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DOCTORAL THESIS

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**Optimal Space Time Adaptive Processing for  
Multichannel Inverse Synthetic Aperture Radar  
Imaging**

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## *Abstract*

### **Optimal Space Time Adaptive Processing for Multichannel Inverse Synthetic Aperture Radar Imaging**

by Alessio BACCI

The thesis deals with the application of ISAR processing to obtain high resolution imaging of non-cooperative moving targets within a SAR scene. The research topic is of great interest nowadays. Modern SAR system can provide high resolution radar images of wide areas with reduced revisiting time. These features make them particularly suited for surveillance application. It is obvious that for these kind of applications the capability of imaging non-cooperative moving targets becomes fundamental. Since conventional SAR processing is unable to focus moving targets because of the lack of knowledge of the target motion a solution based on ISAR processing is proposed. In fact, ISAR systems, do not make any assumption about the target motion, but they exploit it to form the synthetic aperture and to obtain high resolution in the cross-range dimension. Since the ISAR processing must be applied to each target separately a detection step is fundamental. Although this detection step is not a problem when dealing with maritime targets, as the sea clutter return is usually much lower than the target return, it can be a challenge when dealing with ground target. Multichannel information provided by SAR systems with multiple receivers can be exploited to mitigate the return of the static scene. STAP processing is then combined with ISAR technique to produce high resolution images of non-cooperative moving targets after detection within SAR images. The multichannel version of Range Doppler image formation algorithm is derived and analyzed. Then, it is used to define a Space Doppler Adaptive processing to mitigate the strong clutter before the application of ISAR autofocus. Performance are evaluated on simulated data. Results on real data prove the effectiveness of the proposed processing and its applicability on actual systems.



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# Abbreviations

<b>ACE</b>	<b>A</b> daptive <b>C</b> oherent <b>E</b> stimator
<b>AMF</b>	<b>A</b> daptive <b>M</b> atched <b>F</b> ilter
<b>ATI</b>	<b>A</b> long <b>T</b> rack <b>I</b> nterferometry
<b>CFAR</b>	<b>C</b> onstant <b>F</b> alse <b>A</b> larm <b>R</b> ate
<b>CPI</b>	<b>C</b> oherent <b>P</b> rocessing <b>I</b> nterval
<b>CSK</b>	<b>C</b> osmo <b>S</b> Ky <b>M</b> ed
<b>DPCA</b>	<b>D</b> isplaced <b>P</b> hase <b>C</b> entre <b>A</b> rray
<b>DoF</b>	<b>D</b> egree of <b>F</b> reedom
<b>DoA</b>	<b>D</b> irection of <b>A</b> rrival
<b>FFT</b>	<b>F</b> ast <b>F</b> ourier <b>T</b> ransform
<b>FT</b>	<b>F</b> ourier <b>T</b> ransform
<b>GRLT</b>	<b>G</b> eneralized <b>L</b> ikelihood <b>R</b> atio <b>T</b> est
<b>IFFT</b>	<b>I</b> nverse <b>F</b> ast <b>F</b> ourier <b>T</b> ransform
<b>IFT</b>	<b>I</b> nverse <b>F</b> ourier <b>T</b> ransform
<b>IOK</b>	<b>I</b> nverse <b>O</b> mega <b>K</b> ey
<b>IRD</b>	<b>I</b> nverse <b>R</b> ange <b>D</b> oppler
<b>ISAR</b>	<b>I</b> nverse <b>S</b> ynthetic <b>A</b> perture <b>R</b> adar
<b>LCMV</b>	<b>L</b> inearly <b>C</b> onstrained <b>M</b> inimum <b>V</b> ariance
<b>LoS</b>	<b>L</b> ine of <b>S</b> ight
<b>LSE</b>	<b>L</b> east <b>S</b> quare <b>E</b> rror
<b>MTD</b>	<b>M</b> oving <b>T</b> arget <b>D</b> etection
<b>MTI</b>	<b>M</b> oving <b>T</b> arget <b>I</b> ndication
<b>NCTI</b>	<b>N</b> on <b>C</b> ooperaiive <b>T</b> arget <b>I</b> maging
<b>PRF</b>	<b>P</b> ulse <b>R</b> epetition <b>F</b> requency
<b>PRI</b>	<b>P</b> ulse <b>R</b> epetition <b>I</b> ntervall
<b>RD</b>	<b>R</b> ange <b>D</b> oppler
<b>SAR</b>	<b>S</b> ynthetic <b>A</b> perture <b>R</b> adar
<b>SDAP</b>	<b>S</b> pace <b>D</b> oppler <b>A</b> daptive <b>P</b> rocessing

<b>SMI</b>	<b>S</b> ample <b>M</b> atrix <b>I</b> nversion
<b>STAP</b>	<b>S</b> pace <b>T</b> ime <b>A</b> daptive <b>P</b> rocessing
<b>TFT</b>	<b>T</b> ime <b>F</b> requency <b>T</b> ransform
<b>TDC</b>	<b>T</b> ime <b>D</b> omain <b>C</b> orrelation
<b>WVD</b>	<b>W</b> igner <b>V</b> ille <b>D</b> istribution

# Symbols

	<u><b>A</b></u>
$A$	Constraint matrix
	<u><b>B</b></u>
$B$	Signal Bandwidth
$B_D$	Doppler Bandwidth
$b$	Constraint value
	<u><b>D</b></u>
$D_{y_1}/D_{y_2}$	Non-ambiguity region in cross-range/range
$D_{array}$	Array size
$D_{y_1}$	Cross-range non-ambiguity region
$D_{y_2}$	Range non-ambiguity region
	<u><b>E</b></u>
$\mathbf{e}_s(\theta)$	Spatial Steering Vector
$\mathbf{e}_t(F_D)$	Temporal Steering Vector
$\mathbf{e}(\theta, F_D)$	Spatial-Temporal Steering Vector
	<u><b>F</b></u>
$f$	Range frequency
$f_0$	Carrier frequency
$F_S$	Spatial Frequency (beamforming)
$F_D$	Normalized Doppler Frequency
	<u><b>G</b></u>
$\mathbf{G}(n, m)$	Reference signal vector
$\mathbf{G}_i(n, m)$	Reference signal vector ( $i^{th}$ window)
$\tilde{\mathbf{G}}(n, m)$	Reference signal vector
$\tilde{\mathbf{G}}_i(n, m)$	Post-Doppler Reference signal vector ( $i^{th}$ window)
$\mathbf{G}_D(n, m_\nu)$	Space Doppler Reference signal vector
$\mathbf{G}_{D,i}(n, m_\nu)$	Space Doppler Reference signal vector ( $i^{th}$ window)

	<u><b>I</b></u>
$\mathbf{i}_{LoS}$	LoS unit vector
$\mathbf{i}_{LoST_x}/\mathbf{i}_{LoSR_x}$	LoS unit vector TX/RX
$\mathbf{i}_{LoS_{BI}}$	LoS unit vector (bistatic)
$\mathbf{i}_{LoS_x}^{(p,q)}$	LoS unit vector of the element $(p, q)$ with respect $T_x$
$\mathbf{i}_{LoS_\xi}^{(p,q)}$	LoS unit vector of the element $(p, q)$ with respect $T_\xi$
$I(\bullet, \bullet)$	ISAR image
$i$	Window index
	<u><b>K</b></u>
$K_{BI}(t)$	Phase modulation term (bistatic geometry)
$K_{BI_0}(t)$	Phase modulation term $t = 0$ (bistatic geometry)
$(k_{y_1}, k_{y_2})$	Spatial frequencies
	<u><b>L</b></u>
$L$	Window length
$L_f$	Considered Doppler bins
	<u><b>M</b></u>
$m$	Pulse index
$m_\nu$	Doppler frequency index
$M$	Slow time samples
$M_\nu$	Doppler frequency samples
$\mathbf{M}_{\xi x}$	Rotation matrix from $T_\xi$ to $T_x$
	<u><b>N</b></u>
$n$	Range frequency index
$n_\tau$	Delay time index
$N$	Frequency samples
$N_\tau$	Delay time samples
$\mathbf{n}$	Gaussian random vector with zero mean and Identity covariance matrix
$N_d$	Discarded pulses
	<u><b>P</b></u>
$p$	Array element index along $\xi_1$
$P$	Number of array element index along $\xi_1$
$P_c$	Clutter power
	<u><b>Q</b></u>
$q$	Array element index along $\xi_3$
$Q$	Number of array element index along $\xi_3$
	<u><b>R</b></u>

$R_0(t)$	Radar-target distance
$R_0$	Radar-target distance at $t = 0$
$R_{Tx}/R_{Rx}$	Distance between the target and the Tx/Rx
$\mathbf{R}_c$	Clutter space-slow time covariance matrix
$\mathbf{R}_{c,i}$	Clutter space-slow time covariance matrix ( $i^{th}$ window)
$\mathbf{R}_{D,c}$	Clutter cross-power spectral matrix
<b><u>S</u></b>	
$s_T(t_f)$	Transmitted signal
$s_R(t_f, t)$	Received signal
$S_R(f, t)$	Received signal in the frequency/slow time domain
$S_{Rc}(f, t)$	Motion compensated received signal
$S_0(\bullet, \bullet)$	Reference signal
$S_p(f, t)$	Received signal at $p^{th}$ receiver
$S_{t,p}(f, t)$	Received signal at $p^{th}$ receiver (target contribution)
$S_{c,p}(f, t)$	Received signal at $p^{th}$ receiver (clutter contribution)
$\mathbf{s}_s$	data received in the $n^{th}$ frequency bin and the $m^{th}$ pulse
$\mathbf{s}(n)$	stacked signal vector
$\mathbf{s}_T$	stacked signal vector point like target (beamforming)
$S_{ref}(\bullet, \bullet)$	Reference Signal
$\mathbf{S}(n)$	Signal vector
$\mathbf{S}_i(n)$	Signal vector ( $i^{th}$ window)
$\tilde{\mathbf{S}}(n)$	Post-Doppler Signal vector
$\tilde{\mathbf{S}}_i(n)$	Post-Doppler Signal vector ( $i^{th}$ window)
<b><u>T</u></b>	
$T_\xi$	Reference system embedded on the radar
$T_x$	Reference system embedded on the target
$T_y$	Reference system embedded on the target (fixed)
$t$	Slow time
$t_f$	Fast time
$T_R$	Pulse Repetition Interval
$T_{obs}$	Observation time
$T_{win}$	Window length
$T_{step}$	Window step
$\mathbf{T}$	Unitary transformation matrix
$\mathbf{T}_t$	Fourier Matrix
$\mathbf{T}_w$	Unitary windowed transformation matrix

$\mathbf{T}_{w,t}$	Windowed Fourier Matrix
$\tilde{\mathbf{T}}_{w,t}$	Reduced Windowed Fourier Matrix
$\tilde{\mathbf{T}}_w$	Reduced windowed transformation matrix
<u><b>U</b></u>	
$u$	Platform position on the trajectory
$U$	Eigenvectors matrix
$u(\bullet, \bullet)$	Matched Filter Output
$u_i(\bullet, \bullet)$	Matched Filter Output ( $i^{th}$ window)
$u_w(\bullet, \bullet)$	Matched Filter Output (windowed)
$\tilde{u}(n, m_\nu)$	Post-Doppler filtering output
$\tilde{u}_i(\bullet, \bullet)$	Post-Doppler Matched Filter Output ( $i^{th}$ window)
$\tilde{u}_w(\bullet, \bullet)$	Post-Doppler Matched Filter Output (windowed)
$u(\bullet, \bullet)_D$	Space Doppler Processing Output
<u><b>V</b></u>	
$V$	Volume where the target reflectivity is defined
$v_p$	Platform velocity
$\mathbf{V}$	Eigenvalues matrix
$v_{t,y_1}, v_{t,y_1}$	Target velocity along cross-range/range direction
<u><b>W</b></u>	
$W(f, t)$	Frequency/slow time domain where the signal is defined
$W(f, t)$	Spatial Frequencies domain time domain where the signal is defined
$\mathbf{w}(\theta, F_D)$	weightvector
$\mathbf{W}(n, m)$	Optimum weightvector
$\mathbf{W}_i(n, m)$	Optimum weightvector ( $i^{th}$ window)
$\tilde{\mathbf{W}}(n, m_\nu)$	Post-Doppler Optimum weightvector
$\tilde{\mathbf{W}}_i(n, m)$	Post-Doppler Optimum weightvector ( $i^{th}$ window)
$\tilde{\mathbf{W}}_D(n, m_\nu)$	Space Doppler Optimum weightvector
$\tilde{\mathbf{W}}_{D,i}(n, m_\nu)$	Space Doppler Optimum weightvector ( $i^{th}$ window)
<u><b>W</b></u>	
$\mathbf{x}$	Stacked received signal vector
<u><b>Y</b></u>	
$(Y_1, Y_2)$	Spatial frequencies
$y_s(\theta, n, m)$	Steered output in the direction $\theta$
$y_{t,p}(F_D, n)$	Focused output at $F_D$
$y(\theta, F_D, n)$	Focused output at $\theta, F_D$
$y^{(u)}$	useful output

$y^{(c)}$	clutter (disturbance) output
	<u><b>Z</b></u>
$\mathbf{Z}(n_r)$	Target-free data in the $n_r^{th}$ range cell
	<u><b><math>\alpha</math></b></u>
$\alpha$	Angle between $\xi_3$ and $x_3$
	<u><b><math>\beta</math></b></u>
$\beta(t)$	Bistatic angle variation
	<u><b><math>\Delta</math></b></u>
$\Delta_\tau$	Delay time resolution
$\Delta_\nu$	Doppler frequency resolution
$\Delta_{y_1}/\Delta_{y_2}$	Cross-range/range resolution
$\Delta Y_1/\Delta Y_1$	Spacial frequencies spacing
$\Delta\Theta$	Aspect angle variation
	<u><b><math>\Theta</math></b></u>
$\theta_{in}$	Incidence angle
$\theta$	DoA
$\theta_{az}$	Azimuth aperture
	<u><b><math>\nu</math></b></u>
$\nu$	Doppler frequency
	<u><b><math>\rho</math></b></u>
$\rho_s(\Delta_d)$	space correlation
$\rho_t(\Delta_t)$	time correlation
	<u><b><math>\sigma</math></b></u>
$\sigma(\mathbf{y})$	Reflectivity of the scatterer placed in $\mathbf{y}$
$\sigma_k$	Reflectivity of the $k^{th}$ scatterer
$\sigma_T$	Reflectivity of a point like target (beamforming)
	<u><b><math>\tau</math></b></u>
$\tau(\mathbf{y}, t)$	Delay time of the scatterer placed in $\mathbf{y}$ at time $t$
	<u><b><math>\Phi</math></b></u>
$\phi(\mathbf{y}, f, t)$	Phase term (monostatic case)
$\phi_{BI}(\mathbf{y}, f, t)$	Phase term (bistatic case)
	<u><b><math>\Omega</math></b></u>
$\Omega_T(t)$	Total rotation vector
$\Omega_{eff}(t)$	Effective rotation vector