

*Proof.* See the appendix. □

The property that private school households live in the lowest-rent houses is trivial. This is because the tie between public school quality and residential requirement is disintegrated and no other utility can be obtained from living in a specific house or a specific neighbourhood. Households who pay rent in excess of $c$ are those who want to separate themselves to create a better peer-group quality in neighbourhood schools. School zones in this sense facilitate such segregation among public school students. Next consider the equilibrium properties of the private sector.

**Proposition 2.** *For private schools $1, 2, ..., M$,*

*(i) there is a sorting of private school qualities, $\theta_1 > \theta_2 > ... > \theta_M$,*

*(ii) on a boundary locus between two adjacent private schools, $p = EMC$. Within the boundary, $p > EMC$,*

*(iii) if all private schools set $p = EMC$, then every student attends a private school that would maximise utility,*

*(iv) equilibrium is characterised by $SBI$, and*

*(v) if $WSCB$ is satisfied and $\eta_1 \geq \eta_2 \geq ... \geq \eta_M$, then $SBA$ characterises the equilibrium.*

*Proof.* See the appendix.                                                                                            □

Proposition 2 is essentially the main finding in Epple and Romano (1998). The private sector has a strict sorting in school qualities. If there is an equal quality of any two private schools, either of them can increase profits by expelling some of its relatively lower income and ability students and by admitting some relatively higher income and ability students, who value quality more, from the other school. The new student body will have higher quality and the school can charge higher tuition while the total cost remains unchanged. This violates the equilibrium condition that no private school can increase profits by changing its tuition and admission policies. Pricing on a boundary locus ($p = EMC$) between two adjacent schools implies that the slope of the locus is negative in a $(b, y)-$space because $EMC_i(b)$ is decreasing in $b$. The ability to set tuition conditional on both income and ability (price discrimination) ensures that private schools are able to internalise peer externalities into their tuition policy ($p \geq EMC$). Freedom to set tuition allows private schools to extract any utility surplus obtained by some students relative to their best alternative ($p > EMC$). $SBI$ characterises the equilibrium due to $SCI$ while $SBA$ characterises the equilibrium if $WSCB$ holds and $\eta_1 \geq \eta_2 \geq ... \geq \eta_M$. Now it is possible to examine the main contribution of this chapter, whether the existence of cream-skimming private schools is a property of equilibrium when public schools are modeled as neighbourhood schools.

**Proposition 3.** *All private schools have better quality than public schools, $\theta_i > \theta_{0j}, \forall i \in \{1, 2, ..., M\}$ and $\forall j \in \{1, 2, ..., \hat{N}\}$.*

*Proof.* See the appendix.                                                                                            □

It turns out that cream-skimming private schools still dominate neighbourhood schools in terms of quality. Intuitively, tuition and admission policies of private schools are more flexible than those of neighbourhood schools in their ability to

attract relatively higher income and ability students. Peer externalities are internalised into private school tuition, thus creating cross-subsidisation of private schools. With $SCI$, households with relatively higher income but lower ability students are willing to pay higher tuition to subsidise relatively lower income but higher ability students. Private schools then attract relatively high income and ability students, thus producing higher quality than public schools. A neighbourhood school is able to discriminate between students only via the rent of the neighbourhood. This may be seen as an implicit tuition which is fixed for all types in the school. Therefore, neighbourhood schools cannot internalise peer externalities and lose the ability to compete over students with very high ability but too poor to pay the fixed positive rent. If there is a neighbourhood school with a higher quality than a private school, the private school can expel its relatively poorer and lower ability students and admits the same number of students with higher income and ability, who also value quality more, from the neighbourhood school. The new student body of the private school will have higher quality, and then higher tuition can be charged while the total cost remains unchanged. Again, this would violate the equilibrium condition. Hence, cream-skimming private schools are still a property of equilibrium even if school zones are applied.

**Proposition 4.** *The public-private sector equilibrium is not Pareto efficient.*

*Proof.* See the appendix. □

As discussed in Proposition 6 from Epple and Romano (1993), the solution of the planner's problem requires all schools to charge tuition equal to the effective marginal cost. This effective marginal cost pricing is socially optimal because it includes both marginal costs and peer-group externalities. The public-private sector equilibrium with school zones deviates from the effective marginal cost pricing in the following ways. First, the tax rate is set to only cover the total cost of operating

public schools. Second, public schools charge zero tuition, $p = 0$. Third, rents as implicit tuition are set to just make the types on the boundary of two neighbourhood schools indifferent between attending both schools, but not to internalise peer-externalities as everyone in the same neighbourhood pays the same rent. For some public school students in the public-private sector equilibrium, the sum of their income taxes, tuition, and rent is not equal to the effective marginal cost of the school they attend. Hence, the public-private sector equilibrium is not Pareto efficient.

**Corollary 1.** *There is no mixing of private and public school households except possibly in the neighbourhood with the lowest public school quality.*

*Proof.* See the appendix. $\square$

Corollary 1 arises naturally since, under the assumptions on housing supply, identical houses, and the competitive housing market, private-schooling households live only in the lowest rent neighbourhoods. Students in the worst-quality neighbourhood school also have the same incentive to not paying any rent premium beyond $c$. Therefore, the housing capacity of each school zone has to be $\hat{k}$ except the zone of the lowest-quality neighbourhood school. This is so because an increase in the housing capacity of the zone to include houses where private school students live does not alter the lowest rent. This property is somewhat consistent with the result from Nechyba (1999, 2000) whose model features multiple districts and heterogeneous housing types. Private school households who are relatively richer and prefer high-quality housing arise first in the neighbourhood with the lowest public school quality because the property tax burden from supporting such school is lowest. The intuition here is similar. Given the housing attributes, the private school households seek the lowest possible rent to avoid any unnecessary burden once the residential requirement for attending a public school is irrelevant.

25

Figure 2.1 shows an example of the equilibrium rents related to school qualities when there are three $\hat{k}$-size neighbourhood schools (with qualities $\theta_{01}, \theta_{02}, \theta_{03},$) and a single private school (with quality $\theta_1$). A mixing of public and private school households in the neighbourhood with the lowest public school quality is possible, and its size (or the school zone) is bigger than $\hat{k}$.
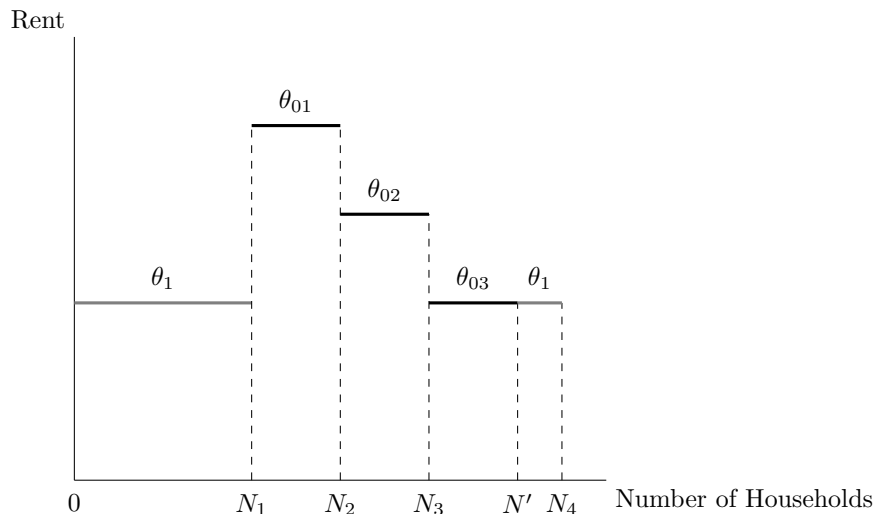


Notes: Here, $\theta_1 > \theta_{01} > \theta_{02} > \theta_{03}$ are school qualities, and the gray and black lines are private and public school households, respectively. The interval $[0, N_1]$ is the neighbourhood of private school households. The interval $[N_1, N_2]$ is the neighbourhood of the highest public school quality. The interval $[N_2, N_3]$ is the neighbourhood of the medium public school quality. Lastly, the interval $[N_3, N_4]$ is the neighbourhood of the lowest public school quality.

Figure 2.1: An Example of a Rent Distribution of Imperfect Housing Segregation

## 2.4 Discussion

The theoretical results of the model confirm that, in equilibrium, private school quality is higher than neighbourhood school quality. Also, a hierarchy of neighbourhood school qualities with differentiated rents and a hierarchy of private school qualities segregated by income and ability are the properties of equilibrium. These results suggest that even if school zones are applied private schools are still able to cream skim relatively richer and higher ability students, thus generating higher

quality than public schools. This cream-skimming private sector is the main result in Epple and Romano (1998). In addition, Epple et al. (2004) have tested the predictions of Epple and Romano (1998)'s model, the model on which this chapter is built upon, and show that the cream-skimming problem is supported by the data. Specifically, the propensity to opt out for a private school rises with income and ability of students. Within the private sector, the propensity to attend the highest-tuition private school also increases with both income and ability. Their empirical findings confirm the cream-skimming result of this chapter.

Two important assumptions are worth discussing. The first assumption is that all schools have the same cost function which depends only on the number of students. In reality, schools may also choose spending per student to attract students. Epple and Romano (2008), based on Epple and Romano (1998) as in this chapter, assume that schools also choose spending per student. School quality is determined by both peer quality and spending per student. An increase in spending per student increases not only school quality but also the total cost. Nevertheless, they show that sortings in school qualities and the superiority of private schools over public schools are still the case. Therefore, to reduce complexity of the model but maintaining the same properties about sorting, this chapter assumes that the cost function depends only on the number of students and school quality is determined only by peer quality as in Epple and Romano (1998). In addition, this assumption points out that even though all schools are equally effective in terms of costs, the cream skimming problem still arises because of residential requirement of public schools and flexible admission and tuition policies of private schools.

The second assumption is identical housing. This assumption is obviously not innocuous because households with different characteristics would demand different housing services and, in reality, there are differences in housing sites in each neighbourhood. This chapter is not aimed to explain the housing market compre-

27

hensively since, under identical housing, rental prices in the model act like membership fees of neighbourhoods. However, a main point made here is that, given a housing quality preference, private school households, who are relatively richer and have higher ability children, seek to live in a neighbourhood with the lowest burden (housing price). Nechyba (1999, 2000) allow for heterogeneity in housing quality and household characteristics while maintaining homogenous school qualities within a jurisdiction. My work obtains similar outcomes as in Nechyba's work. In his papers, high income households demand higher quality education and housing. They are the first group to opt out for private education, and thus living in the high quality housing in the jurisdiction with the lowest property tax burden from supporting unutilised public schools. This chapter assumes identical housing in order to obtain tractability and to obtain analytical results.

Adding an exogenous property tax rate ($\tau$), used to support public schools, to the model will not change the properties of equilibrium qualitatively because when making residential and school choices, households take the income tax rate as given. With the property tax households will just consider the rental prices including taxes, i.e., $R_n = (1 + \tau)r_n$. However, the income tax rate will be different.[9]

Using a numerical model, Epple and Romano (1998) analyse the effects of universal vouchers, financed by a flat tax rate, on various variables such as the number of students in each sector, welfare gains, achievement gains, and voting support for vouchers. They found that, for positive voucher values, a majority experience relatively small losses and a minority experience relatively large gains. Households who lose welfare generally are those left in the public sector. This is because vouchers make some types of public students near the boundary between two sec-

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[9]Specifically, the government balanced budget becomes

$$t \iint_S yf(b,y)db\,dy + \hat{k}\sum_{j=1}^{\hat{N}} \tau r_j + (1 - \hat{N}\hat{k})\tau r_N \quad = \quad \hat{N}(C(\hat{k}) + F).$$

tors able to attend private schools. As private schools are cream-skimmers, they admit relatively high-income and high-ability public school students, resulting in lower peer-group quality in the public sector. Therefore, such a voucher policy is not Pareto improving.

In the presence of school zones as in this chapter, universal vouchers financed by a flat tax rate would not be Pareto improving. It is simply because students in the worst neighbourhood school will be worse off for any positive value vouchers corresponding with a higher tax rate. The lowest rent in the worst quality school neighbourhood is unchanged, but with lower peer-quality they are surely worse-off. Hence, under the setting in this chapter, there is no room for a policy aimed at subsidising private school students without a restriction on tuition that can correct inefficiency in the public-private sector equilibrium. In recent research on voucher design, Epple and Romano (2008) extend the baseline model in Epple and Romano (1998) by allowing private schools to have higher efficiency in providing school quality and show that Pareto improvement can be achieved by restricting private schools to accept ability-linked vouchers as tuition without any further charges. This motivates the next chapter. It will discuss a possibility of Pareto improving policy in the presence of different effectiveness of public and private sectors in providing educational services.

## 2.5   Conclusion

Equilibrium properties of a model with private and neighbourhood schools are studied in this chapter. The model modifies the work of Epple and Romano (1998) by adding school zones and a frictionless housing market. Households make school and residential choices to maximise utility. Private schools maximise profits by choosing admission and tuition policies. Public schools charge no tuition but require

residency in the school zones. Under the setting that public and private schools have the same effectiveness in providing school quality, the theoretical results confirm sorting in school qualities and sorting in rental prices as found in Epple and Romano (1998) and Epple and Romano (2003), respectively. Private schools are still able to cream skim relatively richer and higher ability students, leaving public schools with relatively poorer and lower peer quality even if school zones are introduced. Private school households pay the lowest rent because residential requirement is irrelevant, all houses have the same attributes, and no other utility can be obtained from living in a specific location. A neighbourhood with high public school quality also has a high rent, implying capitalisation of public school qualities. These results suggest that price subsidisation, such as vouchers, to private school households could not be a Pareto improving policy when public and private schools are equally effective. Thus, research should focus more understanding on what environment leads to a Pareto improving policy aimed at subsidising private education households. One such situation may be when there is a difference in effectiveness of public and private sectors in providing educational services, which is the focus of the next chapter.

## 2.6 Appendix

## Stylised Facts of Public and Private schools

## 1. Students in private schools outperform students in public schools in terms of average achievements measured by standard test scores.

This fact is generally observed in the data. For example, in the U.S., the National Assessment of Education Progress (NAEP), which measures the knowledge of students in a variety of subject areas, has reported that students in private schools have higher average scores than students in public schools over the last 30 years. The latest NAEP report that focuses on student achievement in private schools with details of some important private school types is summarised in Perie et al. (2005). In the report, private schools outperform public schools in reading, mathematics, science, and writing. For example, for grade 8 in 2003, private schools have higher average scale score which is significantly different from public schools in both reading and mathematics (Figure 2.2). Also, the proportion of students in private schools who are at or above Basic and Proficient knowledge levels is higher than of public schools. In addition, Catholic, Lutheran, and Conservative Christian private schools outperform public schools in the same manner to the aggregative private sector. This fact is also observed in OECD and non-OECD countries, and within a jurisdiction.[10] Note that this fact does not claim that all private schools are better than public schools, but rather only when considering at the sector level. In fact, Figure 2.3 partly shows that public and private schools have overlapped qualities in terms of standard test scores.

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[10]The country-level data can be generated from http://pisa2009.acer.edu.au. Within a jurisdiction data, see Table 2.1 for the case of South Australia.

Reading

Scale score

300
250
200

| | Public | Private | Catholic | Lutheran | Conservative Christian |
|---|---|---|---|---|---|
| Scale score | 261 | $282^d$ | $281^{c,d}$ | $281^{c,d}$ | $276^{a,b,d}$ |

Type of school

| % at or above | Public | Private | Catholic | Lutheran | Conservative Christian |
|---|---|---|---|---|---|
| Basic | 72 | $90^d$ | $90^d$ | $90^d$ | $86^d$ |
| Proficient | 30 | $53^d$ | $51^d$ | $52^d$ | $46^d$ |

Mathematics

Scale score

300
250
200

| | Public | Private | Catholic | Lutheran | Conservative Christian |
|---|---|---|---|---|---|
| Scale score | 276 | $292^d$ | $289^{b,d}$ | $296^{a,c,d}$ | $286^{b,d}$ |

Type of school

| % at or above | Public | Private | Catholic | Lutheran | Conservative Christian |
|---|---|---|---|---|---|
| Basic | 67 | $82^d$ | $81^d$ | $86^d$ | $78^d$ |
| Proficient | 27 | $43^d$ | $39^{b,d}$ | $48^{a,c,d}$ | $37^{b,d}$ |

[a] Significant different from Catholic schools

[b] Significant different from Lutheran schools

[c] Significant different from Conservative Christian schools

[d] Significant different from public schools

Note: The NAEP reading and mathematics scale ranges from 0 to 500. Data for Other Private schools are included in the overall Private data but not reported separately.

Source: Figure 5, Perie et al. (2005)

Figure 2.2: Average Scale Scores and Achievement-Level Results by Type of School, Grade 8: 2003, U.S.

## 2. Private schools attract socio-economically advantaged students.

Within countries private school students tend to be socio-economically advantaged. Differences between students' socio-economic backgrounds of public and private schools are significant in a majority of countries in PISA 2009 data.[11] For instance, countries such as Brazil, Columbia, Poland, Argentina, United States, New Zealand, United Kingdom, Spain, Canada, and Australia have the difference at or above OECD average, while countries such as Sweden, Switzerland, Belgium, Germany, Thailand, Japan, and Denmark are below OECD average.

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[11]Students' socio-economic backgrounds are proxied by the PISA (Programme for International Student Assessment) index of economic, social, and cultural status. See OECD (2012) for the significance of the index.

Table 2.1: Average Year-7 NAPLAN Scores of Primary Schools in Metropolitan Areas of South Australia

|  |  |  | Public | Catholic | Independent | Private# | All |
|---|---|---|---|---|---|---|---|
| Average Year-7 NAPLAN Score | Mean | 2008 | 531.3 | 546.2 | 566.8 | 551.1 | 536.3 |
|  |  | 2009 | 536.1 | 538.8 | 527.5 | 536.1 | 536.1 |
|  |  | 2010 | 537.7 | 541.2 | 535.8 | 539.9 | 538.3 |
|  |  | 2011 | 532.7 | 532.6 | 523.0 | 530.3 | 532.1 |
|  | S.D. | 2008 | 30.4 | 18.9 | 18.5 | 20.6 | 29.5 |
|  |  | 2009 | 28.8 | 35.9 | 27.5 | 34.2 | 30.2 |
|  |  | 2010 | 28.8 | 29.8 | 23.7 | 28.4 | 28.6 |
|  |  | 2011 | 26.8 | 32.0 | 28.9 | 31.3 | 28.0 |
| ICSEA* | Mean | 2008 | 1006.9 | 1017.9 | 1038.3 | 1022.8 | 1010.9 |
|  |  | 2009 | 1008.1 | 1026.6 | 995.5 | 1019.2 | 1010.9 |
|  |  | 2010 | 1019.9 | 1028.6 | 1006.3 | 1023.3 | 1020.8 |
|  |  | 2011 | 1018.7 | 1030.8 | 1007.6 | 1025.3 | 1020.4 |
|  | S.D. | 2008 | 86.5 | 66.6 | 63.0 | 65.8 | 81.9 |
|  |  | 2009 | 79.3 | 91.3 | 80.9 | 89.3 | 81.9 |
|  |  | 2010 | 73.4 | 87.3 | 83.9 | 86.3 | 76.7 |
|  |  | 2011 | 69.3 | 86.3 | 80.4 | 84.8 | 73.5 |
| Correlation** |  | 2008 | 0.7 | 0.5 | 0.6 | 0.5 | 0.7 |
|  |  | 2009 | 0.7 | 0.7 | 0.8 | 0.7 | 0.7 |
|  |  | 2010 | 0.8 | 0.8 | 0.7 | 0.8 | 0.8 |
|  |  | 2011 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 |
| Number of Schools*** |  |  | 173 | 45 | 14 | 59 | 232 |

# Catholic and independent schools

* Index of Community Socio-Educational Advantage

** Correlation of average NAPLAN scores and ICSEAs

*** Only primary schools (no combined) that have complete data from 2008-2011, which are 232 out of 292 (the full sample)

Data Source: Australian Curriculum, Assessment and Reporting Authority (ACARA)

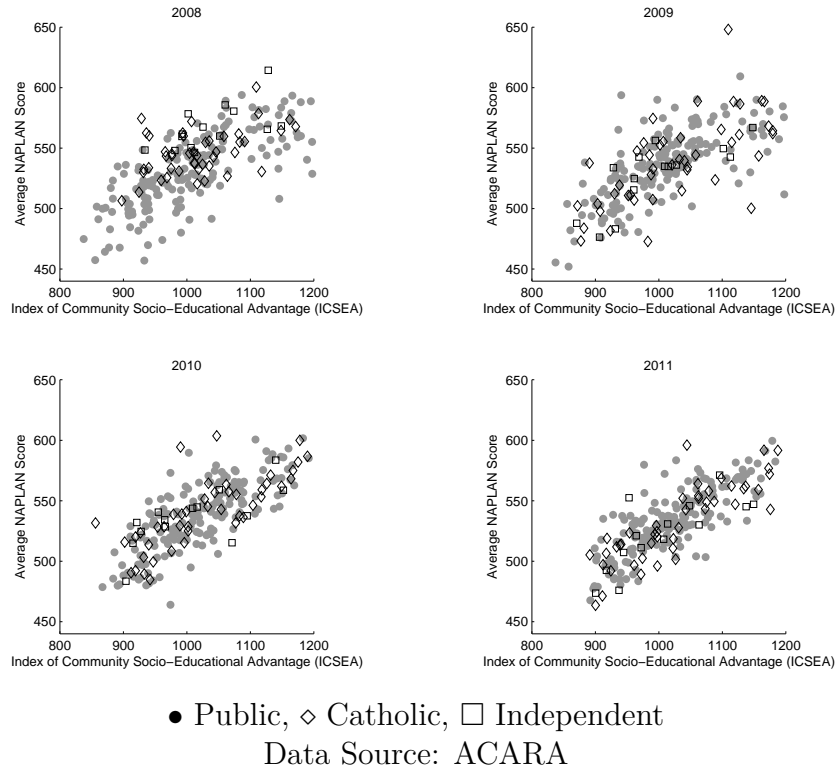● Public, ◇ Catholic, □ Independent
Data Source: ACARA

Figure 2.3: Average Year-7 NAPLAN Scores and ICSEAs of Primary Schools in Metropolitan Areas of South Australia



Data Source: ACARA
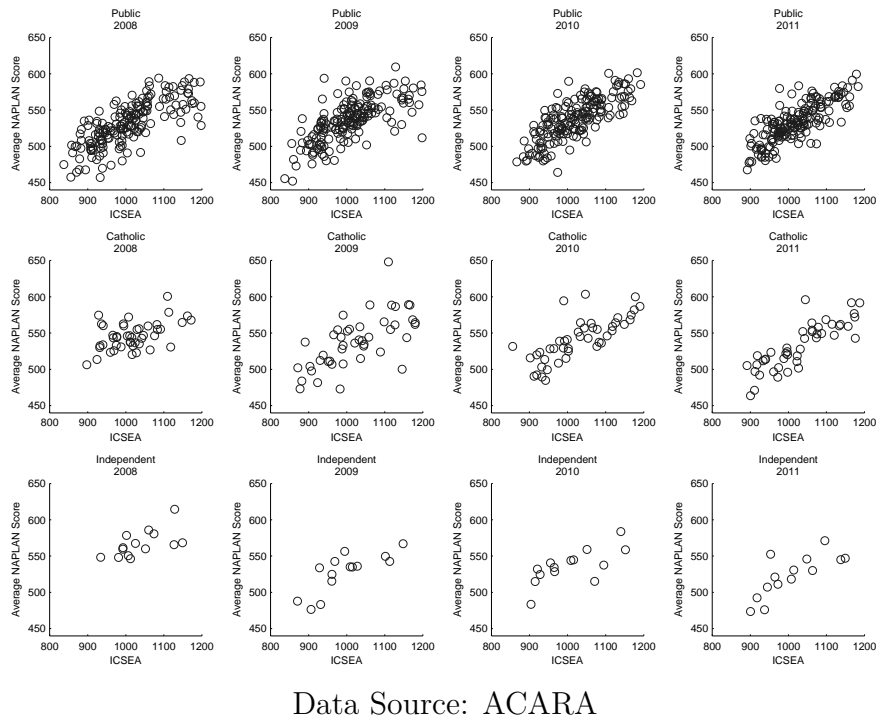
Figure 2.4: Average Year-7 NAPLAN Scores and ICSEAs by Type of School

In the case of the U.S., Figure 2.5 shows that students in private schools tend to have parents with higher education. Only 46% of public school students report that their parents graduated from college while overall, across private schools, the figure is around 72% which is significantly higher than public schools. Students with highly educated parents in Catholic, Lutheran, and Conservative Christian private schools are observed to have similar outcomes. In the case of South Australia, students in private schools also tend to have socio-educational advantage (measured by ICSEA) over students in public schools (Table 2.1).

Students who have socio-economic advantage tend to perform well in schools. In the case of the U.S., looking closely into each parents' education type, students in private schools have higher performance compared to students in public schools (Figure 2.6), and the differences are also significant. In addition, from the data provided by the Australian Curriculum, Assessment and Reporting Authority (ACARA), Figure 2.3 plots average year-7 NAPLAN scores and Index of Community Socio-Educational Advantage (ICSEA) of primary schools in metropolitan areas of South Australia, from 2008-2011.[12] The figure clearly shows a positive correlation between students' performance measured by NAPLAN scores and students' backgrounds measured by ICSEA in all 4 years. This fact carries through within school types as well (Figure 2.4). Table 2.1 shows the summary statistics of NAPLAN scores and ICSEAs and their correlations. With high positive correlations around 0.7 and 0.8, it is clear that schools, whether public or private, that serve higher socio-educationally advantaged students tend to have higher student performances. Again, this fact only points out that private school students tend to have socio-economic advantage, when compared between two sectors. It does not make this claim at the individual or school level. In fact, the levels of socio-economic

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[12]The National Assessment Program – Literacy and Numeracy (NAPLAN) is an annual assessment for students in Years 3, 5, 7 and 9. The tests are in 4 areas, reading, writing, language conventions, and numeracy. In addition, ICSEA is measured by parents' occupation and level of education completed.

## Parents' highest level of education
### (Grade 8:2003, U.S.)

Type of school

| | | | | | |
|---|---|---|---|---|---|
| Public | 7 | 18 | 18 | 46 | 11 |
| Private | $1^d$ $9^d$ | $13^d$ | | $72^d$ | $5^d$ |
| Catholic | $1^d$ $9^d$ | $15^d$ | | $70^d$ | $5^d$ |
| Lutheran | $1^d$ $11^d$ | 15 | | $68^d$ | $6^d$ |
| Conservative Christian | $2^d$ $11^d$ | 16 | | $65^d$ | $7^d$ |

0    20    40    60    80    100

Percent

- Less than high school
- Graduated from high school
- Some education after high school
- Graduated from college
- Unknown

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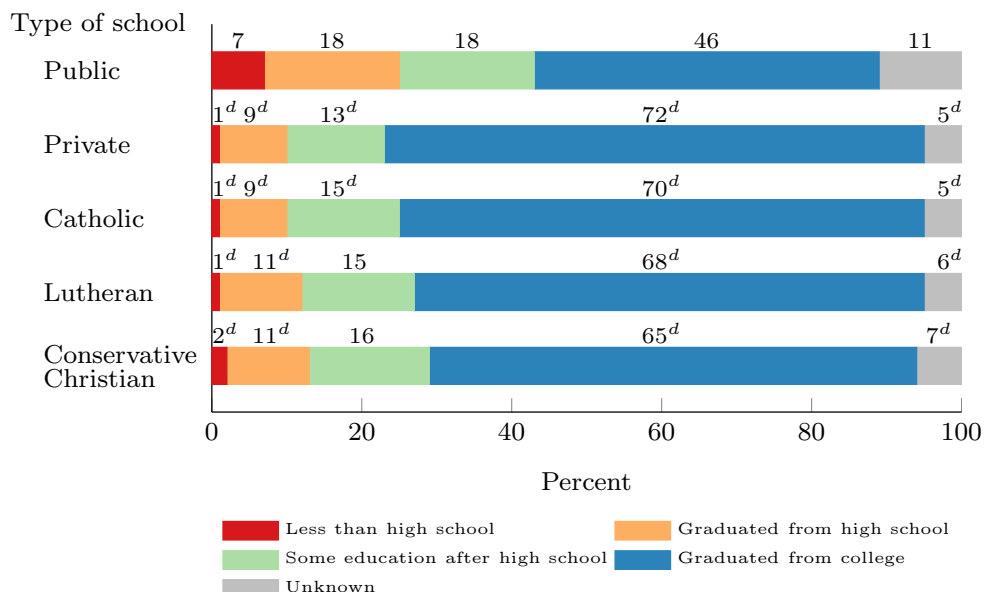
$^d$ Significant different from public schools

Note: Details may not sum to totals because of rounding. Data for Other Private schools are included in the overall Private data but not reported separately.

Source: Figure 2, Perie et al. (2005)

Figure 2.5: Percentage Distribution of Students Who Participated in Reading Assessment, by Student-Reported Parents' Highest Level of Education and Type of School, Grade 8: 2003, U.S.
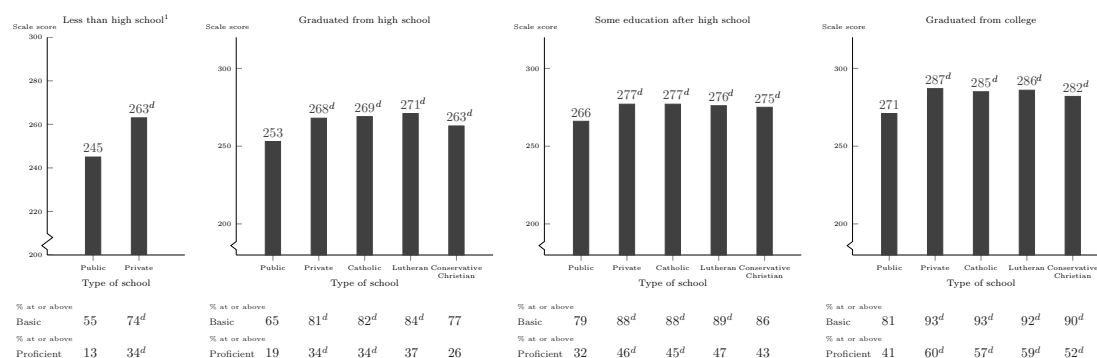


$^d$ Significant different from public schools
$^1$ Data for Catholic, Lutheran, and Conservative Christian are not shown because sample sizes are insufficient to permit reliable estimates of these categories for students whose parents' reported education level is less than high school.
Note: The NAEP reading scale ranges from 0 to 500. Data for Other Private schools are included in the overall Private data but not reported separately.
Source: Figure 13, Perie et al. (2005)

Figure 2.6: Average Scale Scores and Achievement-Level Results in Reading, by Student-Reported Parents' Highest Level of Education and Type of School, Grade 8: 2003, U.S.

advantage of public and private schools are overlapped. Still, this general observation may show some degree of student segregation by household characteristics within and across sectors. The proper test of stratification by income and ability of students within and across sectors requires the data set containing individual-level data as well as geographical data of schools. This chapter does not access such data set and thus can only provide some general observation. For the proper test with such characteristics of the data, see Epple et al. (2004) for a case in the U.S..

## 3. ACARA data description

The data of schools in South Australia used in this thesis are sourced from the Australian Curriculum, Assessment and Reporting Authority (ACARA) and are available from ACARA in accordance with its Data Access Protocols. The filtered data set contains only primary schools (no combined) in metropolitan areas of South Australia and only the schools that have complete data from 2008-2011. This would make sure that these schools compete for students by offering similar educational services in a reasonable locational space.

# Solving a Private School's Maximisation Problem

$$\underset{\alpha_i(b,y),p_i(b,y),k_i,\theta_i}{MAX} \iint_S p_i(b,y)\alpha_i(b,y)f(b,y)db\,dy - V(k) - F \quad (2.13)$$

$$\text{subject to} \quad \alpha_i(b,y) \in [0,1]; \quad\quad\quad\quad\quad\quad (2.14)$$

$$U(y_t - p_i(b,y) - r_n, a(\theta_i,b)) \geq \max U(y_t - p_j(b,y) - r_n, a(\theta_j,b)), \forall(b,y); \quad (2.15)$$

$$j \in \{\{1,...,M\} \cup \Gamma \,|\, j \neq i, \alpha_j(b,y) > 0$$

$$\text{is in the optimal set of j,}$$

$$\Gamma = \{0n\} \text{ if } n \in \{1,...,\hat{N}\},$$

$$\text{otherwise } \Gamma = \varnothing\}$$

$$k_i = \iint_S \alpha_i(b,y)f(b,y)db\,dy; \quad\quad\quad\quad (2.16)$$

$$\theta_i = \frac{1}{k_i} \iint_S b\alpha_i(b,y)f(b,y)db\,dy. \quad\quad\quad\quad (2.17)$$

In equilibrium, admitting a types $(b,y)$ requires (2.15) holds with equality, $U(y_t - p_i^*(b,y,\theta_i) - r_n, a(\theta_i,b)) = U^*(b,y)$. Thus, $p_i^*(b,y,\theta_i)$, the reservation price that makes type $(b,y)$ indifferent between attending school $i$ and her best alternative, is determined.

Using $k_i = \iint_S \alpha_i(b,y)f(b,y)db\,dy$, the Lagrangian is then:

$$\mathcal{L} = \iint_S p_i^*(b,y,\theta_i)\alpha_i(b,y)f(b,y)db\,dy - V(\iint_S \alpha_i(b,y)f(b,y)db\,dy) - F$$

$$+ \iint_S \lambda_i(b,y)\alpha_i(b,y)\left\{U(y_t - p_i^*(b,y,\theta_i) - r_n, a(\theta_i,b)) - U^*(b,y)\right\}f(b,y)db\,dy$$

$$+\beta_i\left[\theta_i \iint_S \alpha_i(b,y)f(b,y)db\,dy - \iint_S b\alpha_i(b,y)f(b,y)db\,dy\right]$$

$$+\Omega_i(b,y)[1 - \alpha_i(b,y)]f(b,y)db\,dy$$

$$+\omega_i(b,y)\alpha_i(b,y)f(b,y)db\,dy.$$

Therefore, the first-order conditions are:[13]

$$\frac{\partial \mathcal{L}}{\partial \alpha_i} = [p_i^* - V'(k) + \lambda_i(U^i - U^*) \tag{2.18}$$

$$+ \beta_i(\theta_i - b) - \Omega_i + \omega_i] \times f = 0;$$

$$\frac{\partial \mathcal{L}}{\partial \theta_i} = \iint_S \frac{\partial p_i^*}{\partial \theta_i} \alpha_i f \, db \, dy + \beta_i k_i = 0; \tag{2.19}$$

$$\frac{\partial \mathcal{L}}{\partial p_i} = \alpha_i f + \lambda_i \alpha_i \frac{\partial U^i}{\partial p_i} f = 0; \tag{2.20}$$

$$\Omega_i[1 - \alpha_i] = 0 \quad \text{and} \quad \Omega_i \geq 0; \tag{2.21}$$

$$\omega_i \alpha_i = 0 \quad \text{and} \quad \omega_i \geq 0. \tag{2.22}$$

From (2.19), $\beta_i = -\frac{1}{k_i} \iint_S \frac{\partial p_i^*}{\partial \theta_i} \alpha_i f \, db \, dy$. Substituting $\beta_i$ into (2.18) yields

$$p_i^* = V'(k_i) - \beta_i(\theta_i - b) + \Omega_i - \omega_i$$

$$= V'(k_i) + \left[\frac{1}{k_i} \iint_S \frac{\partial p_i^*}{\partial \theta_i} \alpha_i f \, db \, dy\right](\theta_i - b) + \Omega_i - \omega_i.$$

Denote $\eta_i = [\frac{1}{k_i} \iint_S \frac{\partial p_i^*}{\partial \theta_i} \alpha_i f \, db \, dy]$, then

$$p_i^* = V'(k_i) + \eta_i(\theta_i - b) + \Omega_i - \omega_i. \tag{2.23}$$

Using (2.21), (2.22), and (2.23), the admission policies can be summarised as follows:

- $\Omega_i = 0, \omega_i = 0$, therefore $\alpha_i \in [0, 1]$ and $p_i^*(b, y, \theta_i) = V'(k_i) + \eta_i(\theta_i - b)$;

- $\Omega_i = 0, \omega_i > 0$ therefore $\alpha_i = 0$ and $p_i^*(b, y, \theta_i) < V'(k_i) + \eta_i(\theta_i - b)$;

- $\Omega_i > 0, \omega_i = 0$ therefore $\alpha_i = 1$ and $p_i^*(b, y, \theta_i) > V'(k_i) + \eta_i(\theta_i - b)$.

These three policies are summarised in (2.11) in the main text.

---

[13]Type indicators are dropped to shorten the derivation.

## Proof of Proposition 1

The proofs here are essentially equivalent to those of Propositions 1 and 2 in Epple and Romano (2003).

(i) Given the same tax rate, it has to be the case that in equilibrium a household attending neighbourhood school $j$ has $U(y_t - r_j, a(\theta_{0j}, b)) \geq U(y_t - r_{j'}, a(\theta_{0j'}, b))$ for all $j \neq j'$. Suppose $\theta_{0j'} > \theta_{0j}$ but $r_{j'} \leq r_j$. Then, $U(y_t - r_{j'}, a(\theta_{0j'}, b)) > U(y_t - r_j, a(\theta_{0j}, b))$, a contradiction. The neighbourhood with the lowest school quality has the lowest possible rent, $r_{\hat{N}} = c$, because of the assumptions of enough supply of houses, identical houses, and the competitive housing market.

(ii) From Proposition 1(i), if $\theta_{0j'} > \theta_{0j}$, then $r_{j'} > r_j$. In the model, indifference curves are upward sloping in the $(\theta, r)$-plane. Utility is increasing when moving an indifference curve to the lower right. As shown in Figure 2.7, condition $SCI$ implies that, with the same student ability, a household with higher income has steeper slopes of indifference curves. No households with lower income than $y$ attend neighbourhood school $j'$. Also, households whose students attend neighbourhood school $j'$ have higher income than $y$ such as $y'$.
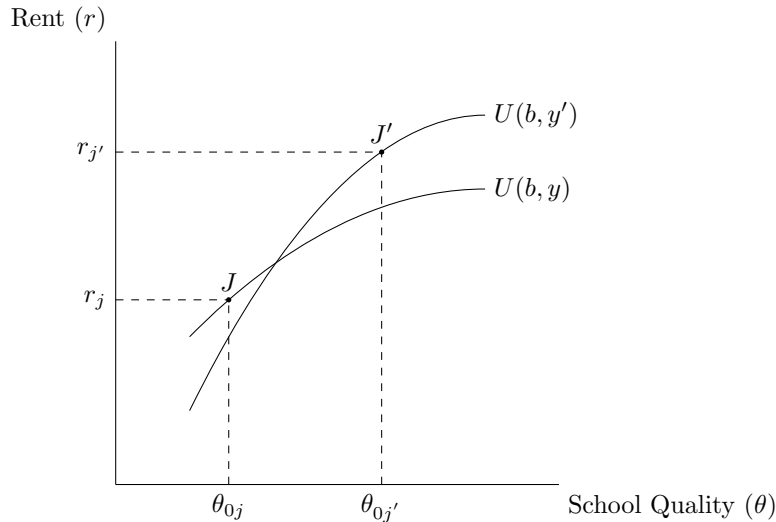


Figure 2.7: $SBI$ in the Public Sector

(iii) This proposition can be analogously proven by the proof of 1(ii) above. Instead of having the same ability but different income, it is the case of having the same income but different ability. If $WSCB$ holds with equality, willingness to pay for school quality does not change with ability, therefore residential choices are invariant to student ability.

(iv) Since the utility function is continuous and all households living in the same neighbourhood pay the same rent, the proof is implied. The size of a boundary locus has to be zero, otherwise $SCI$ will be violated. Suppose the contrary that the size is positive. From (2.1), $SCI$ implies that indifference curves of higher income households are everywhere steeper than of lower income, given the same ability, in a $(\theta, r)$-plane as illustrated in Figure 2.8. Consider a type $(b, y)$ on the boundary who must be indifferent between two neighbourhood schools, say $j$ and $j'$ with $\theta_{0j'} > \theta_{0j}$, and $r_{j'} > r_j$ (Proposition 1(i)). Every student in each school pays the same implicit cost which is the rent. Now pick another type with higher income but the same ability, say, $(b, y')$, $y' > y$, *in the interior* of the locus (since it has a positive size). If $SCI$ holds, type $(b, y')$ would have higher utility when consuming at point $J'$ (that is, moving an indifference curve to the southeast increases utility). Maintaining utility indifference (that is, between point $J$ and $J'$ for type $(b, y')$) on the boundary means that the slope of $U(b, y')$ cannot be greater than that of $U(b, y)$. The strictly single-crossing condition is then violated.

(v) In any neighbourhood school $j$, the mean ability of the school is:

$$
\begin{aligned}
\theta_{0j} &= \frac{\int_{y_{j-1}}^{y_j} \int_{b_m}^{b_x} b f(b, y) db \, dy}{\int_{y_{j-1}}^{y_j} \int_{b_m}^{b_x} f(b, y) db \, dy} = \frac{\int_{y_{j-1}}^{y_j} E[b|y] \times [\int_{b_m}^{b_x} f(b, y) db] \, dy}{\int_{y_{j-1}}^{y_j} \int_{b_m}^{b_x} f(b, y) db \, dy} \\
&= \frac{\int_{y_{j-1}}^{y_j} E[b|y] \times [\int_{b_m}^{b_x} f(b, y) db] \, dy}{\hat{k}}
\end{aligned}
$$

where $y_j$ and $y_{j-1}$ are the maximum and the minimum income types of households
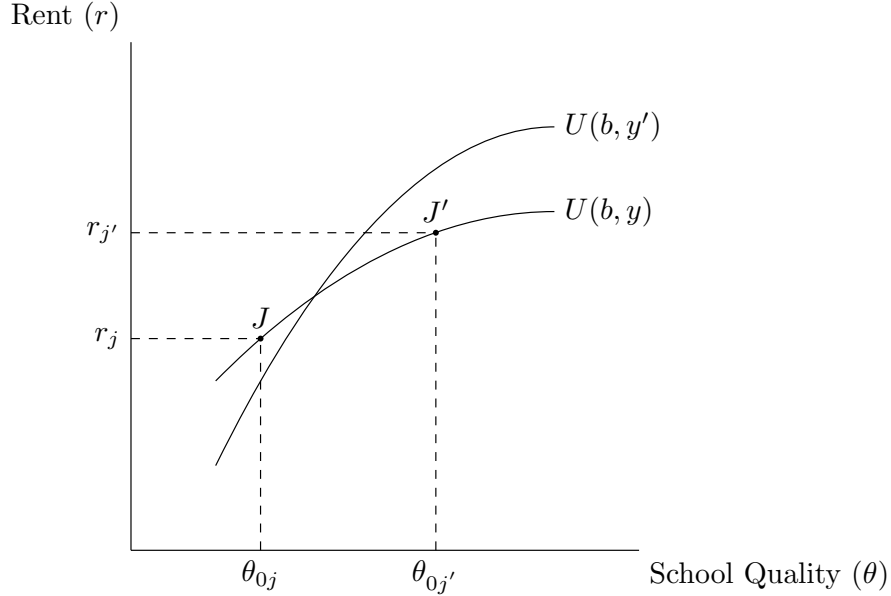
Figure 2.8: $SCI$

living in neighbourhood $j$, and $b_m$ and $b_x$ are the minimum and the maximum ability types, respectively. Since the school quality is determined exclusively only by the mean ability, $\theta_{01} > \theta_{02} > ... > \theta_{0\hat{N}}$ if $E[b|y]$ is increasing in $y$. The types on the boundary between two adjacent neighbourhoods are indifferent between the two neighbourhood schools:

$$U((1-t)y_{j-1} - r_j, a(\theta_{0j}, b)) \;\; = \;\; U((1-t)y_{j-1} - r_{j-1}, a(\theta_{0(j-1)}, b)). \quad (2.24)$$

If $\theta_{0j} > \theta_{0(j-1)}$, then $r_j > r_{j-1}$.

## Proof of Lemma 1

Private school households seek the lowest possible rent because the residential requirement for attending a neighbourhood school is irrelevant. By assumption there is enough housing supply for all households, therefore all private school households accept only the lowest possible rent ($c$) which makes landlords just indifferent between renting and getting the outside option. Such lowest rent is charged in the

$N$-th neighbourhood (with no public school) or the $\hat{N}$-th neighbourhood (with the lowest public school quality, from Proposition 1(i)).

## Proof of Proposition 2

The proofs are essentially equivalent to Proposition 1-3 in Epple and Romano (1998). Therefore, the proofs here focus more on the intuition of their proofs. The reason of equivalent results is because the way the school zones and the housing market are incorporated still does not affect the ability of private schools to cream skim richer and higher ability students.

(i) From Lemma 1, private school households pay $c$ for the rent. Without loss of generality, let $c$ equal zero. Intuitively, if two private schools $i$ and $i'$ have the same quality, then they can both compete among each other for all student types perfectly. Then, a private school, say $i$, can increase profits by expelling type $(b_1, y_1)$ and admit type $(b'_2, y'_2)$ from private school $i'$ by the same number (that is, the total cost $V(k) + F$ does not change), where $b_1 < b'_2$, $y_1 < y'_2$, $y'_2 - y_1 > b'_2 - b_1$ such that it is enough to increase $\theta$, and type $(b'_2, y'_2)$ values quality more than type $(b_1, y_1)$ even though they have different abilities. The profits then increase because private school $i$ can charge the new student body higher tuition since $\theta_i$ and $\eta_i$ are increased. This violates the equilibrium condition that no private school can increase profits further by changing its tuition and admission policies.

(ii) If $p < EMC$ at a point on the boundary, then either school does not admit any students in the locality of that point even inside its own admission space, a contradiction. Likewise, if $p > EMC$, then both schools would like to admit students in the locality outside its own admission space with $\alpha = 1$. This violates the market clearing condition (MC).

Within boundary of private school $i$, suppose $p_i(b, y) = EMC_i(b, y)$. Equation (2.11) implies that the alternative school has $p_{i'}(b, y) = EMC_{i'}(b, y)$. Re-

call that $EMC_{0j} = 0$ and let $c$ equal zero. Then, it has to be the case that $U(y_t - EMC_i(b), a(\theta_i, b)) = U(y_t - EMC_{i'}(b), a(\theta_{i'}, b))$ for some school $i'$. If $\theta_{i'} > \theta_i$, for all $\varepsilon \in (0, \epsilon]$, $\epsilon > 0$, all students with types $(b, y + \varepsilon)$ have $\hat{p}_{i'i} > EMC_{i'}$ by $SCI$, where $\hat{p}_{i'i}$ is defined as the tuition that school $i'$ has to charge to attract student type $(b, y)$ away from school $i$.[14] Then, school $i'$ can attract those students, and thus preventing them from attending school $i$. Therefore, $(b, y)$ cannot be a point within boundary. Also, the proof when $\theta_{i'} < \theta_i$ can be obtained analogously.

(iii) If all schools set $p = EMC$, suppose the contrary that a student attends a school $i$ that gives her lower utility than she would get from another school $j$. According to (2.11), school $i$ has to set tuition at least as high as its $EMC$ to admit the student. However, because the student gets higher utility from school $j$ when $p = EMC$, then school $j$ has to set a higher tuition than $EMC$ of school $i$, leading to having strict preference to admit the student. This violates the market clearing condition (MC).

(iv) The proof is similar to Proposition 1(ii). With the same student ability and with tuition equal to $EMC$, the choice between private school $i$ and $i'$ can be represented by the choice between $\{\theta_i, EMC_i(b)\}$ and $\{\theta_{i'}, EMC_{i'}(b)\}$. If $\theta_{i'} > \theta_i$ and if either type chooses $i$, it has to be the case that $EMC_{i'}(b) > EMC_i(b)$. Again, as illustrated in Figure 2.9, households whose students attend a better private school have a higher income according to $SCI$. For instance, $(b, y')$ chooses school $j'$ while $(b, y)$ chooses school $j$, where $y' > y$.

(v) Recall that $EMC_0 = 0$, $\eta_0 = 0$, and let $c$ equal zero. From Proposition 2(iv) and (2.11), define $\tilde{p}_i = V'(k) + \eta_i \theta_i$ as type-independent tuition, also $\tilde{p}_{0j} = 0$. Then, $EMC_i = \tilde{p}_i - \eta_i b$. The first argument of utility function for private school students becomes $y_t - \tilde{p}_i + \eta b$, therefore a type $(b, y)$ facing $EMC_i(b)$ is equivalent to a type $(b, y_t + \eta_i b)$ facing $\tilde{p}_i$. Moreover, if $\eta_1 \geq \eta_2 \geq ... \geq \eta_M > 0$, there exists

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[14]Specifically, $U(y_t - \hat{p}_{i'i}, a(\theta_{i'}, b)) = U(y_t - EMC_i, a(\theta_i, b))$, assuming that school $i$ charges at $EMC$. Also, Figure 2.9 may help illustrate the point.
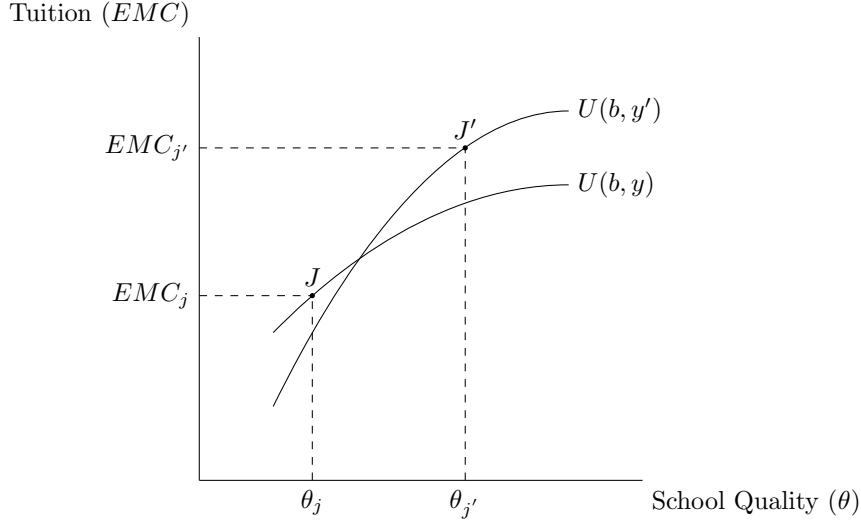
Figure 2.9: $SBI$ in the Private Sector

an infinite number of functions $\tilde{\eta}(\theta) = \eta_i$, $i = 0, 1, ..., M$, which are non-decreasing and differentiable. The maximisation problem of a household can be represented by $\max_{i \in \{0,1,...,M\}} U(y_t - \tilde{p}_i + \tilde{\eta}(\theta_i), a(b, \theta_i))$. Now, to prove that SBA is implied, it has to show that indifference curves in $(\theta, \tilde{p}_i)$ -plane are upward sloping and their slopes are increasing with ability, as a standard single-crossing argument. Indifference curves are upward sloping:

$$\frac{d\tilde{p}}{d\theta}\Big|_{U(.)=\bar{U}} = b\tilde{\eta}' + \frac{(\partial U/\partial a)(\partial a/\partial \theta)}{\partial U/\partial y_t} > 0 \qquad (2.25)$$

since the first term is non-negative and the second term is positive. Differentiating (2.25) again gives

$$\tilde{\eta}' + \tilde{\eta}\frac{\partial}{\partial y_t}\Big[\frac{(\partial U/\partial a)(\partial a/\partial \theta)}{\partial U/\partial y_t}\Big] + \frac{\partial}{\partial b}\Big[\frac{(\partial U/\partial a)(\partial a/\partial \theta)}{\partial U/\partial y_t}\Big]. \qquad (2.26)$$

The first term is weakly positive by $\tilde{\eta}' \geq 0$. By $SCI$, the second term is strictly positive for $\theta > \theta_{0j}$, and weakly positive for $\theta = \theta_{0j}$. The last term is weakly positive by $WSCB$. The slopes of indifference curves are increasing with ability,

45

*SBA* holds.

## Proof of Proposition 3

Let $c$ equal zero, then $r_n \geq c = 0$. From Proposition 2(ii), it can be implied that all private schools are better than the worst neighbourhood school because they pay the same rent, $r = c = 0$. Otherwise, the private sector does not exist since no one would want to pay positive tuition for lower school quality than the neighbourhood school which is free of charge. Now, suppose that there is a neighbourhood school $j$ with a greater or equal quality with a private school $i$, $\theta_{0j} \geq \theta_i$. This neighbourhood school $j$ cannot be the lowest quality with the lowest rent as described above, thus it must be that $r_j > 0$. Next is to show that in equilibrium it cannot be the case that $\theta_{0j} \geq \theta_i$.

First, consider when $\theta_i = \theta_{0j}$. Using (2.11) and UM, $r_j \geq p(b', y') \geq EMC_i(b')$, $\forall (b', y') \in A_i$ and $EMC_i(b) \geq r_j > 0, \forall (b, y) \in A_{0j}$, where $A$ is the admission space of a school. As $r_j$ is equal for all types in $A_{0j}$, then $EMC_i(b) \geq EMC_i(b'), \forall b \in A_{0j}, \forall b' \in A_i$. Given the current student body of private school $i$, this implies that no one in neighbourhood school $j$ has higher ability than those in private school $i$. Therefore, $\theta_i \geq \theta_{0j}$. Hence, for $\theta_i = \theta_{0j}$ to be the case, both schools admit only one and the same type in their student bodies, and private school $i$ operates at the boundary locus where $p = EMC$. Proposition A1 in Epple and Romano (1993) already shows that the size of a boundary locus is zero.[15] Therefore the size of private school $i$ and public school $j$ is zero, meaning that no such schools exist.

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[15]This can be proven analogously by using the proof of Proposition 1(iv). Suppose the contrary that the size is positive. Consider a type $(b, y)$ on the boundary who must be indifferent between two private schools, say $i$ and $j$ with $\theta_i > \theta_j$, that share the locus. From Proposition 2(iii), both schools charge $p = EMC$ on the boundary. Now pick another type with higher income but the same ability, say, $(b, y')$, $y' > y$, *in the interior* of the locus (since it has a positive size). Tuition from the schools sharing the boundary for this type must be the same because $EMC$ depends only on ability. Since both schools have different qualities, maintaining utility indifference of students on the boundary violates the strictly single-crossing condition.

Lastly, consider when $\theta_i < \theta_{0j}$. This implies $r_j > p(b', y') \geq EMC_i(b'), \forall (b', y') \in A_i$ and $EMC_i(b) \lesseqqgtr r_j > 0, \forall (b, y) \in A_{0j}$. For some $(b, y) \in A_{0j}$, $EMC_i(b) \geq r_j$, then $EMC_i(b) > EMC(b')$. These types $(b, y)$ in neighbourhood school $j$ have lower ability than all types in private school $i$. Similarly, for some $(b, y) \in A_{0j}$, $EMC_i(b) < r_j$, then $EMC_i(b) < EMC(b')$. These types $(b, y)$ in neighbourhood school $j$ have higher ability than all types in private school $i$, which means $b > \theta_i$. From $SBI$ (higher income households choose a higher quality school), private school $i$ can increase profits by expelling some type $(b', y')$ and admitting type $(b, y)$ from neighbourhood school $j$ by the same number (that is, $V(k) + F$ does not change), where $b > b'$, $y > y'$, $y - y' > b - b'$, $b \geq \theta_i$ such that it is enough to increase $\theta$ and to ensure that the new student body of private school $i$ will have the quality no less than the original quality of public school $j$, and types $(b, y)$ value quality more than types $(b', y')$ even though they have different abilities. Hence, private school $i$ strictly prefers these types because it can increase profits by charging the new student body higher tuition since $\theta_i$ and $\eta_i$ increase. This violates the market clearing condition.

## Proof of Proposition 4

Assume that the planner does not care about utilities of landlords. Therefore, the planner can just rent all houses at the reservation price ($c$), which is assumed to be zero, and distributes them to households. The social planner problem in this model is then equivalent to the one in Epple and Romano (1993). For the sake of explanation, the equilibrium conditions are derived again here. The proof will show that the public-private sector equilibrium is not Pareto efficient because it is not $N - constrained$ Pareto efficient. Given the number of schools ($N$) and $c = 0$,

for $i = 1, 2, ..., N$, the planner's problem is

$$\underset{\alpha_i(b,y), z_i(b,y), k_i, \theta_i}{MAX} \sum_{i=1}^{N} \iint_S \alpha_i(b,y)\gamma(b,y)U(y - z_i(b,y), a(\theta_i, b))f(b,y)db\,dy \qquad (2.27)$$

subject to

$$\sum_{i=1}^{N} \iint_S z_i(b,y)\alpha_i(b,y)f(b,y)db\,dy \geq \sum_{i=1}^{N}[V(k) + F] \qquad (2.28)$$

$$k_i = \iint_S \alpha_i(b,y)f(b,y)db\,dy \qquad (2.29)$$

$$\theta_i = \frac{1}{k_i} \iint_S b\alpha_i(b,y)f(b,y)db\,dy \qquad (2.30)$$

$$\alpha_i(b,y) \in [0,1], \forall(b,y) \qquad (2.31)$$

$$\sum_{i=1}^{N} \alpha_i(b,y) = 1, \forall(b,y) \qquad (2.32)$$

for some positive weight function $\gamma(b,y)$, and where $z_i(b,y)$ denotes tuition.

The Lagrangian of the above problem is

$$\mathcal{L} = \sum_{i=1}^{N} \iint_{S} \alpha_i(b,y)\gamma_i(b,y)U(y - z_i(b,y), a(\theta_i, b))f(b,y)db\,dy$$

$$+ \lambda \left\{ \sum_{i=1}^{N} \iint_{S} z_i(b,y)\alpha_i(b,y)f(b,y)db\,dy - \sum_{i=1}^{N}[V(k) + F] \right\}$$

$$+ \sum_{i=1}^{N} \beta_i \left[ k_i - \iint_{S} \alpha_i(b,y)f(b,y)db\,dy \right]$$

$$+ \sum_{i=1}^{N} \rho_i \left[ \theta_i k_i - \iint_{S} b\alpha_i(b,y)f(b,y)db\,dy \right]$$

$$+ \sum_{i=1}^{N} \Omega_i(b,y)[1 - \alpha_i(b,y)]f(b,y)db\,dy$$

$$+ \sum_{i=1}^{N} \omega_i(b,y)\alpha_i(b,y)f(b,y)db\,dy$$

$$+ \iint_{S} \phi_i(b,y)[1 - \sum_{i=1}^{N} \alpha_i(b,y)]f(b,y)db\,dy.$$

Denote $U^i = U(y - z_i(b,y), a(\theta_i, b))$, and drop type indicators to shorten the derivation. Then, the first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial \alpha_i} = [\gamma U^i + \lambda z_i - \beta_i - \rho_i b - \Omega_i + \omega_i - \phi] \times f = 0; \quad (2.33)$$

$$\frac{\partial \mathcal{L}}{\partial k_i} = -\lambda V'(k_i) + \beta_i + \rho_i \theta_i = 0; \quad (2.34)$$

$$\frac{\partial \mathcal{L}}{\partial \theta_i} = \iint_{S} \alpha_i \gamma \frac{\partial U^i}{\partial \theta_i} f\, dbdy + \rho_i k_i = 0; \quad (2.35)$$

$$\frac{\partial \mathcal{L}}{\partial z_i} = \alpha_i f[-\gamma \frac{\partial U^i}{\partial(y - z_i)} + \lambda] = 0; \quad (2.36)$$

$$\Omega_i[1 - \alpha_i] = 0 \quad \text{and} \quad \Omega_i \geq 0; \quad (2.37)$$

$$\omega_i \alpha_i = 0 \quad \text{and} \quad \omega_i \geq 0. \quad (2.38)$$

Optimally, (2.28) holds with equality. From (2.35), $\rho_i = \frac{1}{k_i} \iint_{S} \alpha_i \gamma \frac{\partial U^i}{\partial \theta_i} f\, dbdy$. From (2.34), $\beta_i = \lambda V'(k_i) - \rho_i \theta_i = \lambda V'(k_i) - \frac{\theta_i}{k_i} \iint_{S} \alpha_i \gamma \frac{\partial U^i}{\partial \theta_i} f\, dbdy$. Substituting $\rho_i$

and $\beta_i$ into (2.33) yields

$$\gamma U^i + \lambda z_i - \lambda V'(k_i) + \frac{\theta_i}{k_i} \iint\limits_S \alpha_i \gamma \frac{\partial U^i}{\partial \theta_i} f \, dbdy - \frac{b}{k_i} \iint\limits_S \alpha_i \gamma \frac{\partial U^i}{\partial \theta_i} f \, dbdy - \Omega_i + \omega_i - \phi = 0.$$

From (2.36) $\gamma = (\lambda / \frac{\partial U^i}{\partial(y - z_i)})$, then

$$\gamma U^i + \lambda z_i - \lambda V'(k_i) + (\theta_i - b) \frac{1}{k_i} \iint\limits_S \alpha_i \lambda (\frac{\partial U^i}{\partial \theta_i} / \frac{\partial U^i}{\partial(y - z_i)}) f \, dbdy - \Omega_i + \omega_i - \phi = 0$$

$$\gamma U^i + \lambda z_i - \lambda(V'(k_i) + (\theta_i - b)\eta_i) - \Omega_i + \omega_i - \phi = 0$$

$$\gamma U^i + \lambda(z_i - EMC_i) - \Omega_i + \omega_i - \phi = 0.$$

When $\alpha_i > 0$ and all students attend a school, $\Omega_i$, $\omega_i$, and $\phi$ are zeros. Define $I^*(b, y) = \{i \in \{1, 2, ..., N\} | \alpha_i(b, y) > 0 \text{ is optimal}\}$ as the integer set of schools where type $(b, y)$ are efficiently allocated, and $I_c^*(b, y)$ is its compliment set. For all $(b, y)$, it has to be the case that

$$\gamma U^i + \lambda[z_i - EMC_i] = \gamma U^{i'} + \lambda[z_{i'} - EMC_{i'}] > \gamma U^j + \lambda[z_j - EMC_j], \quad (2.39)$$

$\forall i, i' \in I^*$ and $j \in I_c^*$, and

$$\gamma \frac{\partial U^i}{\partial(y - z_i)} = \lambda, \, \forall i \in I^*. \tag{2.40}$$

Conditions (2.39) and (2.40) will be used to prove the inefficiency of the public-private sector equilibrium.

Now turn to the public-private sector equilibrium. Consider a neighbourhood school $j$ and a private school $m$ that share a boundary locus, and also set $c = 0$ thus $r_j \geq 0, \forall j$. Denote the price including the income tax for attending school $i$ as $\tilde{p}_i(b, y) \equiv p_i(b, y) + ty$. Given the rent, then $\tilde{p}_j(b, y) = ty + r_j$, and $\tilde{p}_m(b, y) \geq EMC_m(b) + ty$ for all $(b, y) \in A_m$, and from Proposition 2(iv)

$\tilde{p}_m(b, y) = EMC_m(b) + ty$ for all $(b, y) \notin A_m$, where $A_m$ is the equilibrium admission space of school $m$. Using equation (2.39), the requirement for efficiency is then

$$\gamma(b, y)U(y - \tilde{p}_j(b, y), a(\theta_j, b)) + \lambda[\tilde{p}_j(b, y) - EMC_j(b)]$$

$$\geq \gamma(b, y)U(y - \tilde{p}_m(b, y), a(\theta_m, b)) + \lambda[\tilde{p}_m(b, y) - EMC_m(b)],$$

$$\frac{\gamma(b, y)}{\lambda} [U(y_t - r_j, a(\theta_j, b)) - U(y_t - EMC_m(b), a(\theta_m, b))] \qquad (2.41)$$

$$\geq EMC_j(b) - r_j$$

for some $\frac{\gamma(b,y)}{\lambda}$ above zero (from condition (2.40)).

Consider first when neighbourhood school $j$ has the lowest rent, $r_j = c = 0$. The $LHS$ of (2.41) will be very small (close to zero) for types close to the boundary between school $j$ and $m$. Among these types, there are always some types with $b < \theta_j$ such that $EMC_j(b) > 0$, this implies that (2.41) does not hold, the equilibrium is then not Pareto efficient.

For neighbourhood school $j$ with $r_j > 0$ (that share the boundary with private school $m$), next is to show that (2.41) still does not hold. Analogous to the previous case where $r_j = 0$, if there is a type in school $j$ near the boundary and $EMC_j(b) - r_j > 0$, condition (2.41) is violated by the same reason. If there is no such type, it means that $r_j \geq EMC_j(b)$ for all types near the boundary. Condition (2.41) still does not hold by the following reason. Consider a type $(b, y')$ (with the same ability as before) but in school $m$, on the other side of the boundary. Such type exists because the boundary has a negative slope in $(b, y)$-space (Proposition 2). Using (2.41), the efficient allocation for this type requires

$$\frac{\gamma(b, y')}{\lambda} [U(y'_t - p^*(b, y'), a(\theta_m, b)) - U(y'_t - r_j, a(\theta_j, b))] \qquad (2.42)$$

$$\geq (r_j - EMC_j(b)) - (p^*(b, y') - EMC_m(b)).$$

From Proposition 2, near the boundary $p^*(b, y') - EMC_m(b)$ and the $LHS$ are arbitrarily (positively) small. If $r_j > EMC_j(b)$, again the efficient condition is violated. If $r_j = EMC_j(b)$, a type $(\tilde{b}, \tilde{y})$ near the boundary can be picked such that $\tilde{b} > b$ and $\tilde{y} < y'$. Applying (2.42) to this type gives the efficient condition

$$\frac{\gamma(\tilde{b}, \tilde{y})}{\lambda} \left[ U(\tilde{y}_t - p^*(\tilde{b}, \tilde{y}), a(\theta_m, \tilde{b})) - U(\tilde{y}_t - r_j, a(\theta_j, \tilde{b})) \right]$$
$$\geq (r_j - EMC_j(\tilde{b})) - (p^*(\tilde{b}, \tilde{y}) - EMC_m(\tilde{b})).$$

Similarly, near the boundary $p^*(\tilde{b}, \tilde{y}) - EMC_m(\tilde{b})$ and the $LHS$ are arbitrarily small. The term $r_j - EMC_j(\tilde{b})$ is now positive because $EMC$ is decreasing in $b$. For $\tilde{b}$ sufficiently high on the boundary, the efficient condition is violated. Hence, the proof is complete.

Note that a fully private equilibrium is $N-constrained$ Pareto efficient because, with $p_i^*(b, y) \geq EMC_i(b)$, $\forall (b, y) \in A_i$ and $p_i^*(b, y) = EMC_i(b)$, $\forall (b, y) \notin A_i$, condition (2.39) holds.[16]

## Proof of Corollary 1

From Proposition 1(i), $r_1 > r_2 > ... > r_{\hat{N}} = c$. With the assumption of enough housing supply, it is straightforward that no private school households are willing to live in neighbourhood $j = 1, 2, ..., \hat{N} - 1$. The mixing can happen only in the lowest rent neighbourhood. The equalised public school size is not disturbed in this neighbourhood because including private school households in the school zone of neighbourhood school $\hat{N}$ does not vary the public school demand. Therefore, unequal neighbourhood sizes are possible in equilibrium.

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[16]Assume that when there are no school zones, everyone just pays $c$ (assumed zero) for the rent.

# Chapter 3

# A Model of Price Subsidisation for Private Education

## 3.1   Introduction

Dissatisfaction with the quality of public schools has led to policies aimed at increasing student educational choice and school competition that, hopefully, would drive inefficient public schools to operate efficiently, thus improving the public school quality. Price subsidisation such as private-school vouchers is one such policy. However, this argument is challenged by some studies which show that subsidising private schools via vouchers could lower the quality of public schools because the problems of rent seeking behavior (see Manski (1992); McMillan (2004)) and cream-skimming nature of private schools (see, among others, Epple and Romano (1998, 2003)) are intensified. Such problems cause simple universal voucher policies to create some welfare-loss households. Children of such households are those left behind in the public sector which either has schools with lower productive efforts or students with lower peer quality. In contrast, this chapter argues that price subsidisation for private education can be a Pareto improving policy if private education

53

is more costly per unit and housing capacities of jurisdictions cannot accommodate perfect segregation among heterogeneous households.

This chapter asks the question whether there is a rationale for subsidising private education even though private providers may be less cost effective in providing educational services than public schools. Specifically, private education has higher price per unit.[1] A theoretical analysis of price subsidisation is examined by developing a two-jurisdiction and two-income-type model. Financed by property taxes, public educational services of a jurisdiction is determined by majority voting of its residents. Housing capacities of jurisdictions are fixed and the unit price of education from private providers is higher than the public providers. The main results are as following. First, the laissez-faire equilibrium in which public and private education coexist is inefficient. Only the equilibrium where each jurisdiction contains homogeneous residents without a private sector and a rent premium is efficient. The inefficiency of the equilibrium with private education arises because fixed housing capacities may not accommodate perfect segregation. Specifically, some rich households have to opt out for pricier private education and live in the poor jurisdiction outside the jurisdiction dominated by the rich. The social planner's educational allocation is achieved when there are "just right" housing capacities (that is, corresponding with "just right" jurisdiction boundaries) which induce perfect segregation, allowing each income type to reside in only one jurisdiction and then determine their preferred public educational services which are cheaper per unit. Such allocation has no private sector. Second, income-tax-financed price subsidisation for private education is a Pareto improving policy if only the rich are taxed. Pareto-improving subsidisation exists. The optimal subsidisation is either partial or

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[1]Tsang (2002) reviews studies of school costs and shows that in some countries the unit cost of private schools is higher than public schools, even though this is unusual. He also argues that many studies may lack of the information about the true costs of private schools as well as the problem of omitted variables. These problems would lead to an underestimation of the costs of private schools, and thus overestimating their efficiency when comparing to public schools.

full, which eliminates the unit price difference between two sectors, depending crucially on the unit price difference and the relative size of the rich private-education households. This last result is the main contribution of this chapter.

The main results depend on three important assumptions. The first assumption is that private educational services are more expensive per unit. Given their residence, the majority of each jurisdiction thus want to form a public sector and choose their preferred level of educational services. Since education is a normal good, the rich demand higher educational services than the poor. Therefore, the efficient educational allocation is when the rich and the poor obtain their preferred educational services through public providers. This is equivalent to having perfect segregation among different income types across jurisdictions.

The second assumption, which is fixed housing capacities of jurisdictions, determines whether the efficient allocation can be achieved. If the housing capacities accommodate perfect segregation, each income type will live in the same jurisdiction without type mixing and then determine their own public educational services. In this case, the efficient allocation is achieved. If perfect segregation is not the case, then the model focuses on when some of the rich have to live in the jurisdiction dominated by the poor. Assuming that the rich have income high enough such that they are not satisfied with the level of public educational services, they opt out for private education which is more expensive per unit, even though they also have to pay for unutilised public education. Since these rich households do not obtain the preferred level of educational services from public providers, the equilibrium with public and private education is inefficient.

The inefficient educational allocation arising from these two assumptions creates possibility of Pareto improvement. The third assumption that there is a reservation rent which is assumed to be the rent of the poor jurisdiction simplifies the analysis in the model. Using the equilibrium with public and private education as a starting

point, the central government can introduce an income tax charged only to the rich to subsidise the unit price of private education. Therefore, the public educational services and the rent in the poor jurisdiction remain unchanged, meaning that the disposable income of the opt-out rich is the same after the policy. This policy has possibility to be Pareto improving because a reduction in the unit price causes tax burden that is not fully borne by the opt-out rich since the opt-in rich households in the rich jurisdiction also share the tax burden. When the ratio of the opt-out rich who utilise the policy is sufficiently low, full subsidisation which eliminates the difference between the unit prices of two sectors maximises the welfare gains. When such ratio is sufficiently high, partial subsidisation is optimal. As the welfare of the opt-out rich is improved, the welfare of the opt-in rich also increases. This is so because to have the rich reside in both jurisdictions their utility has to be equivalent. The welfare improvement of the opt-in rich comes from the fact that the new rent premium plus the tax burden are less than the rent premium before subsidies. Hence, this chapter can provide a rationale for price subsidisation for private education that does not rely on rent seeking behavior or peer effects which may be somewhat more difficult to observe than unit costs.

This chapter is related to studies on local publicly provided goods with private alternatives. Existence of private providers may create non-existence of majority voting equilibrium due to non-single-peaked voting preferences (Stiglitz, 1974). Also, mixing communities become possible since some community members may be indifferent between living in a high-service community and living in a low-service community and opting out. Epple and Romano (1996) show that an appropriate single-crossing assumption with a necessary condition can overcome the non-existence problem, but the median income voter may not be pivotal and a standard median-voter theorem may not be applied. This chapter simplifies the problem by using a two-stage game, identical housing sites, and higher unit price of education

56

from private providers to ensure that a voting equilibrium exists and the standard median-voter theorem can be applied.

The model in this chapter is highly stylised and is built on the multiple jurisdiction model in Goldstein and Gronberg (1986), but abstracted as follows. The model considers only two discrete income types and two jurisdictions, instead of a continuum of types with an exogenous distribution and a given number (greater than one) of jurisdictions. Also, the rent in the poor jurisdiction is fixed at the opportunity cost of a house. Other main features of Goldstein and Gronberg (1986) such as identical houses, non-supplementary private educational services, majority-voting public educational services, perfect substitution between public and private education, and higher unit price from private providers are maintained. The model also employs jurisdiction composition and a two-stage game setting from De Bartolome (1990) who models the education output depending on input expenditures and peer group effects in a multiple community model but without private providers. Households in his model decide where to live in the first stage, given the optimal decision of the next stage, and then vote myopically for input expenditures without taking into account the effects on community compositions and housing prices in the second stage. The model utilises such two-stage game but departs from including peer effects to focus on how input expenditures and the difference between unit prices of public and private providers affect multiple-jurisdiction equilibrium.

Studies on price subsidisation in the form of private school vouchers usually show that there are winners and losers from the policy. For example, Nechyba (1999) uses numerical general-equilibrium techniques to predict the effects of private school vouchers in a multi-district model with heterogeneous households and housing quality, together with student peer quality as the measurement of school quality. He finds that vouchers would benefit public schools in poor communities while harming public schools in wealthy communities. This is because migration

of the rich to poor communities raises per pupil spending on public schools which outweighs peer quality reduction from losing relatively high peer quality students to private schools or public schools in other communities. Based on Nechyba (1999)'s model, Ferreyra (2007) adds religious private schools and estimates the model. Using the estimates, two voucher programs, universal and nonsectarian private school vouchers are simulated and their effects on school qualities and residential choices are studied. She finds that households that gain from vouchers are on average less wealthy while those who experience welfare losses are wealthy households who already enjoy high-quality schools before vouchers are introduced. Epple and Romano (1998) develop a model of competition between public and private schools within a district and analyse the effects of universal vouchers. In their public-private sector equilibrium, private schools cream skim relatively richer and higher ability students and generate higher qualities than public schools. Introducing vouchers intensifies this cream-skimming problem and causes public schools to have lower quality because they lose relatively high ability students to private schools. Relatively low ability students who remain in the public sector experience welfare losses while relatively high-ability and low-income students who take the vouchers gain from the policy. These studies share a common feature that public and private schools have the same effectiveness in providing educational quality.

Epple and Romano (2008) abstract from such presumption by assuming that private schools are more effective than public schools in providing any given educational quality. In particular, private schools are less costly to operate and more effective in delivering peer quality. They find that private school vouchers with tuition constraints can improve school quality without worsening the cream-skimming problem. This suggests that Pareto improvement may be achieved if there is a difference in effectiveness of public and private schools. The current chapter employs this idea by exploring an opposite case where private schools are more costly to

operate. A main result in this chapter confirms the idea that Pareto improvement can be achieved under different effectiveness among sectors.

The chapter proceeds by describing the model in the next section. Section 3.3 analyses the laissez-faire equilibrium following by the social planner's problem in Section 3.4. Section 3.5 discusses policies aimed at subsidising the unit price of private education while Section 3.6 illustrates the policy analysis by using simulations. A discussion of the main results is found in Section 3.7 followed by some concluding remarks.

## 3.2 The Model

The model has two jurisdictions, two income levels, and two educational sectors. A household gains utility from consuming the numeraire commodity ($c$) and educational services ($g$). Both $c$ and $g$ are normal goods. All households have the same utility function $U(c, g)$, which is continuous, strictly increasing, strictly quasi-concave, and twice differentiable. Households differ in income, $y^i \in \{y^l, y^h\}$, $0 < y^l < y^h$. There are $N^l$ and $N^h$ numbers of low and high income households, respectively.

Educational services can be provided by public or private providers. Services from both sectors are perfectly substitutable. Public educational services of a jurisdiction are provided only for its residents. Private educational services have no residential requirement and are not supplementary to public services. That is, a household cannot gain any benefits from public education when she opts out. The price per unit of educational services from public and private providers are $p$ and $p'$, respectively. Following Goldstein and Gronberg (1986), assume

**Assumption P**:  $p' > p$.

This assumption imposes that private education is more costly to operate. Assume that public providers produce $g$ with a fixed average cost $p$ while private providers have a fixed average cost $p'$. Under a competitive market with free entry and exit, zero profits arise which results in the price $p'$ per unit of $g$. Public providers are assumed to be benevolent and do not charge higher than $p$.[2]

There are two jurisdictions, 1 and 2. Each jurisdiction has a fixed housing capacity corresponding with a non-overlapping fixed boundary. Let $n_1$ and $n_2$ be housing capacities in jurisdiction 1 and 2, respectively. Assume $n_1 + n_2 = N^l + N^h$; the total number of houses equals the number of households. It is more interesting to focus on a case in which all jurisdictions are not dominated by only one income group. Therefore, the model assumes that low (high) income households always form a majority in jurisdiction 1 (2). It is also assumed that $N^h \geq n_2$, that is, the total number of rich households is at least as large as the housing capacity in jurisdiction 2. This assumption also implies that $N^l \leq n_1$.

$$\textbf{Assumption N}: \quad N^h \geq n_2 \quad \text{and} \quad N^l \leq n_1.$$

All houses in the model are identical. They are owned by landlords who live outside the model and operate in a competitive housing market. The housing market is frictionless. Each household can occupy (by renting) only one house. The opportunity cost of a house is $z$ which is positive. Therefore, an equilibrium rent in any jurisdictions cannot be lower than $z$. Assume $y^l > z > 0$ to allow all households to afford a house in equilibrium. The rental price in the poor jurisdiction is assumed to be fixed at $z$. In this sense, the poor jurisdiction is modeled as the reservation locale assumed here to be dominated by the poor. The concept of the reservation location of housing is used in urban economics when studying housing

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[2]Public school behaviour may take other forms such as maximising surplus or rent seeking (see Manski (1992); McMillan (2004)).

prices (Van Nieuwerburgh and Weill (2010)).

**Assumption R**:   $r_1 = z$, $r_2 \geq z$.

Public education is locally financed by a proportional property tax rate $\tau$. The property tax rate of each jurisdiction is determined by majority voting of the residents who have to pay the property taxes regardless of their educational sector choice. Henceforth, $i$ stands for household with income $y^i$ and $j$ stands for jurisdiction $j$. The local government of $j$ runs a balanced budget such that

$$N_j \tau_j r_j \; = \; N_j p \eta_j g_j \tag{3.1}$$

where $j \in \{1, 2\}$, $g_j$ is majority-voting public educational services, $\eta_j$ is the fraction of households in $j$ that choose public education, $\eta_j \leq 1$, and $r_j$ is the rent in $j$. Immediately from (3.1), the property tax per household is $\tau_j r_j$ which equals $p \eta_j g_j$. Therefore, $i$ living in $j$ pays the after-tax rent $(1 + \tau_j) r_j = r_j + p \eta_j g_j$. The budget constraint of $i$ in $j$ is then either:

$$c_j^i \; = \; y^i - r_j - p \eta_j g_j, \text{ if opt-in, or} \tag{3.2}$$

$$c_j^{i'} \; = \; y^i - r_j - p \eta_j g_j - p' g_j^{i'}, \text{ if opt-out,} \tag{3.3}$$

where $g_j^{i'}$ is the optimal level of private educational services.

Timing of the model has two stages. In the first stage, each household chooses where to live, pays rent, and the housing market clears, given voting preference and the choice of education sector in the next stage. Once residing in a jurisdiction, the household is stuck and has to vote and make the education sector choice given the jurisdiction's characteristics. In the second stage, the level of public educational services in each jurisdiction is determined by majority voting of its residents. Also,

the household decides whether to opt out given other households' choices. Then, the household pays property taxes and consume numeraire goods. This two-stage game is solved backwardly. Before solving the problem, the definition of equilibrium is defined. In jurisdiction $j$, let $\theta_j$ be the fraction of rich households and $\eta_j$ be the fraction of households choosing public education.

**Definition.** An equilibrium is a set $\{\theta_j, r_j, g_j, g_j^{i'}, \eta_j\}$, for all $j \in \{1, 2\}$ and $i \in \{l, h\}$, such that (i) given the residential choice, the educational choice of each household maximises utility, (ii) given the educational choice, no household is better off by migrating to another jurisdiction, and (iii) given the equilibrium allocation, the housing market clears.

### 3.2.1 The Second Stage: Educational Choice

In this stage, $i$ living in $j$ chooses the educational services that maximise her utility given the rent, the fraction of public education households, and the majority-voting public educational services. Therefore, $i$ in $j$ solves

$$\max \begin{cases} U(y^i - r_j - pg_j\eta_j, g_j) \\ \\ V(y^i - r_j - pg_j\eta_j - p'g_j^{i'}, g_j^{i'}) \end{cases} \tag{3.4}$$

where $g_j$ is the majority-voting public educational services, $V(.)$ is the indirect utility function when opting out, and $g_j^{i'} \in \arg\max\left\{U(y^i - r_j - p\eta_j g_j - p'g_j^{i'}, g_j^{i'})\right\}$. Using equation (3.3), private educational services $(g_j^{i'})$ is then determined by the first-order condition:

$$p'U_1(c_j^{i'}, g_j^{i'}) = U_2(c_j^{i'}, g_j^{i'}). \tag{3.5}$$

In each $j$, the existence of private alternatives creates non-single-peaked voting
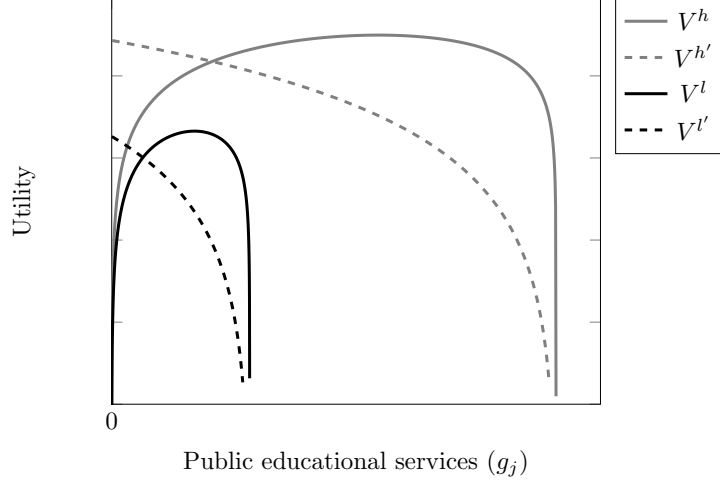
Figure 3.1: Non-Single-Peaked Voting Preferences

preferences as shown in Figure 3.1. The inverted U-shaped curves $V^l$ and $V^h$ plot

utilities obtained by different levels of public educational services ($g_j$) of the poor

and the rich, respectively, given $r_j$. Similarly, the dashed curves $V^{l'}$and $V^{h'}$present

utilities when opting out and choosing $g_j^{i'}$ that satisfies equation (3.5) given the

level of $g_j$. At sufficiently low levels of $g_j$, households are better off by opting out.

For instance, for $y^h$, the gray dashed curve is above the gray solid curve. There are

two peaks for $i$, one at zero public educational services and opting out, another at

the peak of $V^i$. The latter peak gives higher utility because $p < p'$. Also, the latter

peak is at $i$'s optimal $g_j$ as if she is the decisive voter. If $i$ is the decisive voter,

her optimal $g_j^i \in \arg\max \left\{ U(y^i - r_j - p\eta_j g_j^i, g_j^i) \right\}$ is determined by the first-order

condition:

$$p\eta_j U_1(c_j^i, g_j^i) \quad = \quad U_2(c_j^i, g_j^i). \tag{3.6}$$

In this simple two-income-type model, if $i$ has the same income as the majority, $i$

is the decisive voter. Combining with Assumption P, the majority of jurisdictions

therefore never opt out. They vote for positive public educational services at their

optimal level. Moreover, $g_j^i > g_j^{i'}$ and $c_j^i > c_j^{i'}$, results due to the normal goods

63

assumption and Assumption P. Being the decisive voter gives a great benefit of accessing public providers who charge less per unit of educational services. Such benefit guarantees that voting equilibrium with non-zero public educational services exist.[3]

**Lemma 2.** *In $j$, $i$ never opts out if she has the same income type as the majority.*

*Proof.* Consider $i$ as the decisive voter (having the same type as the majority). From Figure 3.1, the highest utility for $i$ when opting out is at $g_j = 0$ and choosing $g_j^{i'} \in \arg\max \left\{ U(y^i - r_j - p'g_j^{i'}, g_j^{i'}) \right\}$. The highest utility when opting in is at $g_j^i \in \arg\max \left\{ U(y^i - r_j - pg_j^i \eta_j, g_j^i) \right\}$. Since $p < p'$ and $\eta_j \leq 1$, then $g_j^i > g_j^{i'}$ and $c_j^i > c_j^{i'}$. Therefore, $i$ never opts out because $U(y^i - r_j - pg_j^i \eta_j, g_j^i) > U(y^i - r_j - p'g_j^{i'}, g_j^{i'})$. $\square$

Now consider the voting subgame. Recall that jurisdiction 1 and 2 are dominated by poor and rich households, respectively. Using Lemma 2, low income households, a majority in jurisdiction 1, would vote public educational services, $g_1$, to satisfy

$$p\eta_1 U_1(c_1^l, g_1) = U_2(c_1^l, g_1). \tag{3.7}$$

Similarly, high income households, a majority in jurisdiction 2, would vote public educational services, $g_2$, to satisfy

$$p\eta_2 U_1(c_2^h, g_2) = U_2(c_2^h, g_2). \tag{3.8}$$

From (3.6) combining with (3.7) and (3.8), it must be the case that $g_1 = g_1^l$ and $g_2 = g_2^h$.

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[3]Instead, if $p = p'$, in $j$ voting equilibrium with non-zero $g_j$ may not exist since the decisive voter is indifferent between opting in and opting out. Later on $p < p'$ is assumed when analysing unit price subsidisation policies, therefore the model maintains Assumption P.

**Lemma 3.** *If there are poor households living in the rich jurisdiction, they never opt out.*

*Proof.* As illustrated by Figure 3.1, the optimal $g_j$ for $y^l$ is lower than the optimal $g_j$ for $y^h$. Since $g_2$ is the choice of $y^h$, opting out obtains lower utility, $U(y^l - r_2 - pg_2^h, g_2^h) > U(y^l - r_2 - pg_2^h - p'g_2^{l'}, g_2^{l'})$, since $g_2^{l'} < g_2^h$. Therefore, the poor never opt out in the rich jurisdiction. $\qquad\square$

To focus on the role of private education, the model assumes further that the difference in the income levels is high enough to make high income households always opt out when living in the poor jurisdiction. Specifically, given $y^l$ and $r_1 \in [z, y^l)$, $y^h$ satisfies

$$\textbf{Assumption Y}: \quad U(y^h - r_1 - pg_1^l - p'g_1^{h'}, g_1^{h'}) > \quad U(y^h - r_1 - pg_1^l, g_1^l).$$

This assumption ensures that private education always exists if the rich have to live in the poor jurisdiction.

**Lemma 4.** *All households in the rich jurisdiction choose public education, $\eta_2 = 1$. Also $\eta_1 = 1 - \theta_1$ if there are (rich) private education households living in the poor jurisdiction, where $\theta_1$ is the fraction of high income households living in the poor jurisdiction.*

*Proof.* From Lemma 3, $\eta_2 = 1$ is direct. Lemma 3 and Assumption Y imply $\eta_1 = 1 - \theta_1$ since only rich households living in the poor jurisdiction will opt out in equilibrium. $\qquad\square$

Lemma 4 points out that the poor benefit from having the rich living in their jurisdiction because the rich do not choose public education. Thus, the net price for the poor is $p(1 - \theta_1) < p$. This allows them to enjoy even higher public educational

65

services, according to (3.7), than the case with perfect segregation where the unit price for public education is $p$.

## 3.2.2 The First Stage: Residential Choice

Now turn to the first stage. Households choose to live in the jurisdiction that gives higher utility given the optimal decision of the second stage. Recall that jurisdiction 1 and 2 are dominated by the poor and the rich, respectively, and $\theta_j$ denotes the fraction of the rich living in $j$. Recall from Lemma 4 and Assumption R, $\eta_2 = 1$, $\eta_2 = 1 - \theta_1$, $r_1 = z$, $r_2 \geq z$. In equilibrium, no households want to move to another jurisdiction.

Hence, for $y^l$, no migration implies either:

$1/2 < \theta_2 < 1$ (when the poor reside in both jurisdictions),

$$U(y^l - r_2 - pg_2^h, g_2^h) \;=\; U(y^l - z - pg_1^l, g_1^l); \qquad (3.9)$$

or $\theta_2 = 1$ (when the poor only reside in the poor jurisdiction),

$$U(y^l - r_2 - pg_2^h, g_2^h) \;\leq\; U(y^l - z - pg_1^l, g_1^l). \qquad (3.10)$$

For $y^h$, no migration implies either:

$0 < \theta_1 < 1/2$ (when the rich reside in both jurisdictions),

$$U(y^h - r_2 - pg_2^h, g_2^h) \;=\; U(y^h - z - p(1 - \theta_1)g_1^l - p'g_1^{h'}, g_1^{h'}); \qquad (3.11)$$

or $\theta_1 = 0$ (when the rich only reside in the rich jurisdiction),

$$U(y^h - r_2 - pg_2^h, g_2^h) \;\geq\; U(y^h - z - p(1 - \theta_1)g_1^l - p'g_1^{h'}, g_1^{h'}). \qquad (3.12)$$

Condition (3.9) is for poor households to reside in both jurisdictions. Condition (3.10) is when there is no poor household in the rich jurisdiction. Condition (3.11) is when rich households are in both jurisdictions. Condition (3.12) is when there is no rich household in the poor jurisdiction. When households with the same income appear in both jurisdictions, no migration conditions require equality in utility. Utility inequality is applied under perfect segregation.

## 3.3 Equilibrium

With the optimising behavior of households in each stage from the previous section, there are 4 cases to be considered.[4]

### 3.3.1 Heterogeneous Residents in the Poor Jurisdiction and Homogenous Residents in the Rich Jurisdiction (Case I)

This case has $\theta_2 = 1$ and $0 < \theta_1 < 1/2$, and solves (3.4)-(3.8), (3.10), and (3.11). Any $r_2 \geq z$ satisfies condition (3.10). $1/2 < \theta_1 < 1$ requires $r > z$ according to condition (3.11). Then, there exists the unique equilibrium rent $r_2^* > z$. This case is certain when the number of rich households is higher than the housing capacity of the rich jurisdiction, $N^h > n_2$. That is, some rich households have to live in the poor jurisdiction because the housing capacity of the rich jurisdiction is not enough to accommodate all rich households. Public and private education households coexist in this case.

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[4]There are actually two more cases where both jurisdictions are dominated by the same income households, either the rich or the poor. However, these cases are ignored here because the focus is on different levels of public educational services among jurisdictions.

### 3.3.2 Homogenous Residents in both Jurisdictions (Case II)

This case has $\theta_2 = 1$ and $\theta_1 = 0$, and solves (3.4)-(3.8), (3.10), and (3.12). Again, any $r_2 \geq z$ satisfies condition (3.10). Also, for condition (3.12), $r_2 \geq z$ is required. Any $r_2 \geq z$ that satisfies (3.12) can be an equilibrium rent. Therefore, there are multiple equilibria. This case corresponds with the "just right" jurisdiction boundaries where $N^l = n_1$ and $N^h = n_2$. Moreover, this case is interesting in the sense that it will be realised if households are allowed to form their preferred community prior to the decision of the level of public educational services. Given the rent, rich households are better off by forming a rich jurisdiction and choose their optimal educational services supplied by public providers who charge less per unit than private providers. Therefore, perfect segregation equilibrium is realised if allowing for endogenously determined jurisdictions.[5]

### 3.3.3 Heterogeneous Residents in both Jurisdictions (Case III)

This case has $1/2 < \theta_2 < 1$ and $0 < \theta_1 < 1/2$, and solves (3.4)-(3.8), (3.9), and (3.11). It has to be the case that to satisfy (3.9) and (3.11), $r_2 < z$ must holds. This violates Assumption R. Therefore, there is no feasible $r_2$ that corresponds with heterogeneous residents in both jurisdictions. Hence, the poor strictly prefer living in jurisdiction 1 where they dominate and choose their own public educational services. With Assumption N, Case III is then ruled out.

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[5]This equilibrium may be called *Tiebout equilibrium*. In Tiebout (1956)'s pioneering paper of fiscal competition, jurisdictions are formed by residents who have the same preference over public services which are financed by a head-tax. Such preferences depend on income levels, that is, higher income households demand higher public services by a normal good assumption. Therefore, different income households choose different jurisdictions. With homogenous housing attributes and freely adjusted jurisdiction boundaries, there is perfect segregation by income among jurisdictions.

### 3.3.4 Heterogeneous Residents in the Rich Jurisdiction and Homogenous Residents in the Poor Jurisdiction (Case IV)

This case has $1/2 < \theta_2 < 1$ and $\theta_1 = 0$, and solves (3.4)-(3.8), (3.9), and (3.12). Again, condition (3.9) requires $r_2 < z$. Condition (3.12) calls for $r_2 \geq z$. Thus, there is no feasible $r_2$ corresponding with this case. With Assumption N, Case IV is also ruled out.

Since public education is financed by property taxes that work as a head-tax, the poor cannot exploit high public educational services in the rich jurisdiction as in the case of financing by income taxes. Such high services come with high tax burden compared to the poor's optimal public educational services. Therefore, Case III and IV where there are heterogeneous residents in the rich jurisdiction cannot arise in equilibrium. This is because the poor have higher utility living in their poor jurisdiction where it is assumed to have no rent premium. To make the poor indifferent between two jurisdictions, the rent in the rich jurisdiction has to be less than the rent in the poor jurisdiction. Such rent is not feasible by Assumption R because it is below the opportunity cost of a house.[6] Hence, in equilibrium, it is either Case I or II. Table 3.1 summarises the equilibrium allocations of Case I and Case II. A proposition of the laissez-faire equilibrium is also concluded below.

**Proposition 5.** *In laissez-faire equilibrium, it is either*

*(i) heterogeneous residents in the poor jurisdiction and homogenous residents in the rich jurisdiction, if $n_2 < N^h$; or*

*(ii) homogenous residents in both jurisdictions, if $n_2 = N^h$.*

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[6]Even if this strong assumption is relaxed, heterogeneous residents in the rich jurisdiction may not be obtained simply because the poor could not afford to live there for any $r_2 \geq z$ since the optimal $g_j$ of the rich might be even higher than the poor's income. Figure 3.1 states this point where $g_j$ that maximises the rich utility (the peak of $V^h$) is higher than the poor's income (the lowest point of $V^l$). This infeasibility is very likely when Assumption Y is made. Since in Case III and IV there is no private education chosen by any household, the model maintains Assumption R and proceeds by focusing on Case I and II.

Table 3.1: The Laissez-Faire Equilibrium

| | Case I: $N^h > n_2$ | Case II: $N^h = n_2$ |
|---|---|---|
| $\theta_1$ | $0 < \frac{N^h - n_2}{n_1} < \frac{1}{2}$ | $0$ |
| $\theta_2$ | $1$ | $1$ |
| $g_1$ | $g_1^l$ (eq.(3.7)) | $g_1^l$ (eq.(3.7)) |
| $g_2$ | $g_2^h$ (eq.(3.8)) | $g_2^h$ (eq.(3.8)) |
| $g_1^{h'}$ | $g_1^{h'}$ (eq.(2)) | - |
| $r_1$ | $z$ | $z$ |
| $r_2$ | $r_2^* > z$ (eq.(3.11)) | $[z, \bar{r}_2]$ ($\bar{r}_2$ satisfies eq.(3.12) with equality) |
| $\eta_1$ | $1 - \theta_1$ | $1$ |
| $\eta_2$ | $1$ | $1$ |
| Private Sector | Yes | No |
| Uniqueness of Equilibrium | Yes | No |

Note: For $j \in \{1,2\}$ and $i \in \{l, h\}$, $n_j$ is the housing capacity in $j$, $N^i$ is the number of households with income $i$, $\theta_j$, is the fraction of rich households in $j$, $g_j$ is the majority-voting educational services, $g_j^i$ is the choice of $g$ of households with income $y^i$ who dominate $j$, $r_j$ is the rent in $j$, and $\eta_j$ is the fraction of households in $j$ who choose public education.

*Proof.* The proof is referred to the analysis of Case I-IV above. □

Proposition 5(i) (or Case I with $n_2 < N^h$) is the focus of this chapter since public and private education coexist. Figure 3.2 illustrates the coexistence of public and private education households in the poor jurisdiction. The net income of the poor is $y^l - z$, depicted the distance $L - 0$, and the net income of the rich without rent premium is $y^h - z$, depicted by the distance $A - 0$. The majority of jurisdiction 1 vote for $g^l$ which is the choice of poor households. Rich households living in jurisdiction 1 opt out since the utility from opting in $(U^{h1})$ is less than from opting out $(U^{h'})$. When choosing private educational services $(g^{h'})$ in the poor jurisdiction, the rich still have to pay $p(1 - \theta_1)g^l$, depicted by $A - B'$, to finance

70
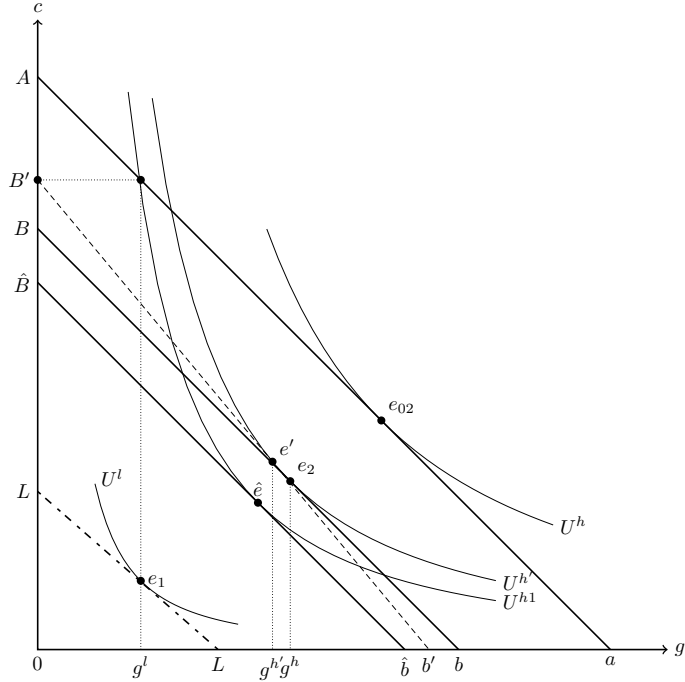
Figure 3.2: Heterogeneous Residents in the Poor Jurisdiction and Homogenous Residents in the Rich Jurisdiction

public education. Therefore, their budget line is $B'b'$. The slope of $B'b'$ is higher than the others because $p < p'$. To make rich households indifferent between living in either jurisdiction, the rent premium, which is $r_2 - z$, has to equal $A - B$. The rent premium lowers the budget line of the rich in jurisdiction 2 to $Bb$, then $g^h$ is the majority-voting choice in jurisdiction 2 and the utility is $U^{h'}$ which is equal to when living in jurisdiction 1 and opting out. Note that the budget line $LL$ and the choice $g^l$ of poor households already take into account that rich households in jurisdiction 1 opt out because the slope of $LL$, which is the net price $p(1 - \theta_1)$, is less than $p$ which is the slope of $Aa$ and $Bb$. The equilibrium in this case is unique as mentioned in Table 3.1.

The equilibrium in Proposition 5(ii) (or Case II with $n_2 = N^h$) is where residents with the same preference of public educational services live in the same jurisdiction. In this case, the jurisdiction boundaries facilitate perfect segregation. Figure 3.3 illustrates this case. With perfect segregation, the poor's budget line now has the
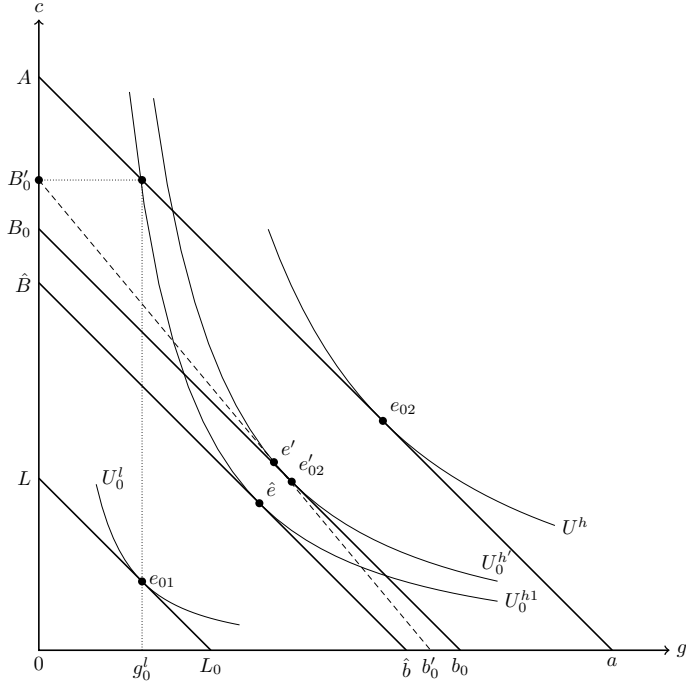
Figure 3.3: Homogenous Residents in both Jurisdictions

slope equal to $p$ since there are no rich private-education households to subsidise public education. The equilibrium allocation of the poor is at $e_{01}$ with $g_0^l$. However, the equilibrium in jurisdiction 2 is not unique. With private alternatives, the equilibrium rent premium in jurisdiction 2 that constitutes perfect segregation can be any level from the set $[0, AB_0]$ because the utility of the rich will be at least no lower than $U_0^{h'}$, which is the utility when rich households instead choose to live in the poor jurisdiction and opt out. The equilibrium allocation of the rich is at $e_{02}'$ when the rent premium is $A - B_0'$, while the allocation is at $e_{02}$ when the rent premium is zero. The allocation when $r \in (0, AB_0)$ is thus somewhere between $e_{02}'$ and $e_{02}$. Hence, when $n_2 = N^h$, there are multiple equilibria. If no private alternatives, the set of the equilibrium rent premiums is larger, $[0, A\hat{B}]$. Therefore, the availability of private education lowers the possible range of the equilibrium rent premiums. This is simply because having private alternatives makes the poor jurisdiction more attractive to live. Consequently, the rent premium does not have to be as high as

72

before to keep the rich preferring their dominated jurisdiction. Notice that the rich get the highest utility when there is no rent premium. As shown in the social planner's problem below, such equilibrium is efficient.

## 3.4 The Social Planner's Problem

The social planner's problem is to choose $g^l$ and $g^h$ for the poor and the rich, respectively. Assume that the planner does not care about landlords' utility. Thus, the planner just rents all houses from the landlords at the rental price $z$ per house and then distributes to households with the same price. Charging a rent premium to the rich without transferring it to the poor brings down the budget line and then lowers the welfare of the rich. Wealth transfer is not considered here to be in line with the laissez-faire setting in which the landlords are assumed to live outside the model. The problem of the planner is then to solve

$$\max_{g^h, g^l} \quad N^h U(y^h - z - pg^h, g^h) + N^l U(y^l - z - pg^l, g^l).$$

The first-order conditions are:

$$pU_1(y^h - z - pg^h, g^h) = U_2(y^h - z - pg^h, g^h); \qquad (3.13)$$

$$pU_1(y^l - z - pg^l, g^l) = U_2(y^l - z - pg^l, g^l). \qquad (3.14)$$

The optimal $g^h$ and $g^l$ are determined by condition (3.13) and (3.14), respectively. The social outcomes are thus equivalent to the laissez-faire equilibrium with homogenous residents and no rent premium. The social outcomes are at $e_{01}$ and $e_{02}$ in Figure 3.3. All the poor (rich) live in jurisdiction 1 (2): there is perfect segregation. The laissez-faire equilibrium with heterogeneous residents ($e_1, e'$ and $e_2$ in Figure 3.2) is not efficient. For the rich, they have higher utility in the social

73

outcomes because there is no rent premium, that is, their net income is higher. For the poor, they have lower utility because the unit price of education in the laissez-faire equilibrium is $p\eta_1 = p(1 - \theta_1)$ which is less than $p$. In other words, there are no rich private-education households who inevitably subsidise the price of public education. Therefore, the poor's budget line is lower in the social outcomes, that is, the budget line $LL_0$ in Figure 3.3 lies below the budget line $LL$ in Figure 3.2 (the distance $L - 0$ in both figures are equal). The inefficiency of the laissez-faire equilibrium with private education comes from the fact that the jurisdiction boundaries (with their corresponding housing capacities) are not adjustable to include all households with the same preference for educational services into the same jurisdiction. Specifically, only the just right boundaries where $n_1 = N^l$ and $n_2 = N^h$ accommodate the efficient allocation. Hence, only the laissez-faire equilibrium with homogenous residents and no rent premium is equivalent to the social outcomes. The second column in Table 3.1 shows that the social optimal equilibrium is only a special case of the laissez-faire equilibrium with homogenous residents. Any other equilibria with positive rent premiums (i.e. $r_2 > z$) are not efficient.

**Proposition 6.** *The laissez-faire equilibrium in which public and private education coexist is inefficient.*

*Proof.* The equilibrium that public and private education coexist is the laissez-faire equilibrium with heterogeneous residents (at $e_1, e'$ and $e_2$ in Figure 3.2). Since the social outcomes are at $e_{01}$ and $e_{02}$ in Figure 3.3, the proof is completed. $\qquad\square$

Proposition 6 motivates the next section which will discuss whether a Pareto improving policy aimed at subsidising the price of private education exists.

## 3.5 Price Subsidisation for Private Education

This section uses the laissez-faire equilibrium with the coexistence of public and private education as the starting point to analyse the effects of price subsidisation for private education. The equilibrium allocation is referred to Case I in the equilibrium section. Recall from Table 3.1 that this case associates with $n_2 < N^h$. As before, private providers charge a higher price per unit, that is $p < p'$. Let the subsidised price be $p' - sF$, where $F = p' - p$ is the price difference between public and private providers, and $s$ is a subsidy rate, $s \in [0, 1]$.[7] The central government is assumed to use income taxes to finance the subsidies. Let an income tax rate $t$ be imposed to only rich households wherever they live.[8] The model maintains Assumption R. The poor households' choices then are not affected by any $s \in [0, 1]$. Therefore, $\theta_1$, $\theta_2$, $g_1^l$, $r_1$, $\eta_1$, $\eta_2$ are constant in any subsidy rates (see their determination in Case I of Table 3.1). This implies that the net income of the rich after paying the rent and the property taxes when living in the poor jurisdiction remains the same, which is $y^h - z - p g_1^l$. Recall that only the rich living in the poor jurisdiction opt out. The number of such rich households is $\theta_1 n_1 = N^h - n_2$, by housing clearing. These rich households are forced to live outside the rich jurisdiction because the housing capacity is not enough to accommodate all the rich.

The central government budget constraint is:

$$tN^h y^h = (N^h - n_2) s F \tilde{g}_1^{h'} \qquad (3.15)$$

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[7]If $s > 1$, the rich living in the rich jurisdiction will have an incentive to consume through private providers because $p' - sF < p$. The corresponding tax burden per rich household will change to be $ty^h = sFg_1^{h'}$ since all the rich now opt out. As shown in the analysis of Figure 3.4 below, the tax burden will be greater than the benefits, causing net welfare loss. Therefore, $s > 1$ cannot be a Pareto improving policy.

[8]Taxing poor households worsens their welfare without any benefits because they never opt out when $p < p''$ and $r_1$ is fixed at $z$. For $p = p''$, poor households are indifferent between forming a public sector and forming a private sector in jurisdiction 1. The model may assume that in such case the poor choose to form a public sector.

where $\tilde{g}_1^{h'} = \arg\max \left\{ U(y^h - z - pg_1^l - (p' - sF)\tilde{g}_1^{h'}, \tilde{g}_1^{h'}) \right\}$ is the optimal private educational services of the rich in jurisdiction 1 under the subsidised price. The tax burden per rich household is $ty^h = \left( \frac{N^h - n_2}{N^h} \right) sF\tilde{g}_1^{h'}$.

Given $s$ and $\tilde{g}_1^{h'}$, tax burden per rich household $(ty^h)$ is low when the fraction of the rich who opt out $\frac{N^h - n_2}{N^h}$ is low. $\frac{N^h - n_2}{N^h}$ may also be interpreted as excess demand of housing in the rich jurisdiction. Therefore, the housing capacity of the rich jurisdiction compared to the total number of rich households is crucial to the performance of the policy.

Before subsidisation, the rich are indifferent between two jurisdictions:

$$U(y^h - r_2 - pg_2^h, g_2^h) = U(y^h - z - pg_1^l - p'g_1^{h'}, g_1^{h'}) = u. \tag{3.16}$$

To evaluate whether a subsidy rate $s$ is a Pareto improving policy, it is enough to just consider how the middle term of equation (3.16) changes, which is the problem of the opt-out rich living in the poor jurisdiction. By duality property, the before-subsidy expenditure function is:

$$m(p', u) = c(p', u) + p'g(p', u) \tag{3.17}$$

where $m(.)$ is the expenditure function, $m(p', u) = y^h - z - pg_1^l$, $g(.)$ is the corresponding Hicksian demand function, and $u$ is the utility before subsidisation.

Next is to find welfare gains from each subsidy rate in terms of resources. Such gains are measured by compensating variation $(CV)$ which is, in this case, the maximum income taxes that a rich household living in the poor jurisdiction can bear while facing the subsidised price and maintaining the utility at $u$:

$$CV(s) = m(p', u) - m(p' - sF, u) = \int_{p'-sF}^{p'} \frac{\partial m}{\partial p} dp = \int_{p'-sF}^{p'} g(p, u)dp. \tag{3.18}$$

From condition (3.15), the corresponding tax burden per rich household from a subsidy rate $s$ is:

$$TB(s) \;\; = \;\; ty^h = \left(\frac{N^h - n_2}{N^h}\right) sF g(p' - sF, u). \qquad (3.19)$$

Using (3.18) and (3.19), the properties of $CV$ and $TB$ curves can be concluded.

**Lemma 5.** *The slope of $CV$ curves is positive and increasing in $s$.*

*Proof.* The slope of $CV$ curves is obtained by differentiating equation (3.18) with respect to $s$, $MCV = \frac{\partial CV}{\partial s} = F g(p' - sF, u) > 0$. The slope is increasing because $\frac{\partial MCV}{\partial s} = -F^2 \frac{\partial g(p' - sF, u)}{\partial p} > 0$, since $\frac{\partial g(p' - sF, u)}{\partial p} < 0$. $\qquad\square$

**Lemma 6.** *The slope of $TB$ curves is positive and increasing in $s$.*

*Proof.* The slope of $TB$ curves is obtained by differentiating equation (3.19) with respect to $s$, $MTB = \frac{\partial TB}{\partial s} = \left(\frac{N^h - n_2}{N^h}\right)\left(F g(p' - sF, u) - sF^2 \frac{\partial g(p' - sF, u)}{\partial p}\right) > 0$ since $\frac{\partial g(p' - sF, u)}{\partial p} < 0$. The slope is increasing because $\frac{\partial MTB}{\partial s} = \left(\frac{N^h - n_2}{N^h}\right)\left[sF^3\left(\frac{\partial^2 g(p' - sF, u)}{\partial p^2}\right) - 2F^2 \frac{\partial g(p' - sF, u)}{\partial p}\right] > 0$, since $\frac{\partial^2 g(p' - sF, u)}{\partial p^2} > 0$ implied by concavity of the utility function. $\qquad\square$

Define $\pi(s) = CV(s) - TB(s) = \int_{p'-sF}^{p'} g(p, u)dp - \left(\frac{N^h - n_2}{N^h}\right) sF g(p' - sF, u)$ as the net welfare gain from a subsidy rate $s$. The subsidy rate $s$ is a Pareto improving policy if $\pi(s) > 0$. Notice that, given the Hicksian demand function $g(p' - sF, u)$, the likeliness of having $\pi(s) > 0$ depends crucially on $\frac{N^h - n_2}{N^h}$, which is exogenously given in the model. Let us consider graphically why this is so.

From Figure 3.4, the maximum tax burden the household can take to finance the subsidy rate $s$ while keeping the utility at the same level as before subsidisation is measured by $CV$ which is the shaded area in the Figure 3.4. The actual tax burden per rich household is $(\frac{N^h - n_2}{N^h}) sF g(p' - sF, u)$. Notice that $sF g(p' - sF, u)$, the tax burden when all rich households opt out, measured by the area $p'bcp''$, is
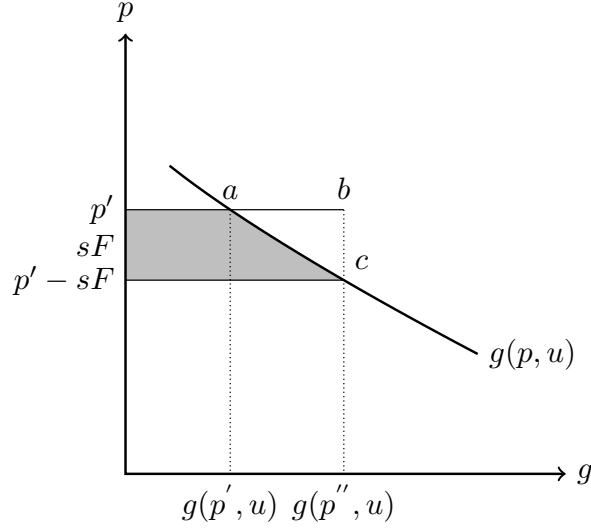
Notes: $CV$ is the shaded area, and $TB$ is $(\frac{N^h - n_2}{N^h})sFg(p' - sF, u)$.

Figure 3.4: Possibility of Pareto-Improving Price Subsidisation

always greater than $CV$. This will be the case when $s > 1$, all rich households will consume through private providers, and then the tax burden will be greater than the gains. Thus, $s > 1$ cannot be a Pareto improving policy. This is the reason why the central government has to restrict to only $s \in [0, 1]$. Since only the rich living in the poor jurisdiction opt out, the actual tax burden per rich household is then weighed by the fraction of private education households to the total number of the rich $(\frac{N^h - n_2}{N^h})$. As $\frac{N^h - n_2}{N^h}$ approaching zero (one), the possibility of Pareto improvement is increasing (decreasing).

The problem of the central government is then to

$$\max_{s} \pi = \int_{p'-sF}^{p'} g(p, u)dp - \left(\frac{N^h - n_2}{N^h}\right)sFg(p' - sF, u) \qquad (3.20)$$

subject to $s \in [0, 1]$. Let $s^*$ is the optimal subsidy rate that solve (3.20), the following proposition is obtained.

**Proposition 7.** *Pareto improving subsidy rates exist and the optimal subsidy rate* $(s^*)$ *is either:*

*(a) full subsidisation ($s^* = 1$) if $MCV(s = 1) \geq MTB(s = 1)$; or*

*(b) partial subsidisation if $MCV(s^*) = MTB(s^*)$, where $0 < s^* < 1$.*

*Proof.* Let $\Omega$ and $\omega$ be the Lagrange multipliers of the Kuhn-Tucker conditions governing the upper-bound (one) and the lower-bound (zero) of $s$, respectively. The problem of the central government becomes

$$\max_{s \in [0,1]} \; \mathcal{L} = \int_{p'-sF}^{p'} g(p, u) dp - \left( \frac{N^h - n_2}{N^h} \right) sFg(p' - sF, u) + \Omega[1 - s] + \omega s.$$

The first-order conditions are:

$$Fg(p' - sF, u) - \left( \frac{N^h - n_2}{N^h} \right) \left( Fg(p' - sF, u) - sF^2 \frac{\partial g(p' - sF, u)}{\partial p} \right)$$
$$-\Omega + \omega = 0; \tag{3.21}$$

$$\Omega[1 - s] = 0 \text{ and } \Omega \geq 0; \tag{3.22}$$

$$\omega s = 0 \text{ and } \omega \geq 0. \tag{3.23}$$

Rearranging (3.21) yields

$$Fg(p' - sF, u) = \left( \frac{N^h - n_2}{N^h} \right) \left( Fg(p' - sF, u) - sF^2 \frac{\partial g(p' - sF, u)}{\partial p} \right)$$
$$+\Omega - \omega, \tag{3.24}$$

or $MCV(s) = MTB(s) + \Omega - \omega$. Using Lemma 5 and 6 along with the first-order conditions, there are 3 cases to consider:

*(i) $0 < s^* < 1$, then $\Omega = 0$, $\omega = 0$, and $MCV(s^*) = MTB(s^*)$;*

*(ii) $s^* = 1$, then $\Omega \geq 0$, $\omega = 0$, and $MCV(s = 1) \geq MTB(s = 1)$;*

*(iii) $s^* = 0$, then $\Omega = 0$, $\omega \geq 0$, and $MCV(s = 0) \leq MTB(s = 0)$.*

Case *(iii)* can be ruled out because, measuring at $s = 0$ and $\omega \geq 0$, condition (3.24) implies $\left( 1 - \frac{N^h - n_2}{N^h} \right) g(p', u) = -\omega \leq 0$. This requires $\frac{N^h - n_2}{N^h} \geq 1$, which violates the assumption that $\frac{N^h - n_2}{N^h} < 1$. At $s$ arbitrarily close to zero, $TB$ lies

below $CV$, Pareto improving subsidy rates always exist. Since both $CV$ and $TB$ have positive and increasing slopes, the optimal subsidy rate is then either the interior solution (case $(i)$) or the corner solution (case $(ii)$).  □

Proposition 7 shows that, in the the laissez-faire equilibrium with public and private education, price subsidisation for private education is a Pareto improving policy. The optimal subsidy rate is either full or partial. Partial subsidisation, or the interior solution, is the case when there exists $0 < s^* < 1$ such that $MCV(s^*) = MTB(s^*)$. This case is likely when $\frac{N^h - n_2}{N^h}$ is sufficiently large (approaching one) to make the tax burden to the rich sufficiently high. Full subsidisation, or the corner solution, is optimal if $MCV(s = 1) \geq MTB(s = 1)$. Similarly, this case is likely when $\frac{N^h - n_2}{N^h}$ is sufficiently low (approaching zero) to make the tax burden sufficiently low. In either case, the welfare of the private-education households increases.

Once $s^*$ is obtained, the welfare of the rich living in the rich jurisdiction along with the rent premium can be calculated. The before and after subsidy conditions for no migration among rich households are equation (3.25) and (3.26), respectively:

$$U(y^h - r_2 - pg_2^h, g_2^h) = U(y^h - z - pg_1^l - p'g_1^{h'}, g_1^{h'}) \tag{3.25}$$

$$U((1 - t)y^h - \tilde{r}_2 - p\tilde{g}_2^h, \tilde{g}_2^h) = U((1 - t)y^h - z - pg_1^l - (p' - s^*F)\tilde{g}_1^{h'}, \tilde{g}_1^{h'}) \tag{3.26}$$

where $t = \left(\frac{N^h - n_2}{N^h}\right) \frac{s^* F \tilde{g}_1^{h'}}{y^h}$.

Pareto improvement from the policy implies $U((1 - t)y^h - z - pg_1^l - (p' - s^*F)\tilde{g}_1^{h'}, \tilde{g}_1^{h'}) > U(y^h - z - pg_1^l - p'g_1^{h'}, g_1^{h'})$. Then, $U(y^h - ty^h - \tilde{r}_2 - p\tilde{g}_2^h, \tilde{g}_2^h) > U(y^h - r_2 - pg_2^h, g_2^h)$, which implies $ty^h + \tilde{r}_2 < r_2$. When the subsidisation policy is Pareto improving, the attractiveness to the rich of living in the poor jurisdiction and opting out increases. Therefore, the rent that balances the utility of the rich does not need to be as high as in the case without subsidies. Hence, the rich living

in the rich jurisdiction are also better off because the rent premium is lower enough to offset the tax burden. The total burden is then lower than the rent premium before subsidies.

Figure 3.5 illustrates the equilibrium allocation when full subsidisation is optimal. The poor's allocation is the same as before taxes at $e_1$. For the rich, the tax burden is $A - A''$, which equals $B' - B''$. The price-subsidised budget line of the rich in the poor jurisdiction is $B''b''$. The line $B''b''$ has the slope $p$ which is lower than the slope of $B''b_0$, the after-subsidy budget line when opting out and no subsidisation. Since this full subsidisation is Pareto improving, the utility of the rich increases from $U^{h'}$ to $U^h_t$. The equilibrium allocation of all the rich is then at $e''$. The rent premium is $A'' - B''$ which is less than the before-subsidy rent premium $A - B$. The rich in the rich jurisdiction also gain from the policy since the tax burden $(A - A'')$ plus the new rent premium $(A'' - B'')$ is less than the before-subsidy rent premium $(A - B)$.

Figure 3.6 illustrates the equilibrium allocation when partial subsidisation is optimal. The poor's allocation is at $e_1$. For the rich, the tax burden is $A - A''$, which equals $B' - B^*$. The price-subsidised budget line of the rich in the poor jurisdiction is $B^*b^*$. Its slope is now less than $p'$ but still lower than $p$ because of partial subsidisation. Thus, the line $B^*b^*$ has flatter slope than $B'b'$ which is the budget line when opting out and no subsidisation. Since this policy is Pareto improving, the utility of the opt-out rich increases from $U^{h'}$ to $U^h_t$. The equilibrium allocation of the opt-out rich is then at $e''$. For the rich living in the rich jurisdiction, no migration implies that their equilibrium allocation is at $e_2$ with the same utility $U^h_t$. The rent premium that accommodates such utility is $A'' - B''$ which is less than the before-subsidy rent premium $A - B$. Notice that the slope of $A''B''$, which is $p$, is less than the slope of $B^*b^*$ because the rich in jurisdiction 2 choose public education which is still cheaper per unit than private education under partial subsidisation.
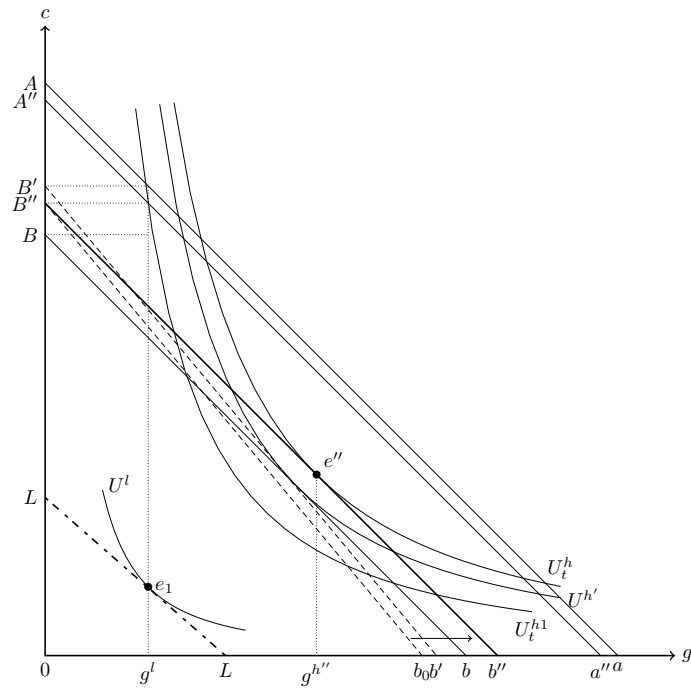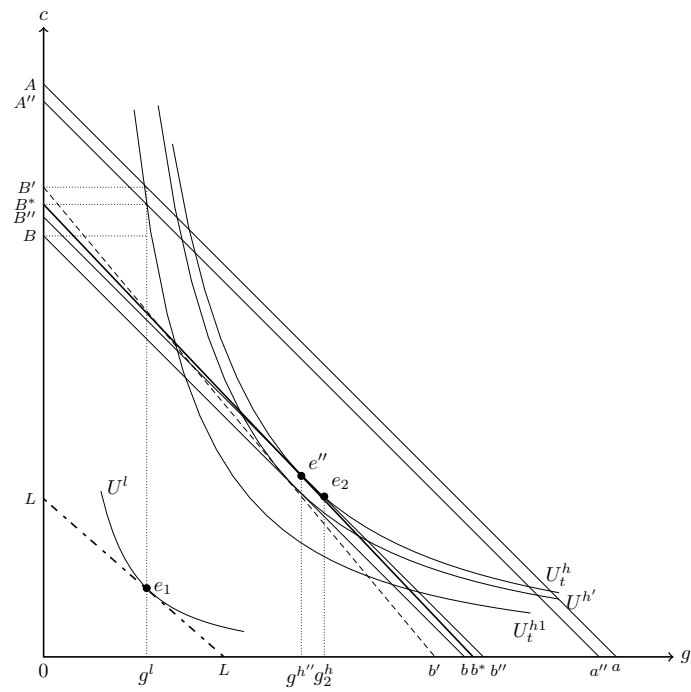
Figure 3.5: Full Subsidisation

Figure 3.6: Partial Subsidisation

They also gain from the policy since the tax burden $(A - A'')$ plus the new rent premium $(A'' - B'')$ is less than the before-subsidy rent premium $(A - B)$.

The next section investigates the comparative statics when the equilibrium is affected by changes in some important parameters.

## 3.6   Simulation Exercises

This section illustrates simulations of the effects of implementing price subsidisation for private education under different values of important parameters. First assume that the utility function is a Cobb-Douglas function of the form:

$$U \;=\; c^{1-\alpha}g^{\alpha}. \tag{3.27}$$

The relevant variables derived from the utility function and used in the following simulations are presented in the appendix. The exogenous variables are set as follows. $y^l = 10$ (corresponding to 10,000\$) and $y^h = 100$ (corresponding to \$100,000), which ignore extremely poor and extremely rich. Also, the difference in these two values is high enough to satisfy Assumption Y. Normalise $p = 1$ and impose $z = 1$. The number of poor and rich households are set to $N^l = 80\%$ and $N^h = 20\%$ of the total number of household $N$, respectively, since in the U.S. it is around 80% of households that have income less than $100,000\$.[9] In 2009, U.S. public school systems spent \$10,499 per pupil, which is around 21% of the median household income in that year.[10]   Therefore, $\alpha$ is set to 0.2. Let the values of $\frac{N^h - n_2}{N^h}$ and $p'$ be varied to illustrate how they affect the optimal subsidy rate. However, only in the simulations of Figure 3.7, $p'$ is set at two.[11]   Henceforth, denote

---

[9]See Table 690, U.S. Census Bureau.

[10]See Table 11, U.S. Census Bureau.

[11]$p'$ is set equal to two for an illustration purpose. Tsang (2002) shows that in some countries the cost per student of private schools is higher than public schools. In such case, the ratio of

$\xi = \frac{N^h - n_2}{N^h}$, the fraction of (rich) private-education households to the total number of rich households. $\xi$ may also be interpreted as excess demand for housing in the rich jurisdiction in the sense that without rent premiums all rich households would like to be in the same jurisdiction and consume public education services which are cheaper per unit than the private education.
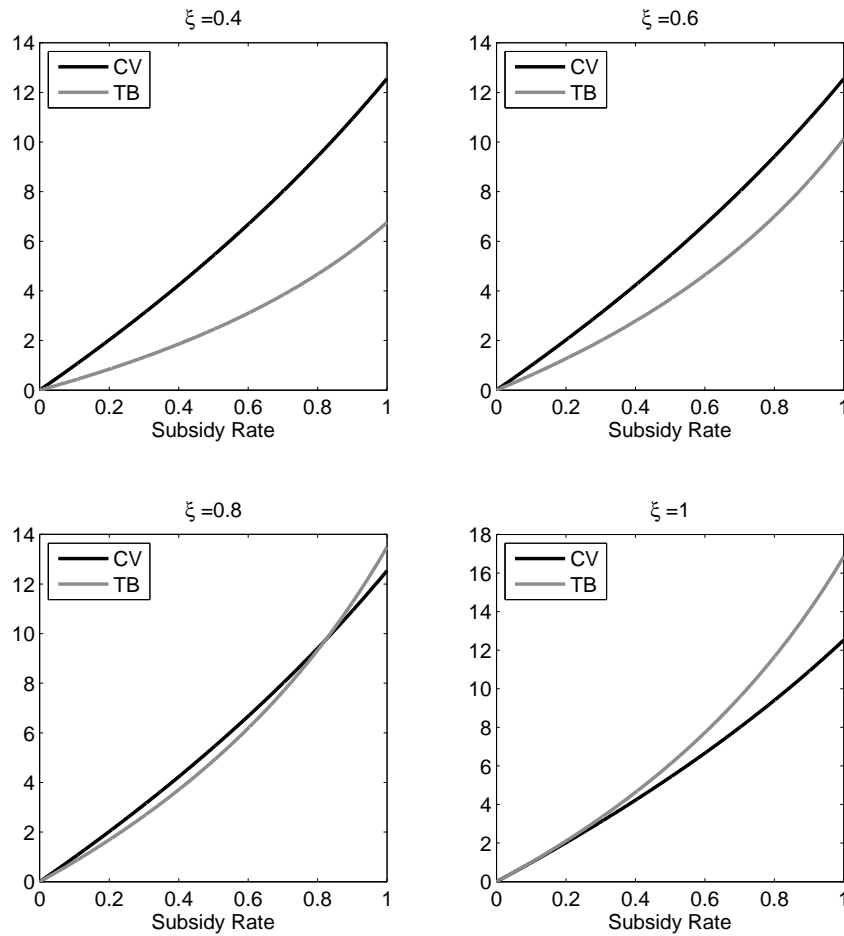


Figure 3.7: $CV$ and $TB$

Firstly, consider the properties of $CV$ and $TB$ as mentioned in Lemma 5 and 6. Figure 3.7 plots $CV$ and $TB$ curves for each given level of $\xi$. The properties in those two lemmas are satisfied, that is, $CV$ and $TB$ have positive and increasing

private to public school cost per student is in the range 1.22 - 1.46. In the simulations of Figure 3.8, $p'$ is then varied from one to two to cover the aforementioned range.

slope with respect to $s$. For $0 < \xi < 1$, at $s$ arbitrarily close to zero, $TB$ lies below $CV$. The difference or gap between $CV$ and $TB$ curves indicates the net welfare gains. The optimal policy is either full or partial subsidisation when $0 < \xi < 1$. If $\xi = 1$, $TB$ lies above $CV$ as shown in the lower right panel of Figure 3.7. In this case, all the rich consume private educational services and the tax burden is too high since there are no rich public-education households to share the burden as mentioned in the analysis of Figure 3.4. No subsidisation is then optimal.

For sufficiently low $\xi$ such as at 0.4, full subsidisation is optimal. In this case, the slope of $CV$ at $s = 1$ is still higher than the slope of $TB$, or the gap between $CV$ and $TB$ is greatest at $s = 1$. For sufficiently large $\xi$ such as at 0.6 and 0.8, partial subsidisation is optimal since there exists $0 < s < 1$ such that $MCV(s) = MTB(s)$. Define the optimal policy curve as a curve that plots the optimal subsidy rate $s^*$ for each given level of $\xi$. The gray solid line in Figure 3.8 summarises the optimal policy of the simulations of Figure 3.7 which sets $p' = 2$. The optimal policy curves are horizontal at $s = 1$ for sufficiently low $\xi$, that is, full subsidisation is optimal. On this part of the curves, $MCV(s = 1) > MTB(s = 1)$. From the cut-off $\xi$ and higher, the curves have negative slopes, meaning that $s^*$ is decreasing in $\xi$ because $\pi(s)$ is decreasing in $\xi$. This part of the curves has $MCV(s^*) = MTB(s^*)$.

To pin down the value of $s^*$ for each set of parameters, let us first consider unconstrained maximisation of the net welfare gain function $\pi(s)$. Denote $\tilde{s}$ as the unconstrained optimal subsidy rate. At $\tilde{s}$, $MCV(\tilde{s}) = MTB(\tilde{s})$. From the appendix, this implies $\tilde{s} = \frac{p'}{p'-p} \times \frac{1-\xi}{1-\alpha\xi}$. The first term on the $RHS$ is greater than one because $p' > p > 0$. The second term on the $RHS$ is less than one since $0 < \alpha, \xi < 1$. Therefore, given $p'$ and $p$, $\tilde{s} > 1$ for a sufficiently small $\xi$ (approaching zero). In this case, the optimal subsidy rate is then at $s^* = 1$ (full subsidisation) because subsidising beyond that cannot sustain in equilibrium. As mentioned in the previous section, any $s > 1$ will cause private education to be cheaper per unit

than public education, so every rich household opts out. The number of households utilising the policy becomes $N^h$ (changed from $N^h - n_2$), thus $\xi = 1$. Obviously, $\tilde{s} = 0$ when $\xi = 1$, no subsidisation is optimal. An interior solution occurs when $0 < \tilde{s} < 1$, which corresponds with sufficiently large $\xi$ (approaching one). Hence, the policy rule for the optimal subsidy rate is:

$$
s^* = \begin{cases} \tilde{s} = \frac{p'}{p'-p} \times \frac{1-\xi}{1-\alpha\xi} & \text{if } \tilde{s} \leq 1 \\\\ 1 & \text{if } \tilde{s} > 1. \end{cases}
\tag{3.28}
$$

Using the policy rule, $s^*$ for $\xi = 0.4$, $0.6$, $0.8$, and $1$, are $1$, $0.909$, $0.477$, and $0$, respectively, given $p' = 2$, $p = 1$, and $\alpha = 0.2$. Also, the optimal subsidy rates for all $\xi \in [0, 1]$, are shown in Figure 3.8. Using equation (3.28), define the cut-off $\bar{\xi}$ such that $\tilde{s} = \frac{p'}{p'-p} \times \frac{1-\bar{\xi}}{1-\alpha\bar{\xi}} = 1$. A policy rule curve thus has zero slope (horizontal) for $\xi \leq \bar{\xi}$ since $\tilde{s} \geq 1$ which implies $s^* = 1$. For $\xi > \bar{\xi}$ which implies $s^* = \tilde{s}$, the slope of the policy rule for these $\xi$ is negative because $\frac{\partial \tilde{s}}{\partial \xi} = \frac{\alpha - 1}{(1 - \alpha\xi)} < 0$, where $\alpha < 1$. The gray solid line in Figure 3.8 plots the policy rule of the above simulations (i.e. $p' = 2$).

Now consider how changes in the difference between $p$ and $p'$ affect the optimal policy rule. For simplicity, $p$ is fixed at one and $p'$ is varied between one and two, the difference between private and public prices ranges from 20% to 100%.[12] Figure 3.8 plots the optimal policy rule for each $p'$. Using (3.28), as $p'$ increases, the term $\frac{p'}{p'-p}$ decreases. The cut-off $\bar{\xi}$ falls. The policy rule curve then shifts to the left for the negative slope part of the curve. In other words, the policy rule curve shifts to the left for all $\xi \leq \bar{\xi}$ before the change. The possibility of full subsidisation to be the optimal policy decreases as $p'$ increases.

---

[12]Holding other parameters constant, the simulations where $p'$ is slightly greater than two were conducted and in these cases the rich living in the poor jurisdiction opt in. Since the focus is on public and private education, the analysis of changes in $p'$ is restricted to $p' \in (1, 2]$.
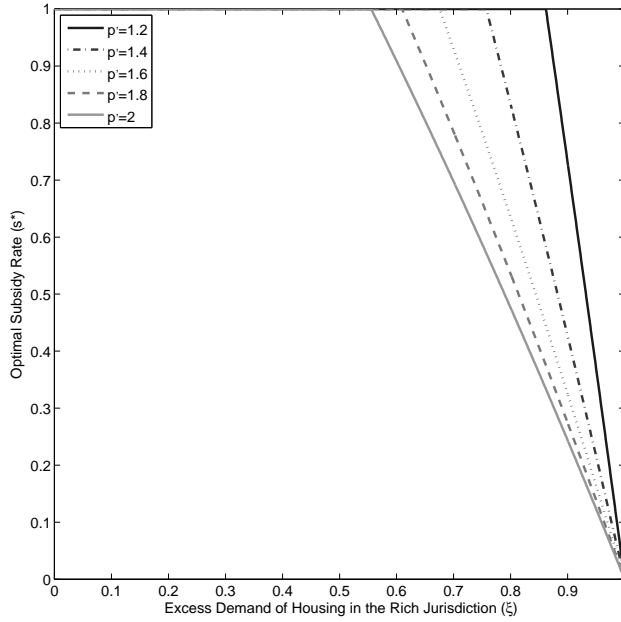
Figure 3.8: $s^*$ for Each $\xi$ when Varying $p'$

Lastly, how subsidy rates affect the opt-in rich living in the rich jurisdiction is presented in Figure 3.9. The details of variables used for the simulations are in the appendix. For a given $\xi$, an increase in the subsidy rate lowers the unit price of private education, the opt-out rich then demand higher educational services $g_1^{h'}$ and thus increasing utility. On the other hand, it also increases tax burden which in turn reduces disposable income and thus lowers both consumption and educational services as well as utility. A further increase in $s$ gives higher net gains if the former outweigh the latter effects. As explained in the previous section, the former effects are measured in terms of resources by compensating variation ($CV$) and the latter effects are measured by tax burden. In order to avoid replicating the same method, the simulations simply use Marshallian demand to compute the effects of subsidy rates on utilities, rent premiums, and tax burdens. The policy rule derived from this method is obviously equivalent to equation (3.28) (see the appendix). Tax burden per rich household is increasing in $s$ (panel ($b$)) because $s$ affects the burden both directly through the subsidy rate and indirectly through an increase in private
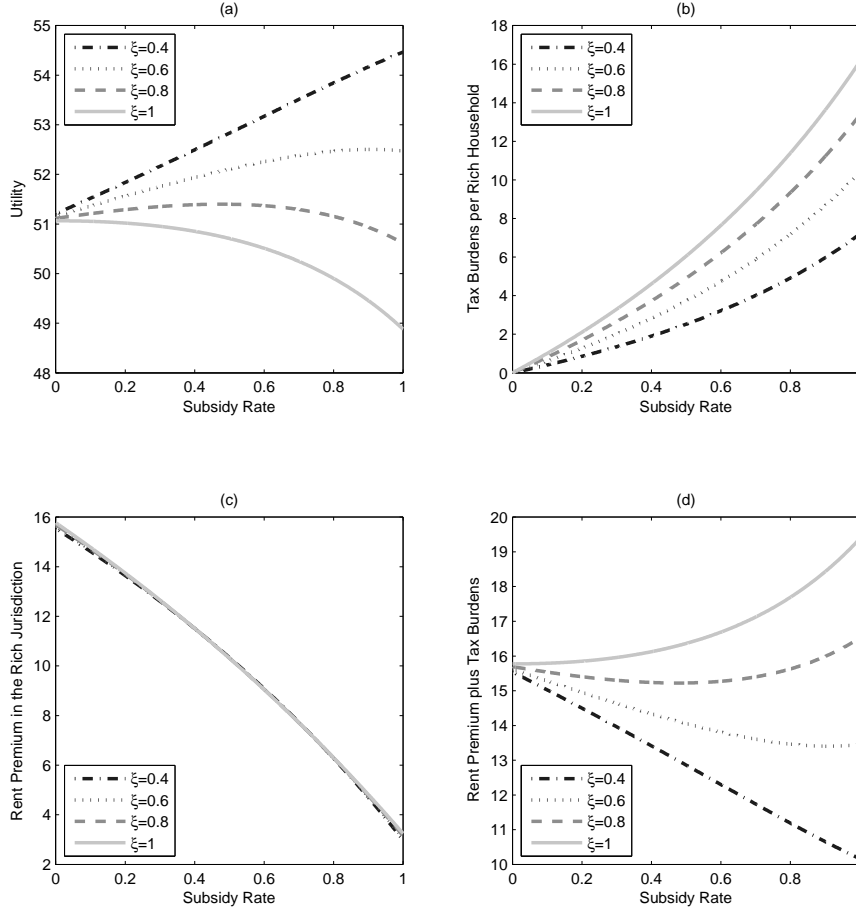
Figure 3.9: The Effects of Subsidy Rates on Rent Premiums and Tax Burden

educational demand $(g_1^{h'})$. For sufficiently low $\xi$, the benefits from the increased $s$ outweigh the tax burden, therefore the indirect utility of the private-education rich $V(s)$ increases (see panel $(a)$). This implies that the disposable income of the public-education rich $(y^h - ty^h - r_2)$ increases to make them better-off since they need to have the same utility as the private-education rich in equilibrium. Therefore, for Pareto improving subsidy rates, the tax burden plus rent premium $(ty^h + r_2)$ are decreasing in such subsidy rates. If the subsidy rates are too high such that there is no positive net welfare gain, the total burden is increasing in such subsidy rates. Panel $(d)$ shows total burden to the public-education rich.

The results from the simulations in this section confirm the analysis in the previous section that price subsidisation for private education is a Pareto improving policy. The optimal subsidisation that maximises the net welfare gains is either full or partial depending crucially on $\xi$ and $F$.

## 3.7  Discussion

An interesting result of this chapter is that price subsidisation for private education can be a Pareto improving policy. Pareto-improving subsidy rates exist and the optimal subsidy rate is either full or partial for a given set of parameters. Such subsidisation has to be financed by taxing income only of the rich since the poor never opt out in the model. This points out that a non-linear tax system is more appropriate than a linear one because the benefits of the policy are not evenly distributed among different types of households. Such a result of public-private sector equilibrium depends on the important assumptions that (i) private education is pricier per unit, (ii) all houses are identical and housing capacity in the rich jurisdiction cannot accommodate all rich households ($n_2 < N^h$), and (iii) the rent in the poor jurisdiction is fixed at the opportunity cost of a house ($r_1 = z$). The last assumption may be strong and not realistic. However, this concept of the reservation location is used in urban economics when studying housing prices. Moreover, one may think of the poor jurisdiction as a jurisdiction where undeveloped land is abundantly available. In this sense, if $z$ is assumed to be the opportunity cost of a house, any household then can always just pay $z$ for a new house to avoid paying any housing price greater than $z$ to the landlords. The concept of reservation price and undeveloped land is used in De Bartolome and Ross (2003), though in different context.

Relaxing the last assumption by allowing $r_1 \geq z$ (while keeping the reservation

price) will not change the analysis qualitatively. To see this, recall that in the public-private education equilibrium all the poor live in the poor jurisdiction while the rich live in both jurisdictions. In this equilibrium, $r_1$ has to satisfy condition (3.10) and $r_2$ has to satisfy condition (3.11) with equality (replacing $z$ on the $RHS$ of no migration conditions by $r_1$). For a given level of $r_1$ condition (3.11) is satisfied by only one value of $r_2$ which is higher than $r_1$ since $p' > p$. From the fact that the poor prefer their own choice of public educational services, any $r_1 \leq r_2$ satisfies condition (3.11). Therefore, a pair of $r_1$ and $r_2$, such that $r_2 > r_1$, satisfying condition (3.11) also satisfies condition (3.10) because the poor always prefer to live in their poor jurisdiction under such pair of rents. Moreover, because the analysis of price subsidisation takes the equilibrium with public and private education as the starting point, the analysis of Pareto improvement from the policy will not be altered qualitatively. In this sense, fixing $r_1$ at the reservation price does not affect the uniqueness of the equilibrium where public and private education coexist.

In a more complex setting such as in Nechyba (1999, 2000)'s model where households are also homeowners and houses are heterogeneous, the behavior of private education households is, to some extent, similar as in the model of this chapter. In his model, private education households who are rich and demand high quality housing seek to live in a jurisdiction which has their preferred housing quality and has the lowest property tax burden from financing local public education. Thus, private education households arise first in the poorest jurisdiction. In this chapter, with only two income types, the mixing of public and private education households is also in the poor jurisdiction, even though the main reason for the existence of private education comes from the fact that the housing capacity in the rich jurisdiction cannot accommodate all high demanders for educational services. The model here is also simple enough to provide analytical equilibrium while Nechyba's model has to rely on numerical results due to complexity.

## 3.8 Conclusion

This chapter provides a theoretical analysis of price subsidisation for private education. A simple multiple jurisdiction model is developed in which a majority of the residents in a jurisdiction determine the level of public educational services, and private providers charge the price per unit of education higher than the public providers. Price subsidisation for private education is Pareto improving. The optimal subsidisation is either partial or full, which eliminates the price difference between public and private sectors, depending crucially on the fraction of the private-education rich to the total rich households and on the difference between the unit prices of the two sectors. Even though this chapter gives a rationale to support price subsidisation for private education, a quantitative research with more complex settings, as in Nechyba (1999, 2000), may be conducted by incorporating cost disadvantage of private providers, and by considering a non-linear income tax system to finance the subsidisation as suggested in this chapter. The results would give a clearer picture of how the policy would affect households' education and residential choices when housing attributes and the initial distribution of homeowners vary across jurisdictions.

# Appendix

## Variables used in the simulations of Figure 3.7

Recall that $\theta_1 = \frac{N^h - n_2}{n_1}$, $\eta_1 = 1 - \theta_1$, $F = p' - p$, $\xi = \frac{N^h - n_2}{N^h}$, $s \in [0, 1]$, and $U = c^{1-\alpha} g^\alpha$. Immediately, the following variables are obtained:

$$
\begin{aligned}
g_1^l &= \frac{\alpha(y^l - z)}{p(1 - \theta_1)} \\
g_1^{h'} &= \frac{\alpha(y^h - z - pg_1^l)}{p'} \\
U_1^{h'} = u &= \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}(y^h - z - pg_1^l)}{(p')^\alpha} \\
m(p', u) &= \frac{u(p')^\alpha}{\alpha^\alpha(1-\alpha)^{1-\alpha}} = y^h - z - pg_1^l = y_1^h \\
m(p' - sF, u) &= \frac{u(p' - sF)^\alpha}{\alpha^\alpha(1-\alpha)^{1-\alpha}} = \frac{y_1^h(p' - sF)^\alpha}{p'^\alpha} \\
g(p' - sF, u) = \frac{\partial m(p' - sF, u)}{\partial(p' - sF)} &= u\left(\frac{\alpha}{(1-\alpha)(p' - sF)}\right)^{1-\alpha} = \frac{\alpha y_1^h}{p'^\alpha(p' - sF)^{1-\alpha}} \\
CV(s) = m(p', u) - m(p' - sF, u) &= \frac{u[(p')^\alpha - (p' - sF)^\alpha]}{\alpha^\alpha(1-\alpha)^{1-\alpha}} \\
&= y_1^h\left(1 - \frac{(p' - sF)^\alpha}{p'^\alpha}\right) \\
MCV(s) = \frac{\partial CV(s)}{\partial s} &= \frac{\alpha F y_1^h}{p'^\alpha(p' - sF)^{1-\alpha}} \\
TB(s) = \xi s F g(p' - sF, u) &= \xi s F u\left(\frac{\alpha}{(1-\alpha)(p' - sF)}\right)^{1-\alpha} \\
&= \alpha\xi s y_1^h\left(\frac{F}{p'^\alpha(p' - sF)^{1-\alpha}}\right) \\
MTB(s) = \frac{\partial TB(s)}{\partial s} &= \frac{\alpha y_1^h \xi F}{p'^\alpha}\left(\frac{p' - \alpha s F}{(p' - sF)^{2-\alpha}}\right) \\
\pi(s) = CV(s) - TB(s) &= \frac{u}{\alpha^\alpha(1-\alpha)^{1-\alpha}} \times \\
&\quad \left[(p')^\alpha - (p' - sF)^\alpha - \frac{\alpha\xi s F}{p' - sF}\right].
\end{aligned}
$$

Consider unconstrained maximisation of $\pi(s)$. Denote $\tilde{s}$ as the unconstrained

optimal subsidy rate. $\tilde{s}$ satisfies $\frac{\partial \pi(s)}{\partial s} = 0$. This implies that $\tilde{s} = \frac{p'}{p'-p} \times \frac{1-\xi}{1-\alpha\xi}$ as presented in the main text. The policy rule for the optimal subsidy rate is shown in equation (3.28).

## Variables used in the simulations of Figure 3.9

Recall that $\theta_1 = \frac{N^h - n_2}{n_1}$, $\eta_1 = 1 - \theta_1$, $F = p' - p$, $\xi = \frac{N^h - n_2}{N^h}$, $s \in [0, 1]$, and $U = c^{1-\alpha}g^\alpha$. Using the concept of Marshallian demand, all of the following variables are obtained:

$$
\begin{aligned}
g_1^l &= \frac{\alpha(y^l - z)}{p(1 - \theta_1)} \\
g_1^{h'} &= \frac{\alpha(y^h - z - pg_1^l)}{p' - sF(1 - \alpha\xi)} \\
ty^h &= \xi s F g_1^{h'} \\
c_1^{h'} &= \frac{(1 - \alpha)(p' - sF)(y^h - z - pg_1^l)}{p' - sF(1 - \alpha\xi)} \\
g_2^h &= \frac{\alpha(y^h - r_2 - \xi s F g_1^{h'})}{p^\alpha} \\
V(s) &= \frac{\alpha^\alpha(1 - \alpha)^{1-\alpha}(p' - sF)^{1-\alpha}(y^h - z - pg_1^l)}{p' - sF(1 - \alpha\xi)} \\
\frac{\partial V(s)}{\partial s} &= \frac{\alpha^\alpha(1 - \alpha)^{1-\alpha}(y^h - z - pg_1^l)}{(p' - sF)^\alpha (p' - sF(1 - \alpha\xi))^2} \times [p'(1 - \xi) - sF(1 - \alpha\xi)] \\
W(s) &= \frac{\alpha^\alpha(1 - \alpha)^{1-\alpha}(y^h - r_2 - \xi s F g_1^{h'})}{p^\alpha}
\end{aligned}
$$

where $V$ and $W$ are the indirect utilities of the rich living in the poor and the rich jurisdictions, respectively. The rent premium $r_2$ then solves $V(s) = W(s)$. Consider the maximisation of the opt-out rich who obtain $V(s)$. Denote $\hat{s}$ as the unconstrained optimal subsidy rate. $\hat{s}$ satisfies $\frac{\partial V(s)}{\partial s} = 0$. This implies $\hat{s} = \frac{p'}{p'-p} \times \frac{1-\xi}{1-\alpha\xi}$, which is equivalent to $\tilde{s}$ in the main text. Hence, the policy rule computed by this method is equivalent to equation (3.28).

# Chapter 4

# School and Residential Choices in Frictional Housing Markets

## 4.1  Introduction

Quality of local public schools plays an important role in residential choices of households.[1] In order to be eligible for services from a local public school, residing in the school attendance zone may be required. Studies focusing on school and residential choices generally assume frictionless housing markets. However, it is natural in housing markets that frictions are present. It takes time for buyers to find appropriate sellers and vice versa. Moreover, households may value school services only in the early stages of their lives, such as when they have school-age children. Therefore, frictions could prevent some households from relocating themselves to another location with a better public school, even though they are willing to buy and sell houses at the transacted prices in the housing markets. In addition, the benefits of living in a good school location may be derived not only from school quality but also from the resale value of the house once school services

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[1]See Nechyba and Strauss (1998) as well as references therein.

are no longer valued. This chapter explores these issues by constructing a model of school and residential choices in frictional housing markets. The relocation of households across frictional housing markets due to different life-cycle valuations of local amenities, such as public schools, as well as the resale aspect of housing are the main contribution of this chapter.

This chapter develops an overlapping-generation (OLG) model. Time is discrete. Agents live for two periods. In each period, young and old generations coexist. When young, agents value school quality. But when they become old they do not value school services. When young agents are born, they inherit their residence from their parents. The economy consists of two frictional home-owner housing markets. Each market corresponds with a neighbourhood. In one of these two neighbourhoods, there is a high-quality public school, and in the other a low-quality public school. The school qualities are exogenous and houses are identical in the two neighbourhoods. Residency in the neighbourhood is required to benefit from the local public school. There is also another location where there are no frictions and is composed of abundantly available rental houses. Rental housing is inferior compared to those in the frictional housing markets such that the rental prices and housing utility are normalised to zero. Rental houses are owned by landlords outside the model. It is assumed that rental housing is the location of all house buyers. This means that house-owner agents have to successfully sell their houses and move to rental houses before they can look for a new house in another location. Therefore, sellers in the model are house owners. The locational search decision of buyers is a discrete choice, whether to search in the high-quality school or in the low-quality school neighbourhood, or not to search. The model assumes that, in each period, there are two subperiods. This assumption allows for a possibility that some young agents can relocate their residence from the low-quality school neighbourhood to the high-quality school neighbourhood within a period.

Within a housing market, search and matching is random. Buyers and sellers meet randomly in bilateral pairs. For each pair, the housing price is determined by Nash bargaining over the joint surplus. There are no search costs, moving costs, or transaction costs.

This chapter focuses on steady-state equilibria in which some young agents who live in the low-quality school neighbourhood can sell their houses and move to the rental housing in the first subperiod, and then search and successfully buy houses in the high-quality school neighbourhood in the next subperiod. For some values of parameters, the equilibria exist under sufficiently high differences in public school qualities across locations. For individuals, the benefits of living in a good school location are derived not only from school quality but also the resale value of the house once school services are no longer valued. Therefore, housing prices internalise benefits from school quality as well as from expected utility from future trade potentials. Comparative statics are also studied by varying the parameters related to school quality. Equilibria exist in which increasing the quality of the low-quality school improves total welfare but affects agents across locations differently. The analysis also includes the effects of private alternatives. To show the effects, the model is modified by allowing for non-zero benefits from rental housing of young agents, assuming that private schools do not impose residential requirement. When relatively good quality private schools are available, the young buyers are better off while some old house owners are worse off due to a reduction in the returns from house sales. In addition, this chapter essentially shows the dual role of owner-occupied housing, that is, a house is a consumption good and an investment good (asset) at the same time. From the consumption aspect, owner-occupied housing gives consumption benefits in terms of housing attributes and, as featured in this chapter, local amenities. On the other hand, owner-occupied housing also has an asset aspect: the housing value internalises not only consumption benefits for the

occupant but also the expected value of future trade which could occur because of different valuations among young and old agents over local amenities such as public schools. Thus, the motive to acquire a house is driven by both consumption and investment incentives.[2]

There exists indeterminacy of equilibria. Indeterminacy arises as a by-product of the assumption that search costs are zero. Under indeterminacy, the search behavior of some buyers is indeterminate, that is, they are indifferent between searching and not searching in equilibrium. These buyers cannot gain from trade in any meetings in the housing markets. Therefore, under a given set of parameters, there are many equilibria depending on how many of these buyers decide to search. The result shows that the total welfare is highest if such agents do not search. The reason is that allowing them to search generates externalities as these agents create congestion in the housing markets and thus affecting the distribution of agents across locations.

It is worth mentioning that, in the context of this chapter, housing frictions, in combination with differences in qualities of neighbourhood schools, may result in inequality over an individual's lifetime. The baseline equilibrium highlights this point. Housing frictions prevent some young agents from moving into the better school neighbourhood. Therefore, some young agents get trapped in the bad neighbourhood and gain lower lifetime utility, compared to young agents who are inherited housing in the good neighbourhood. Even though this chapter does not model human capital and labour market explicitly, it is likely that an extension including human capital accumulation and a labour market would result in that young agents who cannot access to the better school would have lower level of human capital, and thus lower wages. Since agents in the same cohort are identical

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[2]However, this chapter does not address owner-occupied housing as a portfolio choice. For such topic, see, among others, Poterba (1984), Brueckner (1997), and Flavin and Yamashita (2002). Most households in the U.S. and other countries have owner-occupied housing as the dominant asset in their portfolios. The dual role of housing plays a crucial part for such choice.

except their housing locations, this chapter point out that heterogeneity between locations and housing search frictions alone can generate inequality over lifetime.[3]

This chapter relates to the literature on school and residential choices. Most papers assume frictionless housing markets and focus on how endogenous public school or educational quality affects residential choices. A common result is that households segregate themselves by their valuations of school qualities across locations. These valuations generally depend on levels of household income and ability of children. Also, equilibrium housing prices internalise school or educational quality such that there is no incentive for households to move to another location given the transacted prices. Studies in this strand of literature include, for example, Benabou (1993), Nechyba (1999, 2000), Epple and Romano (2003), and Hanushek et al. (2011). Benabou (1993) links residential choice, human capital investment, and productivity in a multi-community city. The model features local complementarities in human capital investment such that the costs of education in a community decrease with the fraction of high-skill individuals. Such complementarities induce occupational segregation. Epple and Romano (2003) provide an analysis of the effects of finance policies on neighbourhood schools. A school's quality is determined by the average ability of the student body (peer-group quality). Their model predicts a sorting of neighbourhood school qualities. A higher quality school locates in a higher rent neighbourhood which composes of relatively richer and higher ability students. Unlike the former two papers, Nechyba (1999, 2000) includes private schools into his multi-community model. In Nechyba (1999), the focus is on the impacts of private school vouchers on migration and segregation patterns under different systems of public school finance, while in Nechyba (2000),

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[3]Benabou (1993) shows how heterogeneity between locations can generates inequalities. Also, there is a number of papers exploring the determinants of inequalities over lifetime. One set of papers focuses on mimicking the wealth distribution (see, for examples, Keane and Wolpin (1997) and Huggett et al. (2011)), while another set of papers digs deeper in the process that generates lifetime inequalities (see, for example, Li and Yao (2007)).

the focus is on the impacts of different private school voucher policies. Hanushek et al. (2011) develop a multiple-jurisdiction model to study the impacts of private schools on educational outcomes. The housing price of a location is determined by auction. The auction winner is the agent with the highest bid for the location which gives utility derived from its housing size and school quality. They show that private school choice improves the welfare of all households.

This chapter abstracts from these papers by taking school quality as exogenous. This assumption is used in some of the studies below that apply a search-theoretical framework to understand the housing market. These studies tend to assume an exogenous common component of dividends or instantaneous utilities of housing services. In addition, private schooling is not modeled explicitly in this chapter, but rather implicitly interpreted from changes of some school-quality parameters. These simplifications are made in order to focus on frictions in the housing markets while keeping the multi-community aspect.

The chapter also relates to the literature that applies search models to understand the frictional housing market. Most papers assume a single-market environment but comprehensively explain key characteristics of the housing market such as vacancy rates, price dispersion, and liquidity (time in the market of units for sale). Agents are assumed to either live forever but subject to shocks on their states or leave the market and get replaced by new agents once they trade. The chapter differs from this strand of literature by its ability to explain the relocation of households across heterogeneous housing markets driven by differences in local amenities such as public schools.

A set of papers assumes that search is random. Buyers and sellers meet bilaterally in pairs and housing prices are determined by either Nash bargaining or take-it-or-leave-it offers by either sellers or buyers. For examples, see Wheaton (1990), Krainer (2001), Albrecht et al. (2007), and Head and Lloyd-Ellis (2012).

Wheaton (1990) builds a model of the house-owner housing market to explain some stylised facts such as structural vacancy rates, vacancy duration for selling units, and switching states of market participants (from matched to mismatched and vice versa). A household becomes mismatched by random shocks and is assumed to search for a new house prior to selling the old house. Consequently, trade occurs between mismatched households who own one house and matched households who own two houses. Krainer (2001) constructs a search model to explain the correlations among prices, average selling times (liquidity) and volume of transactions. Agents are heterogeneous in their idiosyncratic component of utility from housing. Further to that, all houses have a common component of dividends which varies between high and low states. The paper incorporates only aggregate demand shocks. Albrecht et al. (2007) develop a search model in which the flow values of being in the market change over time. Specifically, buyers and sellers enter the market with a relaxed state which gives a high flow value. Over time, if an agent does not trade, she eventually changes her state to be desperate which yields a lower flow value from being in the market. Unlike the papers that consider only one market, Head and Lloyd-Ellis (2012) build a multiple-city model to link labour and housing markets. They explain how liquidity in the housing markets affects the decision of house owners to accept job offers from other cities. Such decision also depends on the labour market of the current location. Without facing the liquidity problem, renters accept job offers at a higher rate. In this way, the paper links home-ownership and unemployment not only at the city level but also at the aggregate level.

This chapter applies a random search framework into a simple multiple-location model. The crucial difference between this chapter and Head and Lloyd-Ellis (2012) is that whereas the relocation motivation in this chapter comes from the differences in valuations and qualities of public schools, the main motivation to relocate in

Head and Lloyd-Ellis (2012) comes from random job offers from other locations. In addition, their paper assumes that the flow values derived from housing services are constant in any locations but agents have different states of employment and home-ownership. This chapter, on the other hand, allows for different valuations of housing services among locations but abstracts from labor markets and home-ownership. Thus, the work in this chapter may be seen as a complement to their paper in the sense that, if local amenities vary in terms of quality, the relocation decision of agents whether to accept a job offer and move to other city would be affected. Such effects are likely to be less prevalent if private alternatives such as private schools are available.

Another set of papers assumes that search is directed. Sellers post prices first, then buyers observe them and decide which seller to visit. For examples, see Carrillo (2012), Stacey (2012), Albrecht et al. (2012) and Díaz and Jerez (2013). Carrillo (2012) develops a search model and estimates it. In the model, sellers deal with one buyer at a time. The buyer may make a counteroffer which is the reservation value of the seller. Therefore, the seller either gets the asking price or her reservation value. Stacey (2012) explores when sellers cannot commit to their asking prices. This allows the terms of trade to be determined by different processes. Houses are sold sometimes through auctions with multiple bidders, and sometimes through bilateral bargaining. In addition, he incorporates real estate agents who can improve matching quality, segment the market, and alleviate the information problem. This multiple price-mechanism feature is carried through in Albrecht et al. (2012). The difference is that in Stacey (2012) sellers signal their type by the type of real estate services they use, while in Albrecht et al. (2012) sellers signal by limited commitment to their asking price. Following Wheaton (1990), Díaz and Jerez (2013) construct a competitive search model to study business cycle properties in the housing market. Households are subjected to idiosyncratic shocks which

affect the valuation of their residence. Díaz and Jerez (2013) differ from others by studying the effects of both aggregate demand and supply shocks on key housing market variables such as vacancies and sales.

This chapter abstracts from directed search, multiple price mechanisms, real estate agents, and all sorts of shocks, as well as other important features of the housing market, such as heterogeneous housing attributes, housing construction (see, for example, Glaeser and Gyourko (2006)), credit constraints (see, for example, Stein (1995); Ortalo-Magne and Rady (2006); Kiyotaki et al. (2011)), and price dispersion (Van Nieuwerburgh and Weill (2010)), to focus on the relocation of households across frictional housing markets in response to different quality of local amenities.

The chapter proceeds by developing the model in the next section. Section 4.3 solves for equilibria and Section 4.4 analyses comparative statics. Results and indeterminacy of equilibria are discussed in Section 4.5 and Section 4.6 concludes.

## 4.2   The Model

Consider an overlapping-generation (OLG) model in which agents live for two periods. In each period young ($y$) and old ($e$) generations coexist. There is a continuum of agents with unit mass in each cohort. There is no population growth. Time is discrete and continues forever. All agents are assumed to maximise expected utility.

The economy consists of 3 locations: neighbourhood $a$, neighbourhood $b$, and rental housing, $r$. Each neighbourhood has a fixed number of houses. Let $A$ and $B$ be the total number of houses in $a$ and $b$, respectively. Housing is an indivisible good. All houses in $a$ and $b$ are identical and occupied by their owners. Assume that $A + B < 2$, that is, there are always some agents who have to live in rental housing. Therefore, it is always the case that the total number of house owners is

$A + B$, and the total number of renters is $2 - A - B$. Assume further that there are abundantly available rental houses such that there are no uncertainty nor any frictions in getting a rental house. Rental houses are owned by landlords outside the model. Housing in $r$ however is inferior in terms of utility from housing services. It is assumed that the housing prices in $r$ are fixed and equal to the utility from rental housing services which is normalised to zero. Therefore, the focus is on the housing markets in $a$ and $b$ where frictions will be present.

Neighbourhoods $a$ and $b$ each have one public school of an exogenous quality. When young, agents care about school quality and do not care when they are old. As houses are identical in $a$ and $b$, young agents obtain higher utility from housing services than old agents when living in both neighbourhoods. In this sense, housing services include both benefits from housing attributes and local amenities such as public schools which, relatively, benefit the young more than the old. This is essentially the consumption aspect of housing.

Utility is transferable, and there are no financial constraints. Agents obtain utility from housing services only at the end of each period. Assume that the quality of the school in $a$ is better than in $b$, and both are better than in $r$. In contrast, old agents obtain equal housing service utility if living in $a$ and $b$, but higher than if living in $r$. Therefore, the ranking of housing service utilities is:

$$U_a^y > U_b^y > \quad U_a^e = U_b^e > \quad U_r^y = U_r^e = 0, \tag{4.1}$$

where the superscripts indicate types of agents and the subscripts denote locations.

In each period, the housing markets have two subperiods. This assumption allows for relocation of some young house owners within a period. Agents can occupy only one house at a time. House owners in $a$ and $b$ need to successfully sell their houses and then move to rental housing before they can search for a new house

in the next subperiod. This means that in this model sellers are house owners and buyers are renters.

There are frictions in the housing markets in $a$ and $b$ but no search costs and transaction costs. Uncertainty arises from the assumption that it takes time for buyers to find suitable sellers and vice versa, perhaps because of imperfect information. Given the number of sellers and buyers in a housing market, buyers and sellers meet randomly in bilateral pairs. The total number of meetings is governed by a meeting technology which is assumed to be equivalent in both $a$ and $b$. When a seller and a buyer meet, the housing price is determined by Nash bargaining over the total bilateral surplus. If the bargaining price is acceptable to both seller and buyer, trade occurs. If agents do not meet any tradable partners, they stay in their current houses.

Assume a Pissarides (2000) style meeting technology $M(u, v)$, where $v$ denotes the number of sellers, and $u$ is the number of buyers. Given $u$ and $v$, $M(u, v)$ gives the total number of meetings. The meeting technology is increasing, concave, differentiable in each argument, and constant returns to scale as in standard random search models. Also, $M(u, v) \leq \min\{u, v\}$. Let $\theta = u/v$ be the tightness of a housing market. The meeting probability for a buyer is $p(\theta) = M(u, v)/u = M(1, 1/\theta)$ and the meeting probability for a seller is $q(\theta) = M(u, v)/v = M(\theta, 1)$. Moreover, $p(\theta)$ is decreasing in $\theta$, $q(\theta)$ is increasing in $\theta$, and $q(\theta) = \theta p(\theta)$. Assume that $q$ is concave, $q(0) = 0$, and $q(\infty) = 1$. Note that $q(\theta)$ is only the probability that a seller will meet a buyer, but not the trading probability. The latter probability additionally depends on "what type" of the buyer the seller actually meets since not all meetings lead to trade under heterogeneity in buyers and sellers. The trading probability will be explained below. The same logic can be applied to the trading probability for a buyer.

In the numerical results that follow, the meeting technology takes the form:

$$m(u,v) = \frac{uv}{[u^\alpha + v^\alpha]^{1/\alpha}}. \tag{4.2}$$

The meeting probability for a buyer is then $p(\theta) = \frac{1}{[\theta^\alpha + 1]^{1/\alpha}}$, and the meeting probability for a seller is $q(\theta) = \frac{\theta}{[\theta^\alpha + 1]^{1/\alpha}}$. This channel meeting technology satisfies the properties discussed above and gives the meeting probabilities within the range of zero and one.

The model will focus only on steady-state equilibria, therefore time-period indicators are ignored and only subperiod indicators are used. Henceforth, $i$, $\hat{i} \in \{y,\, e\}$ denote types of agents, $k \in \{a,\, b\}$ denotes a neighbourhood, $r$ denotes rental housing, and $l \in \{1, 2\}$ denotes a subperiod. Let $V_{kl}^i$ denote the value function of type-$i$ sellers living in $k$ at the beginning of subperiod $l$. For convenience, let $kl$ denote neighbourhood $k$ in subperiod $l$ and $rl$ denote rental housing in subperiod $l$.

For each meeting, the housing price $w$ is determined by Nash bargaining. Let $\sigma$ be the bargaining power parameter of the seller. In the first subperiod, $w$ is acceptable for a type-$\hat{i}$ buyer only if

$$V_{k2}^{\hat{i}} - w \geq V_{r2}^{\hat{i}}. \tag{4.3}$$

The net benefit from moving into $k$, the left-hand side (LHS), has to be at least the buyer's reservation value function, the right-hand side (RHS).

For a type-$i$ seller, $w$ is acceptable only if

$$V_{r2}^{i} + w \geq V_{k2}^{i}. \tag{4.4}$$

The benefit from moving into $r$ plus the housing price (LHS) has to be at least the seller's reservation value function (RHS).

The bargaining price when type-$\hat{i}$ buyer meets with type-$i$ seller in $k1$ is:

$$w_{k1}^{i\hat{i}} = \arg\max_{w} (V_{r2}^{i} - V_{k2}^{i} + w)^{\sigma}(V_{k2}^{\hat{i}} - V_{r2}^{\hat{i}} - w)^{1-\sigma}.$$

The first-order condition gives the solution:

$$w_{k1}^{i\hat{i}} = V_{k2}^{i} - V_{r2}^{i} + \sigma S_{k1}^{i\hat{i}} \tag{4.5}$$

where $S_{k1}^{i\hat{i}} = V_{k2}^{\hat{i}} - V_{r2}^{\hat{i}} - V_{k2}^{i} + V_{r2}^{i}$ denotes the joint surplus of the meeting. If trade occurs, the buyer gets

$$V_{k2}^{\hat{i}} - w_{k1}^{i\hat{i}} = V_{r2}^{\hat{i}} + (1-\sigma)S_{k1}^{i\hat{i}}, \tag{4.6}$$

and the seller gets

$$V_{r2}^{i} + w_{k1}^{i\hat{i}} = V_{k2}^{i} + \sigma S_{k1}^{i\hat{i}}. \tag{4.7}$$

Obviously from (4.3)-(4.7), trade can occur only when the joint surplus is non-negative.

In the second subperiod, $w$ is acceptable for the buyer only if

$$U_{k}^{\hat{i}} + \beta V_{k1}^{\hat{i}'} - w \geq U_{r}^{\hat{i}} + \beta V_{r1}^{\hat{i}'}, \tag{4.8}$$

and is acceptable for the seller only if

$$U_{r}^{i} + \beta V_{r1}^{i'} + w \geq U_{k}^{i} + \beta V_{k1}^{i'}, \tag{4.9}$$

where the prime superscripts indicate the next period valuation. Obviously in this OLG model, $V^{e'} = 0$ and $V^{y'} = V^{e}$ because old agents die at the end of the period

and young agents become old in the next period. Notice that the value function of an agent consists of housing service utility plus the discounted value function of the next period in the location she lives.

In the second subperiod, the bargaining price is:

$$w_{k2}^{i\hat{i}} = \arg\max_{w} (U_r^i + \beta V_{r1}^{i'} - U_k^i - \beta V_{k1}^{i'} + w)^\sigma (U_k^{\hat{i}} + \beta V_{k1}^{\hat{i}'} - U_r^{\hat{i}} - \beta V_{r1}^{\hat{i}'} - w)^{1-\sigma}.$$

The first-order condition gives the solution:

$$w_{k2}^{i\hat{i}} = (U_k^i - U_r^i) + \beta(V_{k1}^{i'} - V_{r1}^{i'}) + \sigma S_{k2}^{i\hat{i}} \tag{4.10}$$

where $S_{k2}^{i\hat{i}} = (U_k^{\hat{i}} - U_r^{\hat{i}} - U_k^i + U_r^i) + \beta(V_{k1}^{\hat{i}'} - V_{r1}^{\hat{i}'} - V_{k1}^{i'} + V_{r1}^{i'})$ is the joint surplus of the meeting. If trade occurs, the buyer gets

$$U_k^{\hat{i}} + \beta V_{k1}^{\hat{i}'} - w_{k2}^{i\hat{i}} = U_r^{\hat{i}} + \beta V_{r1}^{\hat{i}'} + (1-\sigma)S_{k2}^{i\hat{i}}, \tag{4.11}$$

and the seller gets

$$U_r^i + \beta V_{r1}^{i'} + w_{k2}^{i\hat{i}} = U_k^i + \beta V_{k1}^{i'} + \sigma S_{k2}^{i\hat{i}}. \tag{4.12}$$

Using (4.8)-(4.12), again trade can occur only when the joint surplus is non-negative. This points out that the willingness to trade is tied to the sign of the joint surplus of the meeting. Moreover, the definition of joint surpluses implies that

$$S_{kl}^{i\hat{i}} = -S_{kl}^{\hat{i}i}. \tag{4.13}$$

This means that the joint surplus of meetings between type-$i$ sellers and type-$\hat{i}$ buyers is equal in absolute terms but has a different sign to the joint surplus of meetings between type-$\hat{i}$ sellers and type-$i$ buyers. If $i = \hat{i}$, $S_{k1}^{i\hat{i}} = S_{k1}^{\hat{i}i} = 0$, when the

same-type agents meet, the joint surplus is zero and thus both trading counterparts are neither better off nor worse off from trading. Trade could occur in this case but it will not affect the distribution of types across subperiods and locations. In contrast, if $i \neq \hat{i}$, trade will affect the distribution of agents and there is a possibility that sellers would be better off when the joint surplus is positive. Thus, the model will proceed by focusing on when different types of agents meet.

Let $D_{kl}^{i\hat{i}}$ denote the decision rule of type-$i$ sellers in $k$ when they meet type-$\hat{i}$ buyers in subperiod $l$, $i \neq \hat{i}$, $D_{kl}^{i\hat{i}} \in \{0, 1\}$, where $D_{kl}^{i\hat{i}} = 1$ means that type-$i$ sellers choose to sell their houses when they meet with type-$\hat{i}$ buyers. Therefore, the optimal decision rule is:

$$
D_{kl}^{i\hat{i}} = \begin{cases} 1, & \text{if } S_{kl}^{i\hat{i}} \geq 0 \\ 0, & \text{otherwise.} \end{cases} \tag{4.14}
$$

Let $\mathcal{D}$ denote the set of all decision rules and $\mathcal{S}$ denote the set of all joint surpluses.

Given (4.14), the trading probability for a seller equals the meeting probability for a seller times the fraction of buyers that have a different type. Similarly, the trading probability for a buyer equals the meeting probability for a buyer times the fraction of sellers that have a different type. Denote $Q_{kl}^i$ and $P_{kl}^{\hat{i}}$ as the trading probabilities for type-$i$ sellers and type-$\hat{i}$ buyers in $kl$, respectively. Therefore,

$$
Q_{kl}^i = D_{kl}^{i\hat{i}} q(\theta_{kl}) \frac{n_{kl}^{\hat{i}r}}{u_{kl}} \tag{4.15}
$$

$$
P_{kl}^{\hat{i}} = D_{kl}^{i\hat{i}} p(\theta_{kl}) \frac{n_{kl}^i}{v_{kl}}, \tag{4.16}
$$

where $n_{kl}^i$ denotes the number of type-$i$ agents in $kl$ and $n_{kl}^{ir}$ denotes the number of

type-$\hat{i}$ agents in $r$ who search in $kl$. Given $\theta p(\theta) = q(\theta)$,

$$Q_{kl}^i = D_{kl}^{i\hat{i}}\theta p(\theta_{kl})\frac{n_{kl}^{\hat{i}r}}{u_{kl}} = D_{kl}^{i\hat{i}}p(\theta_{kl})\frac{n_{kl}^{\hat{i}r}}{v_{kl}}. \tag{4.17}$$

Since sellers can turn down any bargaining price that gives the returns lower than their reservation value, under no search and transaction costs, all houses in neighbourhoods $a$ and $b$ are assumed to be up for sale. This means that, in each neighbourhood, the number of sellers is equal to the total number of houses:

$$v_{a1} = v_{a2} = A \tag{4.18}$$

$$v_{b1} = v_{b2} = B. \tag{4.19}$$

In this model, there are no vacancies in the sense that all houses for sale are occupied by their owners.

Now turn to the distribution of agents. In steady state, there are two subperiod distributions of agents to consider:

$$H_1 = \{n_{a1}^y, n_{a1}^e, n_{b1}^y, n_{b1}^e, n_{a1}^{yr}, n_{a1}^{er}, n_{b1}^{yr}, n_{b1}^{er}\}$$

$$H_2 = \{n_{a2}^y, n_{a2}^e, n_{b2}^y, n_{b2}^e, n_{a2}^{yr}, n_{a2}^{er}, n_{b2}^{yr}, n_{b2}^{er}\}$$

where $H_1$ and $H_2$ are the distributions *at the beginning* of the first and the second subperiods, respectively. Let $\mathcal{H} = \{H_1, H_2\}$ denote the set of steady-state equilibrium distributions. In equilibrium, $\mathcal{H}$ must clear the numbers of house owners

(4.20)-(4.21), the numbers of renters (4.22), and the population sizes (4.23)-(4.24):

$$n_{a1}^{y} + n_{a1}^{e} = n_{a2}^{y} + n_{a2}^{e} = A \qquad (4.20)$$

$$n_{b1}^{y} + n_{b1}^{e} = n_{b2}^{y} + n_{b2}^{e} = B \qquad (4.21)$$

$$n_{a1}^{yr} + n_{b1}^{yr} + n_{a1}^{er} + n_{b1}^{er} = n_{a2}^{yr} + n_{b2}^{yr} + n_{a2}^{er} + n_{b2}^{er} = 2 - A - B \qquad (4.22)$$

$$n_{a1}^{y} + n_{b1}^{y} + n_{a1}^{yr} + n_{b1}^{yr} = 1 \qquad (4.23)$$

$$n_{a2}^{y} + n_{b2}^{y} + n_{a2}^{yr} + n_{b2}^{yr} = 1. \qquad (4.24)$$

Moreover, in steady state, $\mathcal{H}$ has to be constant overtime and therefore must satisfy the law of motions which will be derived below. Denote $\Delta_{kl}^{y} = P_{kl}^{y} n_{kl}^{yr} - P_{kl}^{e} n_{kl}^{er}$ as the net inflow number of young agents from $r$ to $k$ in subperiod $l$. The first term on the $RHS$ shows the inflow of young buyers if they trade with old sellers. The second term indicates the outflow of young agents from $k$ to $r$ if they trade with old buyers. An immediate implication is that

$$\Delta_{kl}^{e} = -\Delta_{kl}^{y}, \qquad (4.25)$$

that is, the net outflow of old agents equals the net inflow of young agents. When $\Delta_{kl}^{y} > 0$, the number of young (old) agents in $k$ increases (reduces) when starting the next subperiod. Another interpretation of $\Delta_{kl}^{y}$ (in absolute terms) is the total number of traded houses resulting from meetings between young buyers and old sellers.

In equilibrium, both the decision rule of sellers and the search behavior of buyers determine whether there are non-zero flows and house trading. Even though some sellers would like to trade with buyers of a different type if they meet (the joint surplus is positive), there will be no trade possible in the market where the sellers live if all buyers search elsewhere.

Using (4.25), the conditions for the law of motions in the first subperiod are:

$$n_{a1}^{y} = n_{a2}^{e} - \Delta_{a2}^{y} \tag{4.26}$$

$$n_{b1}^{y} = n_{b2}^{e} - \Delta_{b2}^{y} \tag{4.27}$$

$$n_{a1}^{e} = n_{a2}^{y} + \Delta_{a2}^{y} \tag{4.28}$$

$$n_{b1}^{e} = n_{b2}^{y} + \Delta_{b2}^{y}. \tag{4.29}$$

Given that old agents get replaced by their offsprings, conditions (4.26) and (4.27) simply state that the number of young agents in each neighbourhood at the beginning of subperiod 1 equals the number of old agents at the end of subperiod 2. This last number equals the number of old agents at the beginning of subperiod 2 less the net outflow. Since old agents at the beginning of a period are the young agents in the last period, conditions (4.28) and (4.29) indicate that the number of old agents in each neighbourhood at the beginning of subperiod 1 is equal to the number of young agents in the neighbourhood at the end of subperiod 2. This last number equals the number of young agents at the beginning of subperiod 2 plus the net inflow.

In the second subperiod, the conditions for the law of motions in the second subperiod are:

$$n_{a2}^{y} = n_{a1}^{y} + \Delta_{a1}^{y} \tag{4.30}$$

$$n_{b2}^{y} = n_{b1}^{y} + \Delta_{b1}^{y} \tag{4.31}$$

$$n_{a2}^{e} = n_{a1}^{e} - \Delta_{a1}^{y} \tag{4.32}$$

$$n_{b2}^{e} = n_{b1}^{e} - \Delta_{b1}^{y}. \tag{4.33}$$

These four conditions are straightforward. The number of each type in each neighbourhood at the beginning of subperiod 2 equals the number of such type at the

beginning of subperiod 1 plus the net inflow. The law of motions for agents in rental housing can be ignored because it is automatically satisfied when the numbers of house owners and renters clear.[4]

Next, the value functions of agents across locations and subperiods are derived. First consider the value functions of house owners in $a$ and $b$. Using (4.7) and (4.12), the value functions for type-$i$ agents in $k$ are:

$$
\begin{aligned}
V_{k1}^i &= Q_{k1}^i [V_{r2}^i + w_{k1}^{i\hat{i}}] + (1 - Q_{k1}^i) V_{k2}^i \\
&= V_{k2}^i + Q_{k1}^i \sigma S_{k1}^{i\hat{i}} \quad &(4.34) \\
V_{k2}^i &= Q_{k2}^i [U_r^i + \beta V_{r1}^{i'} + w_{k2}^{i\hat{i}}] + (1 - Q_{k2}^i)[U_k^i + \beta V_{k1}^{i'}] \\
&= U_k^i + \beta V_{k1}^{i'} + Q_{k2}^i \sigma S_{k2}^{i\hat{i}}. \quad &(4.35)
\end{aligned}
$$

Let $\mathcal{V}_k = \{V_{kl}^i\}, \forall i, k, l$, denote the set of all value functions of house owners. In addition, the above two equations show that the value of a house for its occupant may exceed its housing services if there is trade potential. For example, consider $V_{a2}^e$ (such that $k = a$). Without trade potential ($Q_{a2}^e = 0$), the old house owners only enjoy $U_a^e$. In contrast, if trade is possible, such as when $Q_{a2}^e > 0$ and $S_{a2}^{ey} > 0$, the expected value of the house becomes $U_a^e + Q_{a2}^e \sigma S_{a2}^{ey}$ which is greater than $U_a^e$. Therefore, housing in different locations would provide a different value for buyers. Occupying a house in the location with future trade potential would be a good asset. These point out the asset aspect of owner-occupied housing.

Now consider buyers who need to decide upon their search decision. It is as-

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[4]The conditions for the law of motions of agents in rental housing are:

$$
\begin{aligned}
n_{a2}^{yr} + n_{b2}^{yr} &= n_{a1}^{yr} + n_{b1}^{yr} - \Delta_{a1}^y - \Delta_{b1}^y \\
n_{a2}^{er} + n_{b2}^{er} &= n_{a1}^{er} + n_{b1}^{er} + \Delta_{a1}^y + \Delta_{b1}^y \\
n_{a1}^{yr} + n_{b1}^{yr} &= n_{a2}^{er} + n_{b2}^{er} + \Delta_{a2}^y + \Delta_{b2}^y \\
n_{a1}^{er} + n_{b1}^{er} &= n_{a2}^{yr} + n_{b2}^{yr} - \Delta_{a2}^y - \Delta_{b2}^y.
\end{aligned}
$$

One can easily see that combining the first two conditions and combining the last two conditions yield the condition (4.22).

sumed that if buyers choose to search, they can search only one market at a time during a subperiod. Using (4.6) and (4.11), the expected values of searching in the first and the second subperiods are:

$$
\begin{aligned}
V_{k1}^{\hat{i}r} &= P_{k1}^{\hat{i}}[V_{k2}^{\hat{i}} - w_{k1}^{i\hat{i}}] + (1 - P_{k1}^{\hat{i}})V_{r2}^{\hat{i}} \\
&= V_{r2}^{\hat{i}} + P_{k1}^{\hat{i}}(1 - \sigma)S_{k1}^{i\hat{i}} \qquad\qquad (4.36) \\
V_{k2}^{\hat{i}r} &= P_{k2}^{\hat{i}}[U_k^{\hat{i}} + \beta V_{k1}^{\hat{i}'} - w_{k2}^{i\hat{i}}] + (1 - P_{k2}^{\hat{i}})(U_r^{\hat{i}} + \beta V_{r1}^{\hat{i}'}) \\
&= U_r^{\hat{i}} + \beta V_{r1}^{\hat{i}'} + P_{k2}^{\hat{i}}(1 - \sigma)S_{k2}^{i\hat{i}}. \qquad\qquad (4.37)
\end{aligned}
$$

Buyers decide whether they want to search in $a$ or $b$ or not to search, given the decisions of other agents. Therefore, the value functions of buyers across subperiods are:

$$
\begin{aligned}
V_{r1}^{\hat{i}} &= \max\{V_{a1}^{\hat{i}r}, V_{b1}^{\hat{i}r}, V_{r2}^{\hat{i}}\}, \qquad\qquad (4.38) \\
V_{r2}^{\hat{i}} &= \max\{V_{a2}^{\hat{i}r}, V_{b2}^{\hat{i}r}, U_r^{\hat{i}} + \beta V_{r1}^{\hat{i}'}\}. \qquad\qquad (4.39)
\end{aligned}
$$

From (4.36)-(4.39), notice that without search costs, searching weakly dominates staying idle. Let $\mathcal{V}_r = \{V_{kl}^{ir}, V_{rl}^{\hat{i}}\}, \forall i, k, l$, denote the set of all value functions associated with buyers. Also, let $\mathcal{V} = \{\mathcal{V}_k, \mathcal{V}_r\}$ be the set of all value functions.

## 4.3 Equilibrium

I am only interested in symmetric equilibria in which agents of the same type behave the same way, that is, sellers (buyers) adopt the same selling (searching) rule.

First consider when searching is strictly better than not searching. According to (4.38) and (4.39), if $n_{al}^{\hat{i}r} = n_{bl}^{\hat{i}r} > 0$, it implies that $V_{al}^{\hat{i}r} = V_{bl}^{\hat{i}r}$, buyers randomly choose their search location since both locations give the same expected utility. In other words, when there are buyers of the same type searching in different housing

113

markets, it has to be the case that the values of searching in both markets are equal. If $n_{al}^{\hat{i}r} > 0$ and $n_{bl}^{\hat{i}r} = 0$, it implies that $V_{al}^{\hat{i}r} > V_{bl}^{\hat{i}r}$. Similarly, if $n_{al}^{\hat{i}r} = 0$ and $n_{bl}^{\hat{i}r} > 0$, it implies that $V_{al}^{\hat{i}r} < V_{bl}^{\hat{i}r}$. When buyers of the same type all search in the same housing market, the expected utility from searching in that market is strictly better than searching in the other market. Since an agent is atomic, her deviation does not affect the valuations and thus yields lower expected utility. If all buyers of a particular type are indifferent between searching and not searching, searching in any housing markets gains nothing more than their reservation valuation. This could happen when all possible meetings in the housing markets for such type of buyers have non-positive joint surpluses. The search behavior is then indeterminate. For now, it is assumed that buyers do not search in such case. This assumption will be relaxed later on when discussing indeterminacy of equilibria.

Therefore, for buyers, the two subperiods can be summarised as: the first subperiod,

$$
\left\{
\begin{array}{c}
n_{a1}^{\hat{i}r} > 0 \text{ and } n_{b1}^{\hat{i}r} = 0 \\[4pt]
n_{a1}^{\hat{i}r} = n_{b1}^{\hat{i}r} > 0 \\[4pt]
n_{a1}^{\hat{i}r} = 0 \text{ and } n_{b1}^{\hat{i}r} > 0 \\[4pt]
n_{a1}^{\hat{i}r} = n_{b1}^{\hat{i}r} = 0
\end{array}
\right\}
\quad \text{if} \quad
\left\{
\begin{array}{c}
V_{a1}^{\hat{i}r} > V_{b1}^{\hat{i}r} \geq V_{r2}^{\hat{i}} \\[4pt]
V_{a1}^{\hat{i}r} = V_{b1}^{\hat{i}r} > V_{r2}^{\hat{i}} \\[4pt]
V_{b1}^{\hat{i}r} > V_{a1}^{\hat{i}r} \geq V_{r2}^{\hat{i}} \\[4pt]
V_{a1}^{\hat{i}r} = V_{b1}^{\hat{i}r} = V_{r2}^{\hat{i}}
\end{array}
\right\},
\tag{4.40}
$$

and the second subperiod,

$$
\left\{
\begin{array}{c}
n_{a2}^{\hat{i}r} > 0 \text{ and } n_{b2}^{\hat{i}r} = 0 \\[4pt]
n_{a2}^{\hat{i}r} = n_{b2}^{\hat{i}r} > 0 \\[4pt]
n_{a2}^{\hat{i}r} = 0 \text{ and } n_{b2}^{\hat{i}r} > 0 \\[4pt]
n_{a2}^{\hat{i}r} = n_{b2}^{\hat{i}r} = 0
\end{array}
\right\}
\quad \text{if} \quad
\left\{
\begin{array}{c}
V_{a2}^{\hat{i}r} > V_{b2}^{\hat{i}r} \geq U_r^{\hat{i}} + \beta V_{r1}^{\hat{i}'} \\[4pt]
V_{a2}^{\hat{i}r} = V_{b2}^{\hat{i}r} > U_r^{\hat{i}} + \beta V_{r1}^{\hat{i}'} \\[4pt]
V_{b2}^{\hat{i}r} > V_{a2}^{\hat{i}r} \geq U_r^{\hat{i}} + \beta V_{r1}^{\hat{i}'} \\[4pt]
V_{a2}^{\hat{i}r} = V_{b2}^{\hat{i}r} = U_r^{\hat{i}} + \beta V_{r1}^{\hat{i}'}
\end{array}
\right\}.
\tag{4.41}
$$

**Definition.** A steady-state equilibrium is a set of $\mathcal{H}$, $\mathcal{D}$, $\mathcal{V}$, and $\mathcal{S}$ such that

(i) each agent maximises her utility, given $\mathcal{H}$, $\mathcal{D}$, $\mathcal{V}$, $\mathcal{S}$ and other agents' decisions,

(ii) $\mathcal{V}$ and $\mathcal{S}$ satisfy (4.14), (4.40), and (4.41), given $\mathcal{H}$ and $\mathcal{D}$,

(iii) $\mathcal{H}$ satisfies the law of motions across locations and subperiods (4.26) - (4.33), and

(iv) $\mathcal{H}$ clears the numbers of house owners and renters and the population sizes (4.20) - (4.24).

By the above definition, no buyers (sellers) can gain higher expected utility by changing their search decisions (selling rules), given other agents' decisions. An equilibrium is also symmetric in the sense that agents of a particular type in a particular location behave the same way. A steady-state equilibrium is characterised by constant relocation and trading activity.

To solve for equilibria, it is useful to summarise the possible meetings across locations and subperiods as in Table 4.1. As mentioned earlier, when sellers and buyers of the same type meet, they can trade but that will not affect the distribution of types. The small letter $t$ denotes this situation. In contrast, if the meeting partners have a different type, trade will depend on whether the joint surplus is non-negative. A bold ? can be replaced by $T$ (trade) or $N$ (no trade).

Table 4.1: Meetings among Buyers and Sellers

Subperiod 1

| | | r | |
|---|---|---|---|
| | | **y** | **e** |
| **a** | **y** | t | ? |
| | **e** | ? | t |
| **b** | **y** | t | ? |
| | **e** | ? | t |

Subperiod 2

| | | r | |
|---|---|---|---|
| | | **y** | **e** |
| a | **y** | t | ? |
| | **e** | ? | t |
| b | **y** | t | ? |
| | **e** | ? | t |

To find equilibria, the algorithm is as follows. For a set of parameters, begin by conjecturing the tradability of meetings, replacing each ? with $T$ or $N$. Given the

115

conjecture, $\mathcal{S}$ is obtained. Then, $\mathcal{D}$ and conditions (4.40) and (4.41) are implied as well as some restriction on some elements of $\mathcal{H}$. Using the definition of each element in $\mathcal{V}$ and $\mathcal{S}$, check whether there exists an $\mathcal{H}$ that supports the conjecture, clears the numbers of house owners and renters as well as the law of motions conditions, without any contradiction to (4.1). If so, then an equilibrium is obtained.

## 4.3.1   The Baseline Equilibrium

It is not in the interest of this chapter to characterise all possible combinations of tradable meetings in equilibrium. Rather, the focus will be on when it is possible for young buyers to move to $a$ where the good public school is. Moreover, it is interesting to include when young house owners in $b$ may be able to sell their houses then move to the rental housing in the first subperiod, and then possibly move to $a$ in the next subperiod. A conjecture that supports this situation is presented in Table 4.2. The relocation of young agents into the good-school neighbourhood will be the focus of the following sections.

Table 4.2: Tradability of Meetings in the Baseline Model

Subperiod 1

|   |   | **r** | |
|---|---|---|---|
|   |   | **y** | **e** |
| **a** | y | t | **N** |
|   | e | **T** | t |
| **b** | y | t | **T** |
|   | e | **N** | t |

Subperiod 2

|   |   | **r** | |
|---|---|---|---|
|   |   | **y** | **e** |
| **a** | y | t | **N** |
|   | e | **T** | t |
| **b** | y | t | **N** |
|   | e | $t^*$ | t |

With the complicated relations of the variables in the model, equilibria are solved numerically. Following the algorithm just described, first assume tradability of meetings. Second, derive the signs of the joint surpluses, the decision rules, and the search behaviors. Third, derive all equilibrium conditions used in the computations. Lastly, given the values of parameters, the computations seek whether there

exist equilibria that satisfy all the conditions.

The tradability of meetings used in the further analysis is summarised in Table 4.2. In the first subperiod, trade occurs only when young buyers meet old sellers in $a$ and when old buyers meet young sellers in $b$, as indicated by $T$s and $N$s. Using (4.13), this implies that, $S_{a1}^{ey} > 0$ and $S_{a1}^{ye} < 0$, and thus $D_{a1}^{ey} = 1$ and $D_{a1}^{ye} = 0$, and $S_{b1}^{ey} < 0$ and $S_{b1}^{ye} > 0$, and thus $D_{b1}^{ey} = 0$ and $D_{b1}^{ye} = 1$. All young buyers search in $a$ and all old buyers search in $b$. Therefore, from (4.38),

$$V_{r1}^{y} = V_{a1}^{yr} \quad \text{and} \quad V_{r1}^{e} = V_{b1}^{er}. \tag{4.42}$$

In the second subperiod, trade occurs only when young buyers meet old sellers in $a$, as indicated by $T$, $t^*$, and $N$s. The letter $t^*$ means that trade could occur if a young agent deviates her searching location from $a$ to $b$. This implies that $S_{a2}^{ey} > 0$ and $S_{a2}^{ye} < 0$, and thus $D_{a1}^{ey} = 1$ and $D_{a1}^{ye} = 0$, and $S_{b2}^{ey} > 0$ and $S_{b2}^{ye} < 0$, and thus $D_{b2}^{ey} = 1$ and $D_{b2}^{ye} = 0$. Note that according to this conjecture, even though $S_{b2}^{ey} > 0$, no young buyers search in $b$ because the expected returns are lower, $V_{a2}^{\hat{i}r} > V_{b2}^{\hat{i}r}$, given that all buyers search in $a2$. Moreover, old buyers cannot trade with any sellers since the joint surpluses when they meet with young sellers in both neighbourhoods are negative. Therefore, from (4.39),

$$V_{r2}^{y} = V_{a2}^{yr} \quad \text{and} \quad V_{r2}^{e} = V_{a2}^{er} = V_{b2}^{er} = U_{r}^{e} = 0. \tag{4.43}$$

Without search costs, the search behavior of old buyers is indeterminate because they are indifferent between searching and not searching. In the analysis below, it is assumed that no old buyers search in any housing markets in the second subperiod. It will be shown later that this assumption actually gives the highest social welfare because there are no externalities caused by such agents in the housing markets.

For the asset aspect of housing, young buyers in the first subperiod do not want

to acquire housing in $b$ because there will be no gains from trade in any meetings in the future once they become old and no longer value school quality. Housing in $a$ is then a better asset since there are trade potentials when the house owners become old.

Next, all equilibrium conditions are derived. First, the signs of joint surpluses are summarised:

$$S_{a1}^{ey} > 0, \quad S_{b1}^{ye} > 0 \quad S_{a2}^{ey} > 0, \text{ and } S_{b2}^{ey} > 0. \tag{4.44}$$

Given the search behaviors described above, it can be concluded that

$$n_{b1}^{yr} = n_{a1}^{er} = n_{b2}^{yr} = 0. \tag{4.45}$$

Moreover, the old buyers in the second subperiod are indifferent between searching in any location and not searching. Let $\gamma_k$ be the fraction of old buyers who search in $k$ in the second subperiod. Thus,

$$n_{a2}^{er} = \gamma_a n_{r2}^e \quad \text{and} \quad n_{b2}^{er} = \gamma_b n_{r2}^e \tag{4.46}$$

where $n_{r2}^e$ is the total number of old buyers (renters) in the second subperiod, and $0 \leq \gamma_a + \gamma_b \leq 1$. In the baseline model, assume that $\gamma_a = \gamma_b = 0$. This gives

$$n_{a2}^{er} = \quad n_{b2}^{er} = 0. \tag{4.47}$$

Using (4.18) and (4.19), the implied market tightnesses are:

$$
\begin{aligned}
\theta_{a1} &= \frac{n_{a1}^{yr}}{A} \\
\theta_{b1} &= \frac{n_{b1}^{er}}{B} \\
\theta_{a2} &= \frac{n_{a2}^{yr} + n_{a2}^{er}}{A} = \frac{n_{a2}^{yr}}{A} \\
\theta_{b2} &= \frac{n_{b2}^{er}}{B} = 0.
\end{aligned}
$$

Using (4.15) and (4.16), the implied trading probabilities for sellers and buyers are:

$$
\begin{aligned}
P_{a1}^y &= p(\theta_{a1}) \frac{n_{a1}^e}{A} \\
Q_{a1}^e &= p(\theta_{a1}) \frac{n_{a1}^{yr}}{A} \\
P_{b1}^e &= p(\theta_{b1}) \frac{n_{b1}^y}{B} \\
Q_{b1}^y &= p(\theta_{b1}) \frac{n_{b1}^{er}}{B} \\
P_{b1}^y = P_{a1}^e &= 0
\end{aligned}
$$

$$
\begin{aligned}
P_{a2}^y &= p(\theta_{a2}) \frac{n_{a2}^e}{A} \\
Q_{a2}^e &= p(\theta_{a2}) \frac{n_{a2}^{yr}}{A} \\
P_{b2}^y &= p(\theta_{b2}) \frac{n_{b1}^e}{B} = \frac{n_{b1}^e}{B} \\
P_{a2}^e = P_{b2}^e &= 0.
\end{aligned}
$$

Also, young house owners in $a1$ and $a2$ as well as old house owners in $b1$ and $b2$ do not trade in equilibrium if they meet with buyers of a different type. Therefore, $Q_{a1}^y = Q_{a2}^y = 0$ and $Q_{b1}^e = Q_{b2}^e = 0$.

Using (4.26)-(4.33), and (4.45), and substituting $\Delta_{kl}^y$ with $P_{kl}^y n_{kl}^{yr} - P_{kl}^e n_{kl}^{er}$, the

conditions for the law of motions are:

$$n_{a2}^{y} = n_{a1}^{y} + P_{a1}^{y} n_{a1}^{yr}$$

$$n_{b2}^{y} = n_{b1}^{y} - P_{b1}^{e} n_{b1}^{er}$$

$$n_{a1}^{y} = n_{a2}^{e} - P_{a2}^{y} n_{a2}^{yr}$$

$$n_{b1}^{y} = n_{b2}^{e}$$

$$n_{a2}^{e} = n_{a1}^{e} - P_{a1}^{y} n_{a1}^{yr}$$

$$n_{b2}^{e} = n_{b1}^{e} + P_{b1}^{e} n_{b1}^{er}$$

$$n_{a1}^{e} = n_{a2}^{y} + P_{a2}^{y} n_{a2}^{yr}$$

$$n_{b1}^{e} = n_{b2}^{y}.$$

The system of equations can be simplified by substituting $n_{b2}^{y}$ and $n_{b2}^{e}$ with $n_{b1}^{e}$ and $n_{b1}^{y}$, respectively. Using the conditions for the number of house owners and renters, the conditions for the law of motions used for the computations are reduced to:

$$n_{a2}^{y} = n_{a1}^{y} + P_{a1}^{y} n_{a1}^{yr} \tag{4.48}$$

$$n_{b1}^{e} = n_{b1}^{y} - P_{b1}^{e} n_{b1}^{er} \tag{4.49}$$

$$n_{a1}^{y} = n_{a2}^{e} - P_{a2}^{y} n_{a2}^{yr}. \tag{4.50}$$

The number of variables in the distribution of agents is reduced to ten:

$$H_1 = \{n_{a1}^{y}, n_{a1}^{e}, n_{b1}^{y}, n_{b1}^{e}, n_{a1}^{yr}, n_{b1}^{er}\}$$

$$H_2 = \{n_{a2}^{y}, n_{a2}^{e}, n_{a2}^{yr}, n_{r2}^{e}\}.$$

The implied conditions for the number of house owners and renters and the

population sizes are:

$$n_{a1}^y + n_{a1}^e = A \tag{4.51}$$

$$n_{a2}^y + n_{a2}^e = A \tag{4.52}$$

$$n_{b1}^y + n_{b1}^e = B \tag{4.53}$$

$$n_{a1}^y + n_{b1}^y + n_{a1}^{yr} = 1 \tag{4.54}$$

$$n_{a1}^e + n_{b1}^e + n_{b1}^{er} = 1 \tag{4.55}$$

$$n_{a2}^y + n_{b2}^y + n_{a2}^{yr} = 1 \tag{4.56}$$

$$n_{a2}^e + n_{b2}^e + n_{r2}^e = 1. \tag{4.57}$$

The condition $n_{b2}^y + n_{b2}^e = B$ is ignored because $n_{b2}^y$ and $n_{b2}^e$ are replaced by $n_{b1}^e$ and $n_{b1}^y$, respectively. Therefore, if $n_{b1}^y + n_{b1}^e = B$ holds, $n_{b2}^y + n_{b2}^e = B$ will hold automatically.

Now the value functions of house owners are derived. From Table 4.2, some value functions can be derived as follows. In the second subperiod,

$$V_{a2}^y = U_a^y + \beta V_{a1}^e \tag{4.58}$$

$$V_{b2}^y = U_b^y + \beta V_{b1}^e \tag{4.59}$$

$$V_{b2}^e = U_b^e \tag{4.60}$$

$$V_{r2}^e = V_{a2}^{er} = V_{b2}^{er} = U_r^e = 0 \tag{4.61}$$

$$V_{b2}^{yr} = U_r^y + \beta V_{b1}^{er} + P_{b2}^y(1 - \sigma)S_{b2}^{ey} \tag{4.62}$$

where $S_{b2}^{ey} = U_b^y - U_b^e + \beta V_{b1}^e$. The first two equations indicate that there are no gains from any meetings for young sellers in $a$, young and old sellers in $b$, and old buyers wherever they search. They all obtain their reservation values. The last equation measures the valuation of a young buyer if she chooses to deviate to search in $b2$. For Table 4.2 to be realised, the computations have to ensure that $V_{a2}^{yr} > V_{b2}^{yr}$ in

equilibrium.

In the first subperiod, using the above four equations and the fact that young buyers search only in $a$ in the second subperiod, it has to be the case that

$$V_{a1}^y = V_{a2}^y = U_a^y + \beta V_{a1}^e \tag{4.63}$$

$$V_{b1}^e = V_{b2}^e = U_b^e \tag{4.64}$$

$$V_{a1}^{er} = V_{r2}^e = 0 \tag{4.65}$$

$$V_{b1}^{yr} = V_{r2}^y = V_{a2}^{yr}. \tag{4.66}$$

These four equations indicate the following. There are no gains from any meetings for young sellers in $a$, old sellers in $b$, an old buyer if she deviates to search in $a$, and a young buyer if she deviates to search in $b$. The number of variables of value functions across locations and subperiods are reduced to six with the other three variables for the joint surpluses:

$$V_{a1}^e = V_{a2}^e + Q_{a1}^e \sigma S_{a1}^{ey} \tag{4.67}$$

$$V_{b1}^y = U_b^y + \beta V_{b2}^e + Q_{b1}^y \sigma S_{b1}^{ye} \tag{4.68}$$

$$V_{a1}^{yr} = V_{a2}^{yr} + P_{a1}^y (1 - \sigma) S_{a1}^{ey} \tag{4.69}$$

$$V_{b1}^{er} = P_{b1}^e (1 - \sigma) S_{b1}^{ye} \tag{4.70}$$

$$V_{a2}^e = U_a^e + Q_{a2}^e \sigma S_{a2}^{ey} \tag{4.71}$$

$$V_{a2}^{yr} = U_r^y + \beta V_{b1}^{er} + P_{a2}^y (1 - \sigma) S_{a2}^{ey} \tag{4.72}$$

where

$$S_{a1}^{ey} = U_a^y + \beta V_{a1}^e - V_{a2}^{yr} - V_{a2}^e \tag{4.73}$$

$$S_{b1}^{ye} = V_{a2}^{yr} + (1 - \beta) V_{b2}^e - U_b^y \tag{4.74}$$

$$S_{a2}^{ey} = U_a^y - U_a^e + \beta V_{a1}^e. \tag{4.75}$$

122

In summary, the system of equations has 19 unknowns:

$$H_1 = \{n_{a1}^y, n_{a1}^e, n_{b1}^y, n_{b1}^e, n_{a1}^{yr}, n_{b1}^{er}\}$$

$$H_2 = \{n_{a2}^y, n_{a2}^e, n_{a2}^{yr}, n_{r2}^e\}$$

$$\mathcal{V} = \{V_{a1}^e, V_{b1}^y, V_{a1}^{yr}, V_{b1}^{er}, V_{a2}^e, V_{a2}^{yr}\}$$

$$\mathcal{S} = \{S_{a1}^{ey}, S_{b1}^{ye}, S_{a2}^{ey}\}$$

with 19 equations, (4.48)-(4.50), (4.51)-(4.57), and (4.67)-(4.75).

The system of equations is solved simultaneously. The computations search for an interior solution with non-zero lower bounds for all variables. After obtaining the solution, other remaining variables can be calculated. The interesting variables are the housing prices,

$$w_{a1}^{ey} = V_{a2}^e + \sigma S_{a1}^{ey} \tag{4.76}$$

$$w_{b1}^{ye} = V_{b2}^y - V_{a2}^{yr} + \sigma S_{b1}^{ye} \tag{4.77}$$

$$w_{a2}^{ey} = U_a^e + \sigma S_{a2}^{ey}, \tag{4.78}$$

and the total welfare $(TW)$ which measures the sum of welfare of all agents at the beginning of the first subperiod,

$$TW = TW_{a1}^y + TW_{b1}^y + TW_{r1}^y + TW_{a1}^e + TW_{b1}^e + TW_{r1}^e \tag{4.79}$$

where $TW_{k1}^i = n_{k1}^i V_{k1}^i$, $TW_{r1}^y = n_{a1}^{yr} V_{a1}^{yr}$, and $TW_{b1}^e = n_{b1}^{er} V_{b1}^{er}$.

## 4.3.2  Computation of the Baseline Equilibrium

This section shows that, for some values of the parameters, there exist equilibria that support the focus of this chapter (Table 4.2). The existence of the focused

equilibria confirms one main point of this chapter, that is, differences in qualities of local amenities such as public schools can create relocation of agents across locations.

The parameters are chosen as follows. The housing service utility in $a$ for young agents $U_a^y$ is normalised to one. The discount factor $\beta$ is set at 0.9. The bargaining power of sellers $\sigma$ is simply set at 0.5. Lastly, the parameter of the meeting technology $\alpha$ is fixed at 1.5. The value of $\alpha$ needs to be sufficiently high for Table 4.2 to exist. Moreover, it allows us to obtain enough equilibria when conducting comparative statics.

The baseline model sets the rest of the parameters as follows. From (4.1), $U_r^y = 0$. The total housing capacities in $a$ and $b$ are set at 0.8, $A = B = 0.8$ and $A + B = 1.6$.[5] Assume that $\gamma_a = \gamma_b = 0$, old agents in rental housing in the second subperiod do not search.

Then, the computations search for the values of $U_k^e$ and $U_b^y$ such that the system of equations is solved. Recall from (4.1) that $U_k^e < U_b^y$. It is expected that the values of $U_k^e$ ($=U_a^e = U_b^e$) and $U_b^y$ have to be sufficiently low to allow young sellers in $b$ in the first subperiod to be better off by selling their houses, with a sufficiently high chance of moving into the better neighbourhood in the next subperiod.

Given $U_r^y = 0$, equilibria are found when setting $U^e \in (0, 0.2692)$ along with $U^e < U_b^y < 0.2769$. Allowing $U_r^y > 0$, equilibria are also found but with an increasing range of $U^e$ and $U_b^y$. The baseline model sets $U_k^e = 0.1$ and $U_b^y = 0.15$. All values of the baseline parameters are summarised in Table 4.3. The baseline equilibrium is also summarised in Table 4.4.

An interior solution was found for the system of equations. Thus, the distribution of agents consists of non-zero numbers of agents across subperiods and locations. The highest value function is $V_{a1}^y$, which is for young agents who in-

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[5]Australia, for an example, has around 20% of households as private renters (see ABS (2008)).

Table 4.3: The Baseline Parameters

| Parameter | Value |
|-----------|-------|
| $\beta$ | 0.9 |
| $\sigma$ | 0.5 |
| $\alpha$ | 1.5 |
| $A$ | 0.8 |
| $B$ | 0.8 |
| $\gamma_a$, $\gamma_b$ | 0 |
| $U_a^y$ | 1 |
| $U_b^y$ | 0.15 |
| $U_a^e$, $U_b^e$ | 0.1 |
| $U_r^y$, $U_r^e$ | 0 |

herit houses in $a$. Not only can they enjoy the highest housing service utility when young, but also, when old, they expect to gain from selling their houses to the next generation of young buyers. This latter benefit is not available for old agents in $b$ since they cannot gain from trade in any meetings in both subperiods, $V_{a1}^e = V_{a2}^e = U_a^e = 0.1$. For young agents in $b$, a possibility of trade in the first subperiod generates higher value than their reservation value, $V_{b1}^y > V_{b2}^y$. Getting matched with an old buyer will make a young agent in $b$ better off because the joint surplus ($S_{b1}^{ye}$) is positive, that is, $V_{b1}^y = V_{b2}^y + Q_{b1}^y \sigma S_{b1}^{ye} = 0.24947 > V_{b2}^y = 0.24$. For young house owners in $b1$, around 13% move out and gain from trade in the first subperiod because $Q_{b1}^y = 0.12895$. Among these fortunate young house owners, around 41% of them can move into neighbourhood $a$ in the second subperiod since $P_{a2}^y = 0.41308$. Suppose agents do not trade with the same type, then around 87% of young house owners in $b$ get stuck in their low-quality school neighbourhood, that is, $1 - Q_{b1}^y = 0.87105$. Being stuck in $b$ not only yields smaller housing service utility when young, but also eliminates any gains from meetings when old. This is indicated by $V_{b2}^y = U_b^y + \beta V_{b1}^e = 0.24$.

To ensure that Table 4.2 is realised in equilibrium, it has to be the case that (i) the search decisions are optimal, that is, $V_{a1}^{yr} > V_{b1}^{yr} (= V_{r2}^y = V_{a2}^{yr})$, $V_{b1}^{er} > V_{a1}^{er} (=$

| Distribution of Agents | | Matching | | Value function | |
|---|---|---|---|---|---|
| $n_{a1}^y$ | 0.27891 | $p(\theta_{a1})$ | 0.87477 | $V_{a1}^y$ | 1.3231 |
| $n_{a1}^e$ | 0.52109 | $p(\theta_{b1})$ | 0.96889 | $V_{a1}^e$ | 0.35898 |
| $n_{b1}^y$ | 0.42757 | $p(\theta_{a2})$ | 0.93392 | $V_{b1}^y$ | 0.24947 |
| $n_{b1}^e$ | 0.37243 | $p(\theta_{b2})$ | 1 | $V_{b1}^e$ | 0.1 |
| $n_{a1}^{yr}$ | 0.29353 | $P_{a1}^y$ | 0.56980 | $V_{a1}^{yr}$ | 0.51667 |
| $n_{b1}^{er}$ | 0.10647 | $Q_{a1}^e$ | 0.32096 | $V_{b1}^{er}$ | 0.03802 |
| $n_{a2}^y$ | 0.44616 | $P_{b1}^e$ | 0.51783 | $V_{a2}^y$ | 1.3231 |
| $n_{a2}^e$ | 0.35384 | $Q_{b1}^y$ | 0.12895 | $V_{a2}^e$ | 0.22951 |
| $n_{b2}^y$ | 0.37243 | $P_{a2}^y$ | 0.41308 | $V_{b2}^y$ | 0.24 |
| $n_{b2}^e$ | 0.42757 | $Q_{a2}^e$ | 0.21178 | $V_{b2}^e$ | 0.1 |
| $n_{a2}^{yr}$ | 0.18141 | | | $V_{a2}^{yr}$ | 0.28683 |
| $n_{r2}^e$ | 0.21859 | | | $V_{a2}^{er}, V_{b2}^{er}$ | 0 |
| | | | | $V_{b2}^{yr}$ | 0.07163 |
| **Housing Price** | | **Joint Surplus** | | $TW$ | 0.85569 |
| $w_{a1}^{ey}$ | 0.63288 | $S_{a1}^{ey}$ | 0.80674 | | |
| $w_{b1}^{ye}$ | 0.02658 | $S_{b1}^{ye}$ | 0.14683 | | |
| $w_{a2}^{ey}$ | 0.71154 | $S_{a2}^{ey}$ | 1.2231 | | |
| | | $S_{b2}^{ey}$ | 0.14 | | |

0), and $V_{a2}^{yr} > V_{b2}^{yr}$. No buyers have incentives to deviate, and (ii) the signs of $S_{a1}^{ey}$, $S_{b1}^{ye}$, $S_{a2}^{ey}$, and $S_{b2}^{ey}$ are all positive. This ensures that the sellers choose to trade according to Table 4.2. As shown in Table 4.4, these requirements are satisfied.

## 4.4 Comparing Steady States

This section focuses on how changes in the school quality parameters affect the steady-state equilibrium. The parameters are changed such that Table 4.2 is still realised in equilibrium. All computations use the same guess. Unfortunately, with the complicated relations of the variables in the model, precise theoretical insights about the direction and the size of the effects cannot be given. However, at least what can be learned from the following exercises is that changes in certain param-

eters affect agents differently depending on their locations and types.

In the following, Model I explores changes in the equilibrium when the public school in the bad neighbourhood is exogenously improved. The total welfare increases in this case. This would give a policy implication that the government could improve social welfare if school quality in the bad neighbourhood can be improved without any disturbance to the school quality in the good neighbourhood. Model II shows changes in the equilibrium when inferior private schools are available and beneficial to only young renters. Model III explores changes in the equilibrium when good quality private schools are available and beneficial to young renters and young agents in the bad neighbourhood. In Model II and III, the total welfare increases. This would give a policy implication that, if Model I is not applicable, the government may have an alternative to improve the social welfare by focusing on private schools. Reducing unnecessary administration costs and restrictions for private schools may constitute to more available private schools with good quality, and thus improving the social welfare as predicted in Model II and III. The effects of changes in housing capacities are also investigated. Comparing the baseline and the modified settings, this last modification yields predictions on potential outcomes if the government were to redraw the school attendance zones.

## 4.4.1   Changes in $U_b^y$ (Model I)

This exercise shows how the steady-state equilibrium changes when the school quality in $b$ increases. As an example, consider the effects of increasing the parameter $U_b^y$ from 0.15 to 0.26. Keeping other parameters constant, the results are presented in Figure 4.1, which shows only the value functions, the housing prices, and the joint surpluses, since the distribution of agents does not change (see Table 4.4 for the distribution). This is simply because when Table 4.2 is realised, the signs of the joint surpluses have to satisfy equation (4.44), which essentially indicates the

willingness to trade. As long as such condition is met, agents do not have incentives to change their decisions. Therefore, the relocation of agents as well as the distribution across subperiods do not change, that is, only value functions and prices are affected. When the distribution is unchanged, the conditions for the law of motions and the number of house owners and renters (4.48)-(4.57) hold. This implies that the changes in the equations (4.67)-(4.75) still support the distribution of agents in the baseline equilibrium. Hence, the equilibria feature an unchanged distribution of agents as well as constant trading probabilities for buyers and sellers across locations and subperiods. This highlights the effects on the value functions and the housing prices that come purely from changes of the school quality parameter not from the change of the distribution of agents.

From the computations, the effects of an increase in the quality of public school in the low-quality school neighbourhood on value functions and housing prices are presented in Figure 4.1. In this model, the value functions of the old in $b$ are unchanged, $V_{b1}^e = V_{b2}^e = U_b^e$ (eq.(4.64)). But the value function of the young in $b$ in the second subperiod is higher since $V_{b2}^y = U_b^y + \beta U_b^e$ (eq.(4.59)). The value function of the young in $b$ at the beginning of a period ($V_{b1}^y$) increases. Using (4.68), this comes from a stronger effect of an increase in $V_{b2}^y$ than a reduction in the joint surplus $S_{b1}^{ye}$. This joint surplus (eq.(4.74)) decreases not only because the benefit of staying in $b$ is now higher for young agents, but also because the value when being a buyer in the second subperiod ($V_{a2}^{yr}$) decreases. Using (4.72), the reduction of $V_{a2}^{yr}$ is explained by the decrease in $V_{b1}^{er}$ which represents the reservation value of being a buyer in the next period once the young buyers become old. Using (4.70), $V_{b1}^{er}$ falls due to the decrease in the joint surplus $S_{b1}^{ye}$: if an old buyer meets with a young seller in $b$, she gets less gains from trade. Therefore, for agents who are involved in the housing market in neighbourhood $b$, only the old buyers are worse off from increasing $U_b^y$, while young sellers are better off and old sellers are unaffected.
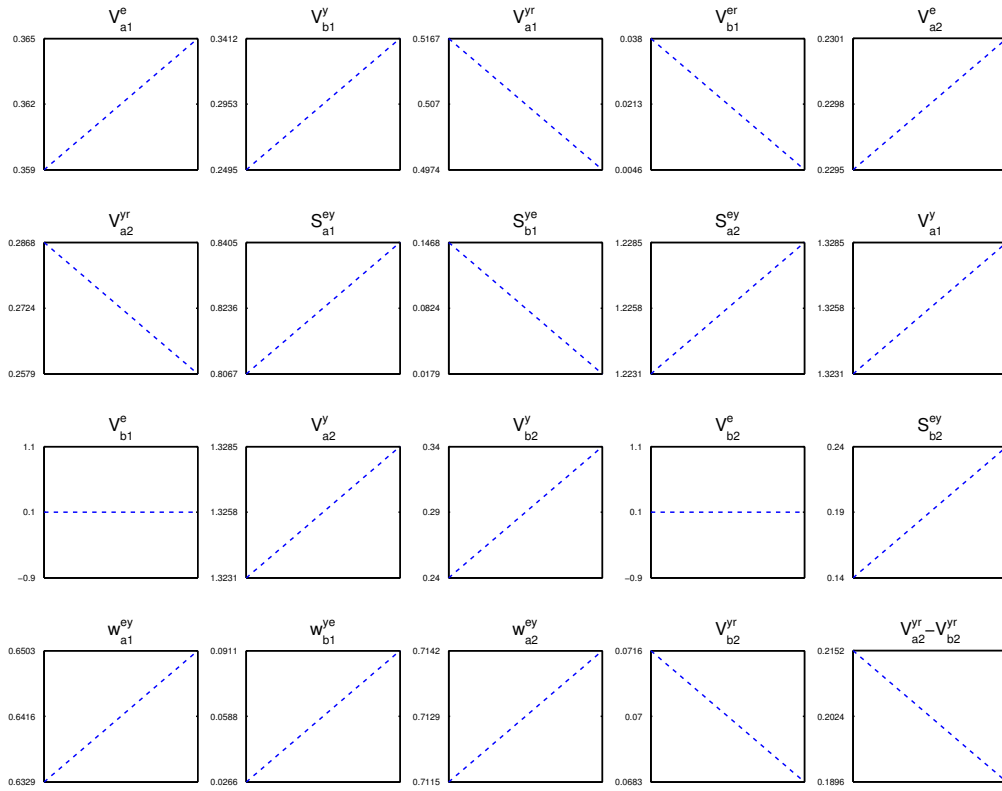
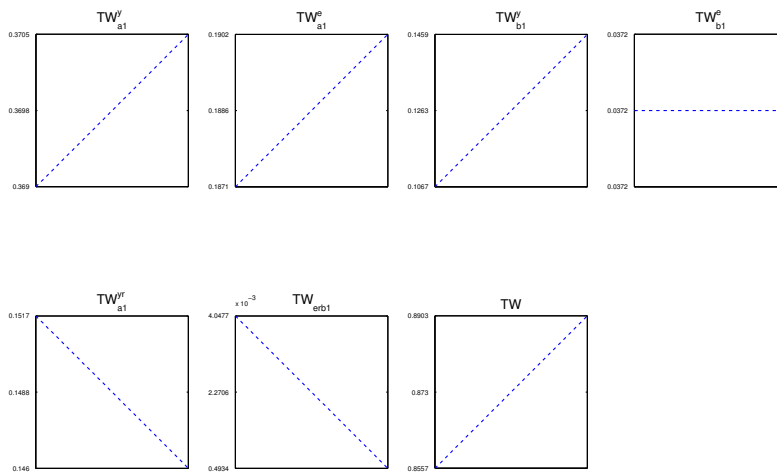Figure 4.1: The Effects of Increasing $U_b^y$ on Value Functions and Housing Prices



Figure 4.2: The Effects of Increasing $U_b^y$ on Welfare

For those who are in the housing market in neighbourhood $a$, only young buyers are worse off since $V_{a1}^{yr}$ and $V_{a2}^{yr}$ decrease. This is as a result of the decline in $V_{b1}^{er}$ that dominates the impacts of greater joint surpluses $S_{a1}^{ey}$ and $S_{a2}^{ey}$ (using (4.69), (4.72), (4.73) and (4.75)). Using (4.67) and (4.71), old sellers in $a$ are better off ($V_{a1}^{e}$ and $V_{a2}^{e}$ increase) because of higher gains from trade which stem from higher joint surpluses $S_{a1}^{ey}$ and $S_{a2}^{ey}$.

The better school in $b$ raises the housing prices in both neighbourhoods. Using (4.77), the housing price in $b$ ($w_{b1}^{ye}$) is higher since the increase in the reservation price for the young sellers ($V_{b2}^{y}$) and the reduction of the benefits from being buyers in the second subperiod ($V_{a2}^{yr}$) dominate the decline in the joint surplus, $S_{b1}^{ye}$. The housing prices in $a$ in both subperiods ($w_{a1}^{ey}$ and $w_{a2}^{ey}$) are rising, that is, $w_{a2}^{ey}$ is greater because of the larger joint surplus ($S_{b2}^{ey}$) while $w_{a2}^{ey}$ increases because both the reservation price of the old sellers ($V_{a2}^{e}$) and the joint surplus ($S_{a1}^{ey}$) rise.

The effects of increasing $U_{b}^{y}$ on the total welfare, measured by the sum of the numbers of agents times the value functions, is shown in Figure 4.2. Only young and old buyers are worse off therefore their welfares $TW_{a1}^{yr}$ and $TW_{b1}^{er}$ are decreasing. The welfare of young sellers in $a$ and $b$ and old sellers in $a$ is rising while the welfare of old sellers in $b$ is unchanged because $V_{b1}^{e}$ is unaffected. Lastly, the total welfare of the economy is increasing.

## 4.4.2 Changes in $U_{r}^{y}$ (Model II)

This exercise shows how the steady-state equilibrium changes when $U_{r}^{y}$ is increasing. This is interesting because it may be interpreted as a case in which inferior private schools are available. The term "inferior" here means that the quality of private schools is lower than the quality of the public school in $b$, and all private schools have the same quality. Therefore, young agents in $b$ do not benefit from such private schools, leaving $U_{b}^{y}$ unaffected. However, since private schools do not impose

residential requirement, the availability of inferior private schools affect only the housing service utility of the young buyers who live in rental houses. This case is illustrated by increasing the parameter $U_r^y$ from 0 to $U_b^y$, which equals 0.1 in the computations. Under such range of $U_r^y$, Table 4.2 is still the steady-state equilibrium. Keeping other parameters constant, the results are presented in Figure 4.3 and 4.4, which show the value functions, the housing prices, and the welfare. The distribution of agents does not change by the same reason as in the previous case (see Table 4.4 for the distribution).

The effects of an increase in the quality of inferior private schools on value functions and housing prices are presented in Figure 4.3. In this model, the value functions of the old in $b$ are unchanged, $V_{b1}^e = V_{b2}^e = U_b^e$. Also, the value function of the young in $b$ in the second subperiod is not affected since $V_{b2}^y = U_b^y + \beta U_b^e$. The value of being young in $b$ at the beginning of the first subperiod $(V_{b1}^y)$ increases. This comes from an increase in the joint surplus $(S_{b1}^{ye})$, which is improving because of the increase in the value of being a buyer in the second subperiod $(V_{a2}^{yr})$. The increase in $V_{a2}^{yr}$ is explained by the domination of the increase in both $U_r^y$ and $V_{b1}^{er}$ over the decrease in the joint surplus $(S_{a2}^{ey})$. $V_{b1}^{er}$ rises due to the increase in the joint surplus $(S_{b1}^{ye})$. Therefore, for agents who are involved in the housing market in neighbourhood $b$, no one is worse off.

For those who are in the housing market in neighbourhood $a$, only young buyers are better off from better inferior private schools since $V_{a1}^{yr}$ and $V_{a2}^{yr}$ increase. $V_{a1}^{yr}$ is higher as a result of the enhancement of $V_{a2}^{yr}$ which dominates the impacts of the lower joint surplus $(S_{a1}^{ey})$. Old sellers in $a$ are worse off ($V_{a1}^e$ and $V_{a2}^e$ decrease), because of lower gains from house sales which stem from declining joint surpluses, $S_{a1}^{ey}$ and $S_{a2}^{ey}$. The young sellers are also worse off since $V_{a1}^e$ is lower.

The availability of inferior private schools in this model lowers the housing prices in both neighbourhoods. Using (4.77), the housing price in $b$ $(w_{b1}^{ye})$ is lower since
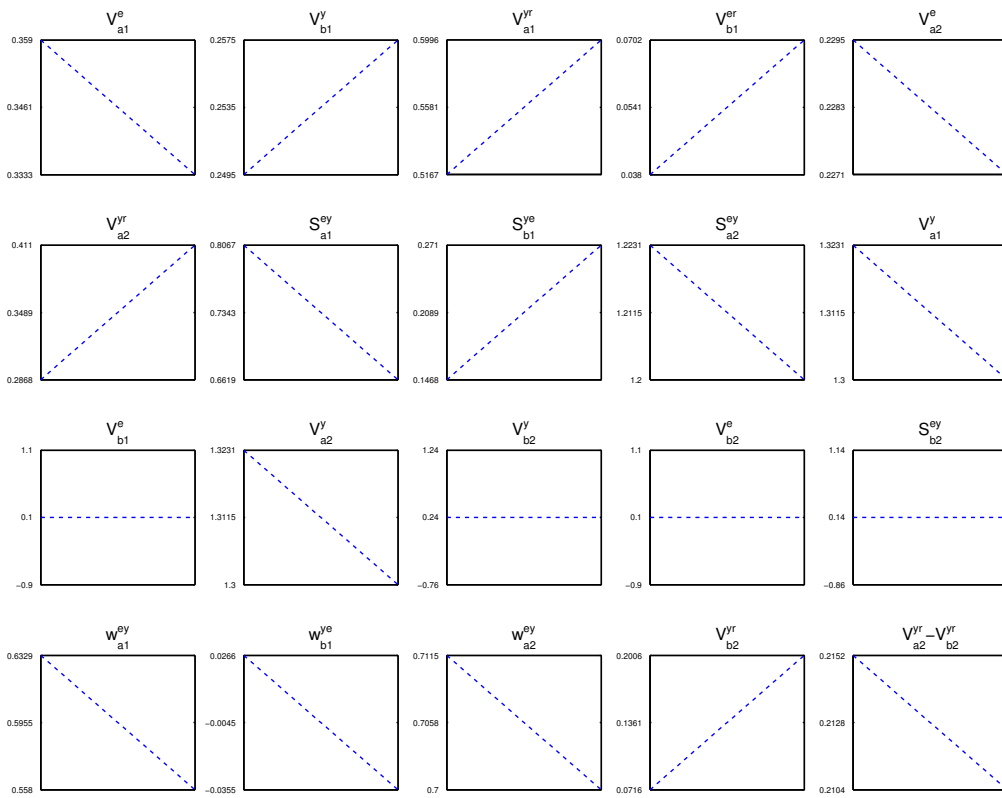
Figure 4.3: The Effects of Increasing $U_r^y$ on Value Functions and Housing Prices
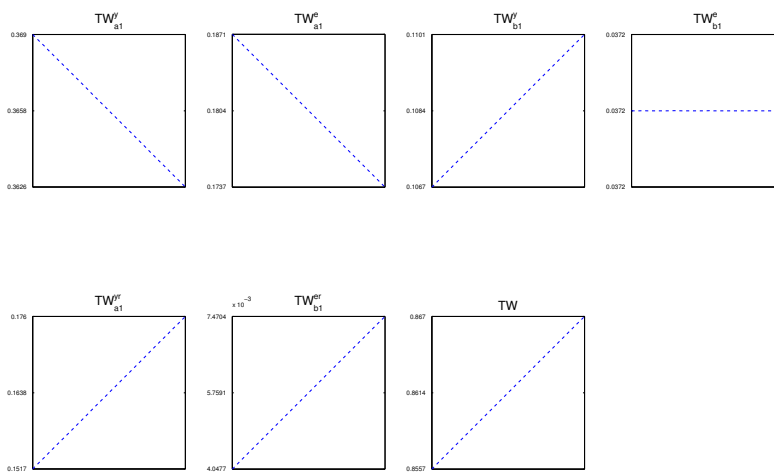


Figure 4.4: The Effects of Increasing $U_r^y$ on Welfare

the decline in the value of being a buyer in the second subperiod ($V_{a2}^{yr}$) dominates the increase in the joint surplus ($S_{b1}^{ye}$), while $V_{b2}^{y}$ is unaffected. The housing prices in $a$ in both subperiods ($w_{a1}^{ey}$ and $w_{a2}^{ey}$) fall. $w_{a2}^{ey}$ is lower because of the smaller joint surplus ($S_{b2}^{ey}$) while $w_{a2}^{ey}$ decreases because both the reservation price of old sellers ($V_{a2}^{e}$) and the joint surplus ($S_{a1}^{ey}$) shrink.

The effects of increasing $U_r^y$ on welfare are shown in Figure 4.4. In the first subperiod described earlier, young and old buyers are better off, therefore their welfares $TW_{a1}^{yr}$ and $TW_{b1}^{er}$ are increasing. The welfares of young and old sellers in $a$, $TW_{a1}^{y}$ and $TW_{a1}^{e}$, are decreasing. In neighbourhood $b$, the welfare of the young sellers ($TW_{b1}^{y}$) is improving, while the welfare of the old sellers ($TW_{b1}^{e}$) is unchanged due to $V_{b1}^{e}$ being unaffected. Lastly, the total welfare of the economy is increasing.

## 4.4.3 Changes in both $U_b^y$ and $U_r^y$ (Model III)

This section shows how the steady-state equilibrium changes when both $U_b^y$ and $U_r^y$ are increasing. This case is examined because it may be interpreted as a case when there are "relatively good" private schools available. The term "relatively good" here means that the quality of private schools is higher than the quality of the public school in $b$, but still lower than the quality of the public school in $a$. Therefore, both young agents in $b$ and those who live in rental houses benefit from such availability. Thus, this case is investigated by increasing the parameter $U_b^y$ and $U_r^y$ while keeping their difference constant. In the computations, $U_b^y = 0.15 + U_r^y$, where the constant 0.15 is the difference between $U_b^y$ and $U_r^y$ in the baseline setting. The range of $U_r^y$ is from 0 to 0.1, as in the previous section. Therefore, $U_b^y$ is varied from 0.15 to 0.25. In this range of $U_r^y$, Table 4.2 is still the steady-state equilibrium. Keeping other parameters constant, the results are presented by dashed lines in Figure 4.5 and 4.6, which show the value functions, the housing prices, and the welfare. The distribution of agents does not change by the same reason as in Model I.
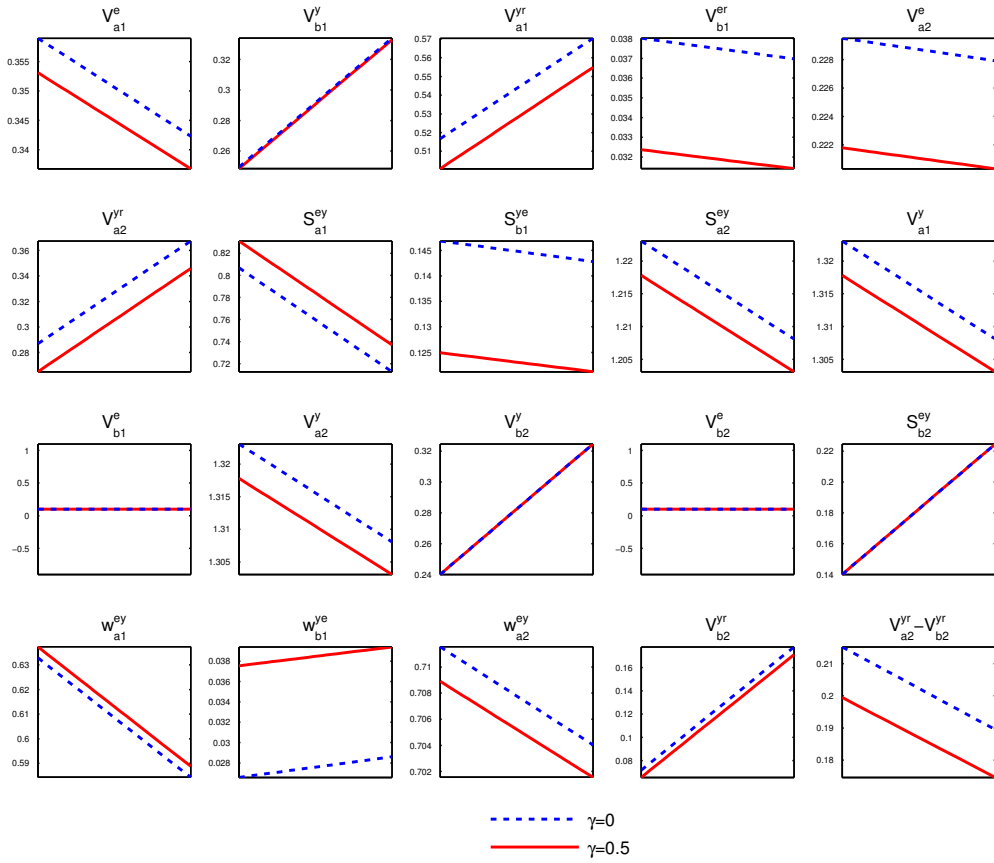
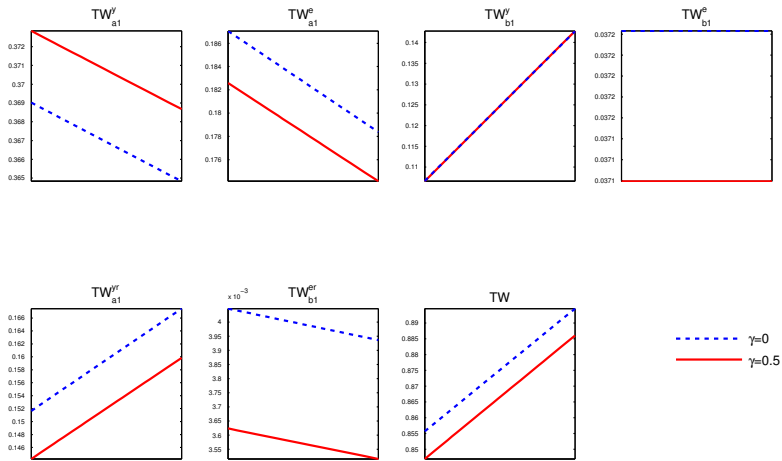Figure 4.5: The Effects of Increasing $U_b^y$ and $U_r^y$ on Value Functions and Housing Prices



Figure 4.6: The Effects of Increasing $U_b^y$ and $U_r^y$ on Welfare

134

The effects of an increase in the quality of relatively good private schools on value functions and housing prices are presented by the dashed lines in Figure 4.5. In this model, the value functions of the old in $b$ are unchanged, that is, $V_{b1}^e = V_{b2}^e = U_b^e$. The value function of the young in $b$ in the second subperiod is higher since $V_{b2}^y = U_b^y + \beta U_b^e$. The value function of the young in $b$ at the beginning of a period ($V_{b1}^y$) increases. This comes from a stronger effect of an increase in $V_{b2}^y$ than a reduction in the joint surplus ($S_{b1}^{ye}$). This joint surplus decreases because the benefits of staying in $b$ is now higher for young agents relative to the increase in the value of being a buyer in the second subperiod ($V_{a2}^{yr}$). The rise in $V_{a2}^{yr}$ is explained by the increase in $U_r^y$ over the decline of both $V_{b1}^{er}$ and the joint surplus ($S_{a2}^{ey}$). $V_{b1}^{er}$ falls due to the decrease in the joint surplus ($S_{b1}^{ye}$). Therefore, for agents who are involved in the housing market in neighbourhood $b$, only old buyers are worse off.

For those who are in the housing market in neighbourhood $a$, only young buyers are better off since $V_{a1}^{yr}$ and $V_{a2}^{yr}$ increase. $V_{a1}^{yr}$ is higher as a result of an enhancement in $V_{a2}^{yr}$ which dominates the impact of the lower joint surplus ($S_{a1}^{ey}$). The old sellers in $a$ are worse off ($V_{a1}^e$ and $V_{a2}^e$ decrease) because of lower gains from trade which stem from declining joint surpluses, $S_{a1}^{ey}$ and $S_{a2}^{ey}$. The young sellers are also worse off since $V_{a1}^e$ is lower.

The availability of relatively good private schools in this model reduces housing prices in neighbourhood $a$ but inflates the prices in neighbourhood $b$. $w_{a2}^{ey}$ is lower because of the smaller joint surplus ($S_{b2}^{ey}$) while $w_{a2}^{ey}$ decreases because both the reservation price of the old sellers ($V_{a2}^e$) and the joint surplus ($S_{a1}^{ey}$) shrink. Using (4.77), the housing price in $b$ ($w_{b1}^{ye}$) is higher since the reservation price for the young sellers increases ($V_{b2}^y$), this outweighs the greater benefits of being a buyer in the second subperiod ($V_{a2}^{yr}$) and the decline in the joint surplus ($S_{b1}^{ye}$).

The effects on welfare are shown in Figure 4.4. The total welfare of the economy is increasing because the increase in the welfare of those who are better off, the

young sellers in $b$ and the young buyers in $a$, dominates the welfare reduction of the young and old sellers in $a$ and the old buyers in $b$.

### 4.4.4 Changes in the Housing Capacities

So far, the numbers of houses in $a$ and $b$ have been set to be equal. It would be interesting to see how the equilibria are affected if the housing supply in the good public school location is relatively limited. In this regard, the following computations set $A = 0.7$ and $B = 0.9$ and use Model III as the starting point. The results are shown in Figure 4.7-4.9.
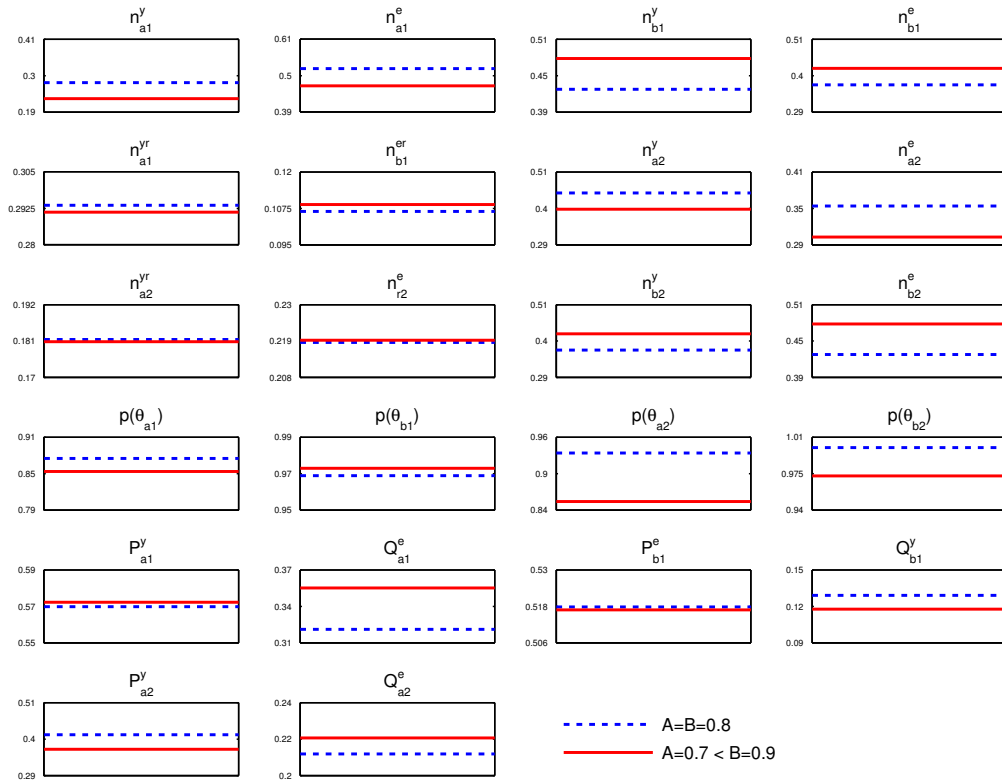


Figure 4.7: The Distributions of Agents under Different Values of Housing Capacities

There are changes in the distribution of agents, the value functions, and the total welfare. However, qualitatively the direction of how each variable responds to changes in the school quality parameters remains the same as in Model III. Only
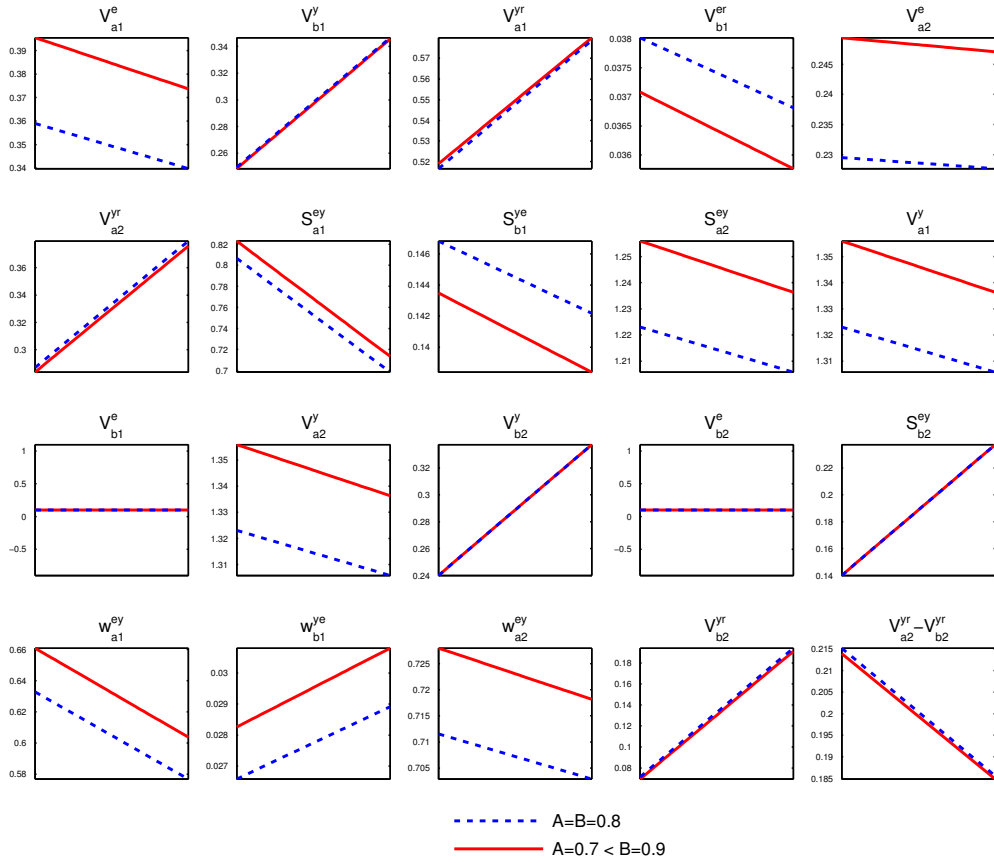
136

Figure 4.8: The Effects of Increasing $U_b^y$ and $U_r^y$ on Value Functions and Housing Prices under Different Values of Housing Capacities
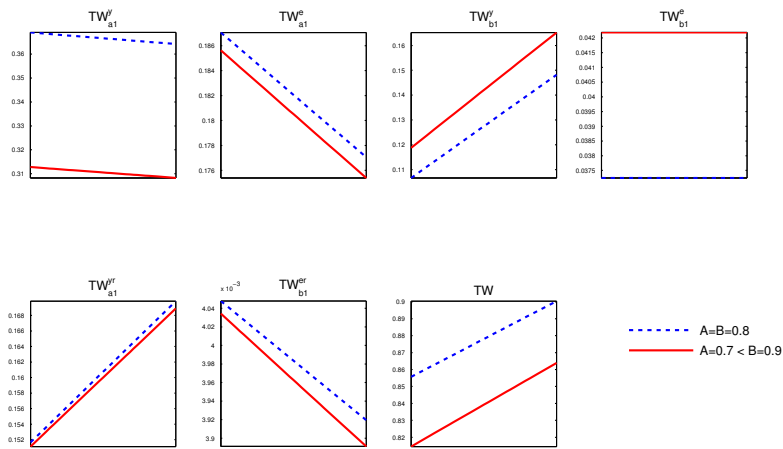


Figure 4.9: The Effects of Increasing $U_b^y$ and $U_r^y$ on Welfare under Different Values of Housing Capacities

sellers in $a$, both young and old, have higher value functions in this case. Basically, the changes in the distribution of agents increase the probability of selling a house as well as the joint surpluses in $a$ in both subperiods. Therefore, the old and young sellers in $a$ are better off since their value functions also include such benefits of resale values once they become old.

## 4.5 Discussion

As mentioned earlier, there is indeterminacy of equilibria because the search behavior of old buyers in the second subperiod is indeterminate since they are indifferent between searching and not searching. This is a by-product of the assumption that search costs are zero. Under a given set of parameters, there could be many equilibria depending on how many of those old agents decide to search in the housing markets. Figure 4.10 emphasises this point. Using Model III as the benchmark model, for a given set of parameters, the numerical procedure found equilibria each corresponding to different numbers of old buyers searching in the housing markets. The results also show that the total welfare is highest if there are no old agents searching in any housing markets. The solid line plots the total welfare against changes in $U_b^y$ when $\gamma_a = \gamma_b = 0$, that is, no old buyers search in the second subperiod. This line is above all other dashed lines which plot cases where all old buyers are in the housing markets, that is, $1 \geq \gamma_a > 0$ and $\gamma_b = 1 - \gamma_a$.

The reason that the total welfare is highest when $\gamma_a = \gamma_b = 0$ is quite simple. Allowing the old buyers in the second subperiod to search in the housing markets generates externalities which affect congestion and redistribution of agents across locations. For example, in Figure 4.5, for a given set of parameters, the value functions of all agents when $\gamma_a = \gamma_b = 0$ (dashed lines) are at least as high as the value functions when $\gamma_a = \gamma_b = 0.5$ (solid lines). Moreover, the directions of
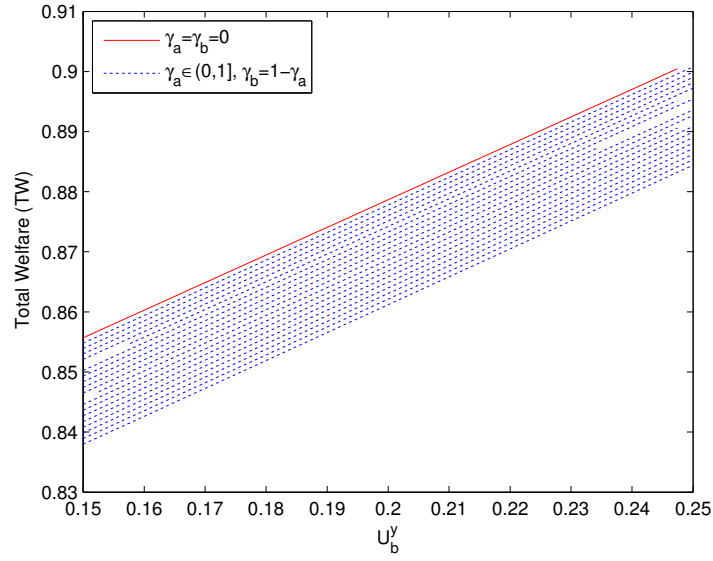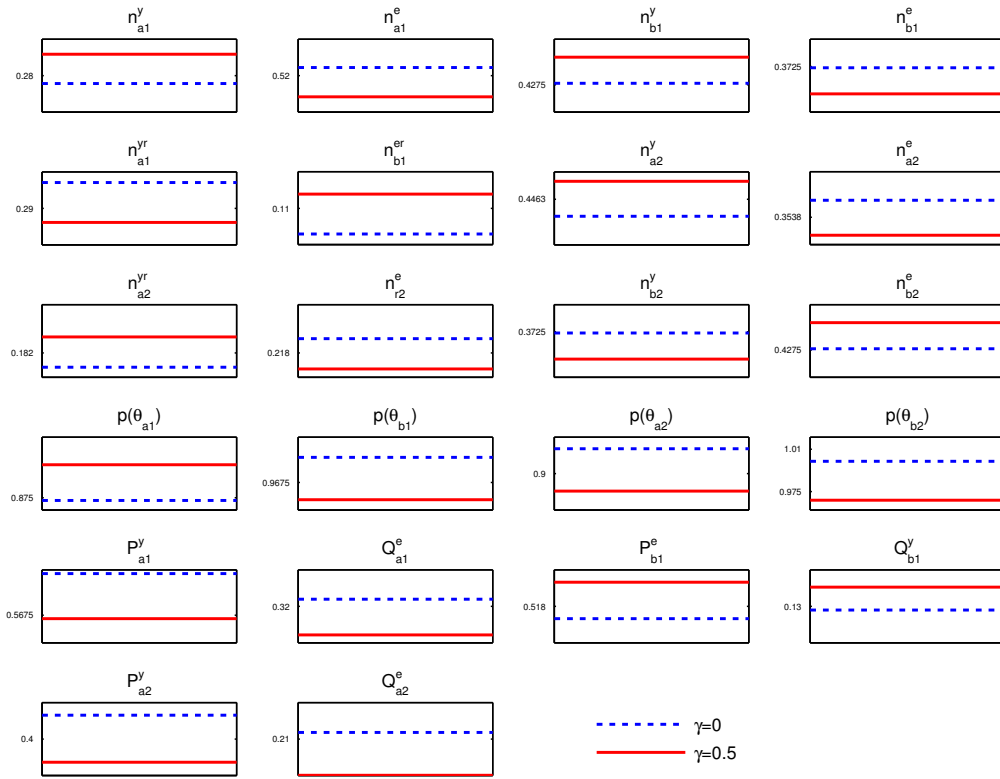
Figure 4.10: Indeterminacy of Equilibria



Figure 4.11: The Effects of Increasing $U_b^y$ and $U_r^y$ on the Distribution of Agents

changes of the value functions and the total welfare are still the same in both cases. After accounting for changes in the distribution of agents (Figure 4.11), the total welfare is higher when $\gamma_a = \gamma_b = 0$.

It is worth mentioning that the result that some young households may get trapped in the low-quality neighbourhood because of housing frictions may have some policy implications for education. Any educational policies that expect households to move their residence to suit their preference of educational quality may not be effective if housing frictions are severe. In addition, improving school quality in bad neighbourhoods may increase the social welfare. Fernandez and Rogerson (1996) show that welfare improvement can be achieved by redistribution toward the poorest and by increasing the educational spending as well as the attractiveness of the poorest community. However, a proper analysis in the context of this chapter is required in order to understand the potential effects of such policies. This is left for future research.

## 4.6   Conclusion

This chapter provides an analysis of school and residential choices when there are frictions in housing markets and agents only value school quality during the early stages of their lives. An overlapping-generation model is developed to show that there exist steady-state equilibria in which young agents who value school quality and live in a location with a low-quality public school can move to another location with a better public school. These equilibria exist under sufficiently high differences in public school quality across locations. For individuals, the benefits of moving into a good school location are not only derived from the school quality but also from the resale value of the house once school services are no longer valued. Equilibria exist in which increasing the quality of the low-quality school improves the social

140

welfare but affects agents across locations differently. In particular, when relatively good quality private schools are available, the young house buyers are better off while some old sellers are worse off due to a reduction in the returns from house sales. Moreover, there is indeterminacy of equilibria because the search behavior of some buyers is indeterminate in equilibrium since they are indifferent between searching and not searching. The indifference comes from the fact that they cannot gain from any meetings in the housing markets. Without search costs, if these buyers decide to search, they create externalities on congestion and redistribution of agents in equilibrium. Note that the model here is not limited to analysing school and residential choices. It may be applied to other kinds of goods and services that are valued only in the early stages of agents' lives and are publicly provided with different quality across locations. An example of such services is child care.

Even though the model in this chapter is simple and highly stylised, it is a starting point of how to apply a search-theoretic framework into a model of school and residential choices. The literature on school and residential choices tend to ignore frictions in housing markets but allow for multiple locations and endogenous school quality. The search-theoretic literature tends to analyse only in a single market environment but comprehensively explains key characteristics of the housing market. To get closer to the former literature, a possible extension would be to allow for endogenous school quality by having heterogeneity in school valuations among young agents as well as measuring school quality by peer-group quality. Moreover, private schools have to be modeled explicitly. With such modification, the composition of types in a location will potentially affect the school quality, which in turn affects the search decisions of buyers and the supply decisions of house owners. It is likely that some young agents who care less about school quality would choose to sell their houses if they live in the high-quality school location, or choose to stay in the same location if they are in the low-quality school location.

There would be cut-off types for being sellers in the housing markets, therefore changes in school quality parameters would potentially affect the distribution of agents, unlike what is shown in Model I-III, as well as the value functions, housing prices, and school qualities. Buyers and sellers who trade in equilibrium in each housing market will be heterogeneous. This heterogeneity could create different traded prices within each housing market as well as endogenous school quality. These features are not possible under the current model. A model of school and residential choices with endogenous school quality and frictional housing markets would be useful for analysing the effects of educational policies. The analysis that incorporates frictions into the housing markets would yield more precise predictions. The relevant modification would increase the complication significantly and is left for an ongoing research.

# Chapter 5

# Conclusion Remarks

This thesis studies school and residential choices when private schooling is available and attending a public school is free of charge but requires a residence in the school attendance zone. Chapter 2 examines the cream-skimming problem: private schools attract relatively richer and higher ability students and produce higher school quality than public schools. The results of Chapter 2 contribute to the debate of private education subsidisation by confirming that the cream-skimming problem persists even when incorporating neighbourhood (public) schools. Subsidising private schools is likely to intensify the cream-skimming problem and worsen the welfare of students who are left in the public sector as public schools lose relatively higher ability students to the private sector.

Chapter 3 argues that price subsidisation for private education can be a Pareto improving policy if (i) school quality is measured by levels of educational services, (ii) private education is more costly per unit, (iii) the level of educational services of public schools within a jurisdiction is determined by majority voting of the residents, and (iv) housing capacities of jurisdictions cannot accommodate perfect segregation among heterogeneous households. Financed by income taxes imposed to all high-income households, Pareto-improving subsidisation exists and it is either

partial or full subsidisation, which eliminates the unit price difference between two sectors, that maximises the total welfare.

Chapter 4 studies school and residential choices when there are frictions in housing markets and agents only value school quality during the early stages of their lives. Chapter 4 shows that agents relocate themselves according to different valuations and qualities of local amenities such as public schools. There exist steady-state equilibria in which young agents who value school quality and live in a location with a low-quality public school can move to another location with a better public school. With frictions, some of such young agents who are willing to relocate get stuck in the low-quality school location and obtain relatively low lifetime utility. For individuals, the benefits of moving into a good school location are not only derived from the school quality but also from the resale value of the house once school services are no longer valued. Increasing the quality of the low-quality school improves the total welfare but affects agents across locations differently. In addition, when relatively good quality private schools are available, young house buyers are better off while some old house owners are worse off due to a reduction in the returns from house sales. These results point out that frictions matter to the effects of changes in school quality on the relocation and welfare of agents. Future research will extend the model by (i) allowing for heterogeneous preference of young agents over school quality, (ii) using peer-group quality as the measurement of school quality, and (iii) modeling private schools explicitly. This would make the schooling aspect of the modified model closer to those in the literature of school and residential choices, and thus potentially providing more precise predictions.

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