

# Chiral-Scale Perturbation Theory

## About an Infrared Fixed Point



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“An idea that is not dangerous is unworthy of being called an idea at all.”

— Oscar Wilde: *The Critic as Artist*

## Abstract

This work explores the infrared behaviour of the strong running coupling  $\alpha_s$  in Quantum Chromodynamics (QCD). We propose that  $\alpha_s$  runs non-perturbatively to an infrared fixed point  $\alpha_{\text{IR}}$  for three light quark flavours  $u, d, s$ . At the fixed point, we show that the quark condensate spontaneously breaks scale and chiral  $SU(3)_L \times SU(3)_R$  symmetry. Consequently, the low-lying spectrum contains nine pseudo-Nambu-Goldstone bosons:  $\pi, K, \eta$  and a scalar-isoscalar QCD dilaton  $\sigma$ . We argue that  $\sigma$  may be identified with the  $f_0(500)$  resonance, a pole at a complex mass with real part  $\lesssim m_K$ . For low-energy expansions in  $\alpha_s$  about  $\alpha_{\text{IR}}$ , we replace chiral  $SU(3)_L \times SU(3)_R$  perturbation theory with a new model-independent theory  $\chi\text{PT}_\sigma$  based on approximate scale and chiral  $SU(3)_L \times SU(3)_R$  symmetry.

We examine the phenomenological consequences which arise from this framework by constructing effective Lagrangians which simulate strong, weak, and electromagnetic interactions. We also study the convergence properties of the effective theory, wherein we find that  $\chi\text{PT}_\sigma$  converges much better than  $\chi\text{PT}_3$  in the presence of both scalar-isoscalar channels and  $O(m_K)$  extrapolations in momentum. We achieve this without spoiling the successful leading order predictions of  $\chi\text{PT}_3$  elsewhere.

In our phenomenological investigations, we show that the  $\Delta I = 1/2$  rule for non-leptonic  $K$ -decays emerges as a consequence of  $\chi\text{PT}_\sigma$ , with a  $K_S\sigma$  coupling fixed by data for  $\gamma\gamma \rightarrow \pi\pi$  and  $K_S \rightarrow \gamma\gamma$ . This constitutes our most important result.

We also apply the electromagnetic trace anomaly to QCD at the infrared fixed point and obtain the estimate  $R_{\text{IR}} \approx 5$  for the non-perturbative Drell-Yan ratio  $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  at  $\alpha_{\text{IR}}$ .

## Statement of Originality

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree. I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968. I also give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library catalogue and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

*List of publications, workshop proceedings, and presentations based on this thesis.*

- R. J. Crewther and L. C. Tunstall, "*Origin of the  $\Delta I = 1/2$  Rule for Kaon Decays: QCD Infrared Fixed Point*", arXiv:1203.1321 [hep-ph] (submitted to Physical Review D).
- R. J. Crewther and L. C. Tunstall, "*Infrared Fixed Point in the Strong Running Coupling: Unraveling the  $\Delta I = 1/2$  Puzzle in K-Decays*", arXiv:1306.4445 [hep-ph] (Contribution to the proceedings of the workshop "Determination of the Fundamental Parameters of QCD",

Nanyang Technological University, Singapore, March 18-22, 2013,  
to be published in Mod. Phys. Lett. A).

- L. C. Tunstall, "*QCD Dilatons and the Origin of the  $\Delta I = 1/2$  Rule*", talk and poster presented at CoEPP Summer School and Workshop, Lorne, VIC, Australia, February 20-24, 2012.
- L. C. Tunstall, "*Origin of the  $\Delta I = 1/2$  Rule for Kaon Decays: QCD Infrared Fixed Point*", talks given at Universidad de los Andes and Universidad Nacional de Colombia, Bogotá, Colombia, August 1-2, 2012.

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Date

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