

# Fast and Unambiguous Direction Finding for Digital Radar Intercept Receivers

Peter Quoc Cuong Ly

A Thesis Submitted for the Degree of  
**Doctor of Philosophy**



School of Electrical and Electronic Engineering  
The University of Adelaide  
Adelaide, South Australia

December 2013



*For my wife, Joy*

*The Light of my Life*



# Contents

<i>List of Figures</i>	v
<i>List of Tables</i>	xiii
<i>Abstract</i>	xv
<i>Declaration</i>	xvii
<i>Acknowledgements</i>	xix
<i>Publications</i>	xxi
<i>Acronyms</i>	xxiii
<i>Notation</i>	xxv
<b>1 Introduction</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Electronic Support . . . . .	1
1.3 The Importance of Direction Finding . . . . .	3
1.3.1 Situational Awareness . . . . .	3
1.3.2 Signal Deinterleaving . . . . .	3
1.3.3 Electronic Attack and Electronic Protection Measures . . . . .	4
1.3.4 Signal Enhancement . . . . .	4
1.3.5 Beaming Manoeuvre . . . . .	4
1.4 Problem Statement . . . . .	5
1.4.1 Design Constraints of a Radar Intercept Receiver . . . . .	5
1.4.2 One-Way Propagation Advantage . . . . .	6
1.4.3 Implications for AOA Estimation . . . . .	7
1.4.4 Assumptions . . . . .	8
1.5 Organisation of this Thesis . . . . .	10
1.6 Original Contributions . . . . .	12
<b>2 Contemporary Direction Finding Techniques</b>	<b>13</b>
2.1 Spinning Antenna . . . . .	13
2.2 Amplitude Comparison . . . . .	14
2.2.1 Loop Antennas . . . . .	15
2.2.2 Adcock Arrays . . . . .	15
2.2.3 Cavity-Backed Spiral Antennas . . . . .	16

2.3	Frequency-Difference-of-Arrival . . . . .	18
2.4	Time-Difference-of-Arrival . . . . .	19
2.5	Interferometry . . . . .	20
2.6	Beamforming and Array Processing . . . . .	20
2.7	Summary . . . . .	22
<b>3</b>	<b>Interferometry</b>	<b>25</b>
3.1	Introduction . . . . .	25
3.2	Signal Model . . . . .	25
3.2.1	Propagation Delays . . . . .	26
3.2.2	Narrowband Signal Model . . . . .	28
3.3	Interferometry . . . . .	29
3.3.1	Relationship to TDOA . . . . .	29
3.4	AOA Estimation in the Presence of Receiver Noise . . . . .	30
3.4.1	Maximum Likelihood Estimate . . . . .	30
3.4.2	Time-Domain Methods . . . . .	31
3.4.3	Cramér-Rao Lower Bound for a Two-Antenna Interferometer . . . . .	32
3.4.4	Performance Comparison . . . . .	33
3.4.5	RMS Error of an Interferometer . . . . .	36
3.5	Long Baseline Interferometry . . . . .	36
3.6	Ambiguity Resolution Using Independent Methods . . . . .	42
3.6.1	Amplitude Comparison . . . . .	43
3.6.2	Short Baseline . . . . .	43
3.7	Ambiguity Resolution Using Multiple Baselines . . . . .	45
3.7.1	The Chinese Remainder Theorem . . . . .	45
3.7.2	Non-Uniform Array Geometry . . . . .	52
3.7.3	Number of Baselines . . . . .	55
3.7.4	Maximum Likelihood Estimator . . . . .	56
3.7.5	Correlative Interferometers . . . . .	58
3.7.6	Common Angle Search . . . . .	60
3.7.7	Line Fitting . . . . .	65
3.8	Performance Comparison . . . . .	69
3.8.1	Cramér-Rao Lower Bound for a Non-Uniform Linear Array . . . . .	69
3.8.2	Array Geometry . . . . .	70
3.8.3	Monte Carlo Simulations . . . . .	71
3.9	Other Considerations . . . . .	75
3.9.1	Multiple Signals . . . . .	75
3.9.2	Optimal Linear Array Geometries . . . . .	75
3.9.3	Field-of-View . . . . .	76
3.10	Summary . . . . .	80
<b>4</b>	<b>Interferometry Using Second Order Difference Arrays</b>	<b>83</b>
4.1	Introduction . . . . .	83
4.2	SODA Interferometry . . . . .	84
4.2.1	Unambiguous AOA Estimation . . . . .	84
4.2.2	Correction for Ambiguous Phase Delay Measurements . . . . .	85
4.2.3	Algorithm Complexity . . . . .	86
4.2.4	Alternate Expressions for the SODA Baseline Constraint . . . . .	86

4.2.5	RMS Error of the SODA Interferometer . . . . .	87
4.2.6	Performance Evaluation . . . . .	89
4.3	SODA-Cued Ambiguity Resolution . . . . .	91
4.3.1	SODA-Based Inference (SBI) Interferometer . . . . .	92
4.3.2	SODA-Cued Correlative Interferometer . . . . .	93
4.3.3	SBI-Cued Correlative Interferometer . . . . .	94
4.4	Performance Comparison . . . . .	95
4.5	Other Considerations . . . . .	99
4.5.1	Other Linear Combinations of First-Order Phase Delays . . . . .	99
4.5.2	Non-Collinear SODA Interferometer . . . . .	101
4.5.3	Field-of-View . . . . .	103
4.6	Summary . . . . .	103
<b>5</b>	<b>Array Processing Using Second Order Difference Arrays</b>	<b>107</b>
5.1	Beamforming and Array Processing . . . . .	107
5.1.1	Signal Model . . . . .	107
5.1.2	Conventional Phaseshift Beamformer . . . . .	109
5.1.3	Optimal Beamformers and Super-Resolution Methods . . . . .	113
5.1.4	Multiple Signal Classification (MUSIC) . . . . .	113
5.2	Sparse Large Aperture Arrays . . . . .	114
5.2.1	Grid Search Resolution . . . . .	118
5.3	Array Processing with SODA Geometries . . . . .	124
5.3.1	SODA-Cued Array Processing . . . . .	124
5.3.2	SBI-Cued Array Processing . . . . .	126
5.3.3	SODA Array Processing . . . . .	127
5.4	Performance Comparison . . . . .	133
5.4.1	Array Beampatterns . . . . .	133
5.4.2	Grid Search Resolutions . . . . .	134
5.4.3	Monte Carlo Simulations . . . . .	134
5.5	Summary . . . . .	141
<b>6</b>	<b>Calibration</b>	<b>143</b>
6.1	Introduction . . . . .	143
6.2	Effect of Channel Imbalances . . . . .	144
6.2.1	Phase, Frequency and Baseline Errors . . . . .	144
6.2.2	Gain Imbalance . . . . .	146
6.3	Signal Models . . . . .	148
6.3.1	Calibrated Signal Model . . . . .	148
6.3.2	Uncalibrated Signal Model . . . . .	149
6.4	Calibration Methodology . . . . .	150
6.4.1	Calibration Tables . . . . .	150
6.4.2	Simple Calibration . . . . .	151
6.4.3	Joint Calibration and AOA Estimation . . . . .	151
6.5	Short-Baseline Calibration . . . . .	151
6.5.1	Implementation Using a 1-D Look-Up-Table . . . . .	156
6.6	Long-Baseline Calibration . . . . .	156
6.6.1	Ambiguity Resolution Using Multiple Baselines and Uncalibrated Data . . . . .	161

6.7	Summary . . . . .	163
<b>7</b>	<b>The Electronic Support Testbed</b>	<b>165</b>
7.1	Introduction . . . . .	165
7.2	Hardware Design . . . . .	166
7.2.1	Design Objectives . . . . .	166
7.2.2	Sampling Architecture . . . . .	166
7.2.3	Hardware Components . . . . .	169
7.2.4	Data Encoding . . . . .	171
7.3	Data Alignment . . . . .	173
7.3.1	Sources of Data Misalignment . . . . .	173
7.3.2	Data Alignment Methodology . . . . .	174
7.4	Summary . . . . .	176
<b>8</b>	<b>Experimental Results</b>	<b>179</b>
8.1	Introduction . . . . .	179
8.2	Experimental Setup . . . . .	179
8.2.1	Experiment Site . . . . .	179
8.2.2	Transmission Source . . . . .	182
8.2.3	Array Geometry . . . . .	184
8.2.4	Data Collection Methodology . . . . .	184
8.3	Calibration . . . . .	186
8.3.1	Correlative Calibration . . . . .	189
8.3.2	SODA Calibration . . . . .	190
8.4	Experimental Results . . . . .	194
8.4.1	Experimental Performance With Correlative Calibration . . . . .	197
8.4.2	Experimental Performance With SODA Calibration . . . . .	199
8.5	Summary . . . . .	200
<b>9</b>	<b>Concluding Remarks</b>	<b>205</b>
9.1	Summary . . . . .	205
9.2	Future Work . . . . .	207
<b>A</b>	<b>Derivations</b>	<b>209</b>
A.1	Signal Model . . . . .	209
A.2	Maximum Likelihood Estimation . . . . .	210
A.2.1	Maximum Likelihood Estimator for a Non-Uniform Linear Array . . . . .	210
A.2.2	Maximum Likelihood Estimator for Two Antennas . . . . .	212
A.3	Cramér-Rao Lower Bounds . . . . .	213
A.3.1	Cramér-Rao Lower Bounds for a Non-Uniform Linear Array . . . . .	213
A.3.2	Cramér-Rao Lower Bounds for Two Antennas . . . . .	216



# List of Figures

1.1	Block diagram of the typical functions performed by a radar intercept receiver. . . . .	2
1.2	Radar intercept receivers have a range advantage over the radar. . . . .	7
1.3	A typical pulsed radar signal has a well-defined leading and trailing edge. . . . .	9
2.1	A mechanical spinning antenna direction finding system. . . . .	13
2.2	The radar intercept receiver may not “see” an illuminating radar signal if it happens to be “looking away” from the radar. . . . .	14
2.3	Beampattern of two orthogonal loop antennas. The orthogonal sinusoidal beampatterns ensure a unique ratio between the measured power levels of each channel for each AOA. . . . .	15
2.4	Beampattern of an Adcock array comprising of 4 dipole antennas with a radius of $\lambda/5$ , where $\lambda$ is the wavelength of the signal of interest. A linear combination of the omnidirectional beampatterns can produce two orthogonal, near-sinusoidal beampatterns that are comparable to the beampatterns of two loop antennas. . . . .	16
2.5	Beampatterns of four cavity-backed spiral antennas with a squint angle of $90^\circ$ between the antennas. . . . .	17
2.6	A FDOA direction finding array comprising of $K$ antennas. . . . .	18
2.7	A signal incident upon a pair of spatially separated antennas must travel a further distance to reach the second antenna after arriving at the first antenna. . . . .	19
2.8	Array processors use AOA-dependent propagation time or phase delays to coherently sum the array output. . . . .	21
3.1	Relationship between the antenna separation and propagation delay of the signal arrival for a linear array. . . . .	26
3.2	The geographical coordinate system. . . . .	27
3.3	The spherical polar coordinate system. . . . .	27
3.4	Comparison of the RMS errors of an interferometer as a function of SNR. Simulation parameters: $\theta = 23.42^\circ$ , $f = 18$ GHz, $d = 8.3333$ mm, $N = 2048$ samples, and $t_s = 750$ ps. A 2048-point FFT was used to calculate the phase delays of the FFT-based MLE. . . . .	34
3.5	Comparison of the RMS errors of an interferometer as a function of SNR. Simulation parameters: $\theta = 23.42^\circ$ , $f = 18$ GHz, $d = 8.3333$ mm, $N = 2048$ samples, and $t_s = 750$ ps. A 2050-point FFT was used to calculate the phase delays of the FFT-based MLE. . . . .	34

3.6	Comparison of the RMS errors of an interferometer as a function of AOA. Simulation parameters: $\eta = 15$ dB, $f = 18$ GHz, $d = 8.3333$ mm, $N = 2048$ samples, and $t_s = 750$ ps. A 2048-point FFT was used to calculate the phase delays of the FFT-based MLE. . . . .	35
3.7	Unambiguous phase delays as a function of AOA for a short and long baseline interferometer. Simulation parameters: $f = 18$ GHz, $\lambda = 16.67$ mm, $d_{\text{short}} = \lambda/2$ and $d_{\text{long}} = 5\lambda$ . . . . .	38
3.8	Peak error in the AOA estimation for a short and long baseline interferometer due to a peak phase error is $\delta\psi_{\text{peak}} = \pm 5^\circ$ . Simulation parameters: $\theta = 0^\circ$ , $f = 18$ GHz, $\lambda = 16.67$ mm, $d_{\text{short}} = \lambda/2$ and $d_{\text{long}} = 5\lambda$ . . . . .	38
3.9	This plot shows that a short baseline interferometer obtains unambiguous AOA estimates. However, the estimation errors (as indicated by the widths of the triangles) are also larger. In this plot, $\theta = 23.42^\circ$ , $f = 18$ GHz, $d_{\text{short}} = \lambda/2$ and the peak phase error is $\delta\psi_{\text{peak}} = \pm 5^\circ$ . . . . .	39
3.10	This plot shows that a long baseline interferometer has lower estimation errors (as indicated by the widths of the triangles) but are ambiguous. In this plot, $\theta = 23.42^\circ$ , $f = 18$ GHz, $d_{\text{long}} = 5\lambda$ and the peak phase error is $\delta\psi_{\text{peak}} = \pm 5^\circ$ . . . . .	39
3.11	Ambiguous phase delays as a function of AOA for a short and long baseline interferometer. Simulation parameters: $f = 18$ GHz, $\lambda = 16.67$ mm, $d_{\text{short}} = \lambda/2$ and $d_{\text{long}} = 5\lambda$ . The black dots represent the ambiguous AOAs that correspond to an ambiguous phase delay measurement of $\hat{\psi} = -4.56^\circ$ . . . . .	41
3.12	This plot shows the improvement in FOV for a given maximum error tolerance as a function of the aperture. . . . .	42
3.13	Successful ambiguity resolution using an independent method requires that the coarse AOA estimate has a RMS error that satisfies $\delta\theta_{\text{RMS,coarse}} \leq \delta\theta_{\text{amb,min}}/6$ . . . . .	43
3.14	A short-baseline interferometer can be used to successively resolve the ambiguities of the longer baselines. In this figure, the width of the triangles indicate the RMS errors associated with the interferometer baselines. The RMS error improves as the coarse AOA estimation method successively resolves the ambiguities of the longer baselines. . . . .	45
3.15	AOA estimation using the CRT ambiguity resolution algorithm on a noiseless signal produces quantised estimates. Simulation parameters: $d_1 = 3\lambda/2$ , $d_2 = 7\lambda/2$ , $f = 18$ GHz, $\lambda = 16.67$ mm. . . . .	52
3.16	For interferometers with non-uniform antenna spacings, there will only be one AOA that is common among the ambiguities of the long baselines. . . . .	53
3.17	A simple set of interferometer baselines comprising of 4 antennas. . . . .	55
3.18	An extended set of interferometer baselines comprising of 4 antennas. . . . .	55
3.19	The correlative interferometer searches for the set of true phase delays that best match the measured, ambiguous phase delays. . . . .	58
3.20	Example of the cosine and least squares cost functions for a correlative interferometer. These cost functions have been normalised to the same scale for visual comparison. . . . .	60
3.21	Example of the cost function for the exhaustive CAS algorithm. . . . .	63
3.22	Plot of the unambiguous phase delays of the $d_2$ baseline against the $d_1$ baseline. . . . .	64

3.23	Plot of the ambiguous phase delays of the $d_2$ baseline against the $d_1$ baseline.	66
3.24	Look-up-table representation of Figure 3.23. Each entry in the look-up-table represents the corresponding ambiguity number for $\rho_2$ for a given combination of ambiguous phase delay measurements, $\tilde{\psi}_1$ and $\tilde{\psi}_2$ . Note that the row address is counted upwards and the column address is counted rightwards. . . . .	68
3.25	Array geometry for the performance comparison. . . . .	70
3.26	RMS error performance of each algorithm as a function of SNR. Simulation parameters: $K = 3$ antennas, $\theta = 23.42^\circ$ , $f = 9410$ MHz, $\varphi = 0^\circ$ , $N = 2048$ samples, $t_s = 750$ ps and $\mathcal{Q} = 10,000$ realisations. The “Opt.” label indicates that Newton’s Method optimisation has been performed.	73
3.27	RMS error performance of each algorithm as a function of SNR. Simulation parameters: $K = 3$ antennas, $\theta = 23.42^\circ$ , $f = 9410$ MHz, $\varphi = 0^\circ$ , $N = 2048$ samples, $t_s = 750$ ps and $\mathcal{Q} = 10,000$ realisations. . . . .	73
3.28	RMS error performance of each algorithm as a function of SNR. Simulation parameters: $K = 4$ antennas, $\theta = 23.42^\circ$ , $f = 9410$ MHz, $\varphi = 0^\circ$ , $N = 2048$ samples, $t_s = 750$ ps and $\mathcal{Q} = 10,000$ realisations. The “Opt.” label indicates that Newton’s Method optimisation has been performed.	74
3.29	RMS error performance of each algorithm as a function of SNR. Simulation parameters: $K = 4$ antennas, $\theta = 23.42^\circ$ , $f = 9410$ MHz, $\varphi = 0^\circ$ , $N = 2048$ samples, $t_s = 750$ ps and $\mathcal{Q} = 10,000$ realisations. . . . .	74
3.30	Comparison of the FOV of an interferometer as a function of SNR at various error tolerances. Simulation parameters: $f = 18$ GHz, $d = \lambda/2$ , $\lambda = 16.67$ mm, $N = 2048$ samples, and $t_s = 750$ ps. . . . .	78
3.31	Comparison of the FOV of an interferometer as a function of frequency at various error tolerances. Simulation parameters: $\eta = 15$ dB, $d = \lambda/2$ , $\lambda = 16.67$ mm, $N = 2048$ samples, and $t_s = 750$ ps. . . . .	78
3.32	Comparison of the FOV of an interferometer as a function of the array aperture with various error tolerances. Simulation parameters: $\eta = 15$ dB, $f = 18$ GHz, $N = 2048$ samples, and $t_s = 750$ ps. . . . .	79
3.33	Linear arrays are unable to distinguish between signals arriving from the “front” or “back” hemispheres due to the geometric symmetry. . . . .	79
3.34	Multiple independent linear arrays are required to obtain a $360^\circ$ field-of-view. . . . .	80
3.35	A circular array geometry. . . . .	81
4.1	A collinear array with three antennas. . . . .	84
4.2	Array geometry for a SODA interferometer. . . . .	85
4.3	A SODA interferometer effectively creates a virtual short-baseline interferometer from a sparse antenna array. . . . .	87
4.4	Comparison of the AOA estimation performance of the SODA interferometer and the equivalent first-order interferometer as a function of AOA. Simulation parameters: $\eta = 15$ dB, $f = 18$ GHz, $\lambda = 16.67$ mm, $d_{21} = 3\lambda/2$ , $d_{32} = 7\lambda/2$ , $N = 2048$ samples, $t_s = 750$ ps, and $\mathcal{Q} = 10,000$ realisations. . . . .	90

4.5	Comparison of the AOA estimation performance of the SODA interferometer and the equivalent first-order interferometer as a function of frequency. Simulation parameters: $\eta = 15$ dB, $\theta = 70^\circ$ , $\lambda = 16.67$ mm, $d_{21} = 3\lambda/2$ , $d_{32} = 7\lambda/2$ , $N = 2048$ samples, $t_s = 750$ ps, and $\mathcal{Q} = 10,000$ realisations. . . . .	90
4.6	The angular accuracy of a SODA interferometer is independent of the physical first-order baselines. . . . .	91
4.7	The unambiguous second-order phase delay can be used to successively resolve the ambiguities of the longer first-order baselines. . . . .	92
4.8	The SODA AOA estimate can be used to reduce the search range of the correlative interferometer. . . . .	94
4.9	RMS error performance of each algorithm as a function of SNR. Simulation parameters: $K = 3$ antennas, $\theta = 23.42^\circ$ , $f = 9410$ MHz, $\varphi = 0^\circ$ , $N = 2048$ samples, $t_s = 750$ ps and $\mathcal{Q} = 10,000$ realisations. . . . .	97
4.10	RMS error performance of each algorithm as a function of SNR. Simulation parameters: $K = 3$ antennas, $\theta = 23.42^\circ$ , $f = 9410$ MHz, $\varphi = 0^\circ$ , $N = 2048$ samples, $t_s = 750$ ps and $\mathcal{Q} = 10,000$ realisations. . . . .	97
4.11	RMS error performance of each algorithm as a function of SNR. Simulation parameters: $K = 4$ antennas, $\theta = 23.42^\circ$ , $f = 9410$ MHz, $\varphi = 0^\circ$ , $N = 2048$ samples, $t_s = 750$ ps and $\mathcal{Q} = 10,000$ realisations. . . . .	98
4.12	RMS error performance of each algorithm as a function of SNR. Simulation parameters: $K = 4$ antennas, $\theta = 23.42^\circ$ , $f = 9410$ MHz, $\varphi = 0^\circ$ , $N = 2048$ samples, $t_s = 750$ ps and $\mathcal{Q} = 10,000$ realisations. . . . .	98
4.13	A 3-antenna non-linear array can be considered as a triangular array. . .	102
4.14	$d_{21}$ baseline rotation angle, $\alpha$ , vs the array aperture for $d_\Delta = \lambda/2$ . . . .	104
4.15	Virtual array rotation angle, $\Theta$ , vs the $d_{21}$ rotation angle, $\alpha$ , for $d_{31} = 50\lambda$ and $d_\Delta = \lambda/2$ . . . . .	104
5.1	Array processing algorithms exploit the propagation delays in a coherent manner. . . . .	108
5.2	Beampattern of an 8-antenna uniform linear array with a $\lambda/2$ antenna spacing and steered at $\theta_s = 0^\circ$ . . . . .	110
5.3	Array output of the CBF algorithm using an 8-antenna uniform linear array with a $\lambda/2$ antenna spacing when $\theta = 23.42^\circ$ . Simulation parameters: $\eta = 15$ dB, $f = 16$ GHz, $N = 2048$ samples, and $\Delta\theta = 0.01^\circ$ . . . . .	110
5.4	Array output of a MUSIC array processor using an 8-antenna uniform linear array with a $\lambda/2$ antenna spacing when $\theta = 23.42^\circ$ . Simulation parameters: $\eta = 15$ dB, $f = 16$ GHz, $N = 2048$ samples, $W = 1$ snapshot, and $\Delta\theta = 0.01^\circ$ . . . . .	115
5.5	Beampattern of an 8-antenna uniform linear array with a uniform antenna spacing of $7.1429\lambda$ ( $50\lambda$ aperture). . . . .	116
5.6	CBF array output using an 8-antenna uniform linear array with a uniform antenna spacing of $7.1429\lambda$ ( $50\lambda$ aperture) when $\theta = 23.42^\circ$ . Simulation parameters: $\eta = 15$ dB, $f = 16$ GHz, $N = 2048$ samples, $W = 1$ snapshot, and $\Delta\theta = 0.01^\circ$ . . . . .	117

5.7	MUSIC array output using an 8-antenna uniform linear array with a uniform antenna spacing of $7.1429\lambda$ ( $50\lambda$ aperture) when $\theta = 23.42^\circ$ . Simulation parameters: $\eta = 15$ dB, $f = 16$ GHz, $N = 2048$ samples, $W = 1$ snapshot, and $\Delta\theta = 0.01^\circ$ . . . . .	117
5.8	An 8-antenna non-uniform linear array. . . . .	118
5.9	Beampattern of a 8-antenna non-uniform linear array with a $50\lambda$ aperture at 16 GHz. . . . .	119
5.10	CBF array output using an 8-antenna non-uniform linear array with a $50\lambda$ aperture when $\theta = 23.42^\circ$ . Simulation parameters: $\eta = 15$ dB, $f = 16$ GHz, $N = 2048$ samples, $W = 1$ snapshot, and $\Delta\theta = 0.01^\circ$ . . . . .	120
5.11	MUSIC array output using an 8-antenna non-uniform linear array with a $50\lambda$ aperture when $\theta = 23.42^\circ$ . Simulation parameters: $\eta = 15$ dB, $f = 16$ GHz, $N = 2048$ samples, $W = 1$ snapshot, and $\Delta\theta = 0.01^\circ$ . . . . .	120
5.12	CBF array output using an 8-antenna non-uniform linear array with a $50\lambda$ aperture when $\theta = 23.42^\circ$ . Simulation parameters: $\eta = 15$ dB, $f = 16$ GHz, $N = 2048$ samples, $W = 1$ snapshot, and $\Delta\theta = 1.146^\circ$ . . . . .	122
5.13	MUSIC array output using an 8-antenna non-uniform linear array with a $50\lambda$ aperture when $\theta = 23.42^\circ$ . Simulation parameters: $\eta = 15$ dB, $f = 16$ GHz, $N = 2048$ samples, $W = 1$ snapshot, and $\Delta\theta = 1.146^\circ$ . . . . .	122
5.14	CBF array output using an 8-antenna non-uniform linear array with a $50\lambda$ aperture when $\theta = 23.42^\circ$ . Simulation parameters: $\eta = 15$ dB, $f = 16$ GHz, $N = 2048$ samples, $W = 1$ snapshot, and $\Delta\theta = 0.573^\circ$ . . . . .	123
5.15	MUSIC array output using an 8-antenna non-uniform linear array with a $50\lambda$ aperture when $\theta = 23.42^\circ$ . Simulation parameters: $\eta = 15$ dB, $f = 16$ GHz, $N = 2048$ samples, $W = 1$ snapshot, and $\Delta\theta = 0.573^\circ$ . . . . .	123
5.16	The SODA AOA estimate can be used to reduce the search range of the conventional beamformer. . . . .	125
5.17	The second-order differences between the physical antenna positions of a sparse large aperture array can be used to synthesise the baselines of an unambiguous virtual uniform linear array. . . . .	128
5.18	The antenna positions of a virtual uniform linear array formed from the second-order differences of the physical antenna positions. . . . .	128
5.19	Beampattern of a 7-antenna virtual uniform linear array derived from an 8-antenna physical SODA geometry with a $50\lambda$ aperture. . . . .	130
5.20	Comparison of the first-order and second-order array outputs for a 8-antenna SODA geometry using the CBF algorithm. Simulation parameters: $\theta = 23.42^\circ$ , $\eta = 15$ dB, $f = 16$ GHz, $N = 2048$ samples, $W = 1$ snapshot, $\Delta\theta_{\text{first-order}} = 0.573^\circ$ and $\Delta\theta_{\text{second-order}} = 9.736^\circ$ . . . . .	132
5.21	Comparison of the first-order and second-order array outputs for a 8-antenna SODA geometry using the MUSIC algorithm. Simulation parameters: $\theta = 23.42^\circ$ , $\eta = 15$ dB, $f = 16$ GHz, $N = 2048$ samples, $W = 1$ snapshot, $\Delta\theta_{\text{first-order}} = 0.573^\circ$ and $\Delta\theta_{\text{second-order}} = 9.736^\circ$ . . . . .	132
5.22	Array beampatterns for the physical and virtual arrays using the 3-antenna array geometry at $f = 9410$ MHz. . . . .	135
5.23	Array beampatterns for the physical and virtual arrays using the 4-antenna array geometry at $f = 9410$ MHz. . . . .	135
5.24	Array beampatterns for the physical and virtual arrays using the 8-antenna array geometry at $f = 9410$ MHz. . . . .	136

5.25	RMS error performance of each algorithm as a function of SNR. Simulation parameters: $K = 3$ antennas, $\theta = 23.42^\circ$ , $f = 9410$ MHz, $\varphi = 0^\circ$ , $N = 2048$ samples, $W = 1$ snapshot, $t_s = 750$ ps and $Q = 10,000$ realisations. . . . .	138
5.26	RMS error performance of each algorithm as a function of SNR. Simulation parameters: $K = 3$ antennas, $\theta = 23.42^\circ$ , $f = 9410$ MHz, $\varphi = 0^\circ$ , $N = 2048$ samples, $W = 1$ snapshot, $t_s = 750$ ps and $Q = 10,000$ realisations. . . . .	138
5.27	RMS error performance of each algorithm as a function of SNR. Simulation parameters: $K = 4$ antennas, $\theta = 23.42^\circ$ , $f = 9410$ MHz, $\varphi = 0^\circ$ , $N = 2048$ samples, $W = 1$ snapshot, $t_s = 750$ ps and $Q = 10,000$ realisations. . . . .	139
5.28	RMS error performance of each algorithm as a function of SNR. Simulation parameters: $K = 4$ antennas, $\theta = 23.42^\circ$ , $f = 9410$ MHz, $\varphi = 0^\circ$ , $N = 2048$ samples, $W = 1$ snapshot, $t_s = 750$ ps and $Q = 10,000$ realisations. . . . .	139
5.29	RMS error performance of each algorithm as a function of SNR. Simulation parameters: $K = 8$ antennas, $\theta = 23.42^\circ$ , $f = 9410$ MHz, $\varphi = 0^\circ$ , $N = 2048$ samples, $W = 1$ snapshot, $t_s = 750$ ps and $Q = 10,000$ realisations. . . . .	140
6.1	AOA bias error due to a $5^\circ$ bias error in the phase delay estimate. The signal frequency is assumed to be $f = 18$ GHz and the antenna separation is $d = \lambda/2$ . . . . .	147
6.2	AOA bias error due to a 1 MHz bias error in the frequency estimate. The signal frequency is assumed to be $f = 18$ GHz and the antenna separation is $d = \lambda/2$ . . . . .	147
6.3	AOA bias error due to a 1 mm bias error in the interferometer baseline. The signal frequency is assumed to be $f = 18$ GHz and the antenna separation is $d = \lambda/2$ . . . . .	147
6.4	A simple calibration method. The signals are calibrated prior to AOA estimation. . . . .	151
6.5	The AOA estimation algorithms can be modified to allow AOA estimation directly from the uncalibrated data. . . . .	152
6.6	Example of a constant phase delay error of $50^\circ$ . . . . .	154
6.7	The relationship between the uncalibrated phase delays and the AOA remains unique when there is a constant phase delay error. . . . .	154
6.8	Example of a monotonically decreasing phase delay error arising from a shorter than expected interferometer baseline. . . . .	155
6.9	The relationship between the uncalibrated phase delays and the AOA remains monotonic and unique and so unambiguous AOA estimation can be performed. . . . .	155
6.10	Example of a monotonically increasing phase delay error arising from a longer than expected interferometer baseline. . . . .	157
6.11	The relationship between the uncalibrated phase delays and the AOA remains monotonic but is not unique, and so unambiguous AOA estimation cannot be performed at all angles. . . . .	157
6.12	Example of a non-monotonic phase delay error. . . . .	158

6.13	Relationship between the uncalibrated phase delays and the AOA is ambiguous if the phase delay error is non-monotonic. . . . .	158
6.14	A look-up-table can be used to map the uncalibrated phase delays to the AOA. . . . .	159
7.1	Simplified block diagram of a bandpass sampling architecture. . . . .	167
7.2	An appropriately selected sampling rate can shift a signal centred at $f_c$ to $f_s/4$ without an explicit frequency shift operation. . . . .	168
7.3	Hardware architecture for the multi-channel ES Testbed. . . . .	170
7.4	Typical data stream of one channel from the ES Testbed. . . . .	172
7.5	Encoding of the ADC data. . . . .	172
7.6	Encoding of the TOB data. . . . .	172
8.1	The Gemini Trial was conducted at St Kilda, South Australia (Marker A) in July 2011. . . . .	180
8.2	Location of the transmitting and receiving sites at St Kilda. . . . .	181
8.3	Instantaneous frequency of a linear FM chirp signal with a chirp rate of 510 MHz per 2.5 ms. . . . .	183
8.4	The average frequency of each observation period is plotted against the instantaneous frequency of the chirp. . . . .	183
8.5	The frequency error in the approximation of the centre frequencies of each observation period by the average instantaneous frequencies. . . . .	185
8.6	AOA bias error due to a 0.3133 MHz frequency error introduced by approximating the slow-changing linear FM chirp signal as a sequence of narrowband, single-tone signals. . . . .	185
8.7	Array geometry for the Gemini Trial. . . . .	186
8.8	Uncalibrated, unambiguous phase delays. . . . .	187
8.9	Calibration values as a function of azimuth. . . . .	188
8.10	Calibration values as a function of the ambiguous, uncalibrated phase delays. . . . .	189
8.11	Plot of the uncalibrated, ambiguous phase delays as a function of AOA for each baseline. . . . .	190
8.12	Calibrated phase delays using correlative interferometry. . . . .	191
8.13	Residual phase delay offsets after calibration using correlative interferometry. . . . .	192
8.14	Look-up-table for SODA AOA estimation using the uncalibrated SODA phase delays. . . . .	193
8.15	Calibrated phase delays using SODA interferometry. . . . .	195
8.16	Residual phase delay offsets after calibration using SODA interferometry. . . . .	196
8.17	RMS errors of the AOA estimation with correlative calibration using the 3-antenna array geometry. The angular values in the labels represent the total RMS error for the algorithms. . . . .	202
8.18	RMS errors of the AOA estimation with correlative calibration using the 4-antenna array geometry. The angular values in the labels represent the total RMS error for the algorithms. . . . .	202
8.19	RMS errors of the AOA estimation with SODA calibration using the 3-antenna array geometry. The angular values in the labels represent the total RMS error for the algorithms. . . . .	203

8.20 RMS errors of the AOA estimation with SODA calibration using the 4-  
antenna array geometry. The angular values in the labels represent the  
total RMS error for the algorithms. . . . . 203



# List of Tables

3.1	Possible candidates for $b_1$ . . . . .	47
3.2	Possible candidates for $b_2$ . . . . .	47
3.3	Possible combinations of $\rho_1$ and $\rho_2$ . . . . .	63
3.4	Relative execution times for the conventional ambiguity resolution algorithms. The “Opt.” label indicates that Newton’s Method optimisation has been performed. . . . .	75
4.1	Relative execution times for the SODA-based algorithms. . . . .	99
5.1	Antenna positions for an 8-antenna SODA geometry with $d_\Delta = \lambda/2$ , $u_1 = 0$ and $u_2 = 3.1429\lambda$ . . . . .	128
5.2	Relative execution time factor for the array processing and interferometric algorithms. . . . .	141
7.1	Data masks used in the encoding of the ES Testbed data. . . . .	172
7.2	Parameters for the control signal used for data alignment. . . . .	175
8.1	Experimental performance of the AOA estimation algorithms using correlative calibration at 5 dB SNR. †The fourth auxiliary antenna is unused by this algorithm. . . . .	199
8.2	Experimental performance of the AOA estimation algorithms using SODA calibration at 5 dB SNR. †The fourth auxiliary antenna is unused by this algorithm. . . . .	200



# Abstract

This thesis considers the problem of angle-of-arrival (AOA) estimation in the context of its application to electronic surveillance systems. Due to the operational requirements of such systems, the AOA estimation algorithm must be computationally fast, accurate and will need to be implemented using sparse, large aperture arrays.

Interferometry is proposed as a suitable algorithm that meets the operational requirements of electronic surveillance systems. However, for sparse array geometries, phase wrapping effects introduce ambiguities to the phase measurements and so unambiguous AOA estimation requires the use of computationally intensive ambiguity resolution algorithms using three or more antennas.

Beamforming and array processing techniques are another class of AOA estimation algorithms that can unambiguously estimate the AOA using sparse, large aperture arrays. While these techniques generally offer better AOA estimation performance than interferometric techniques, they are also comparatively more computationally intensive algorithms. Furthermore, by virtue of using very sparse arrays, high sidelobes in the array beampattern may cause incorrect AOA estimation.

This thesis will introduce the concept of using second-order difference array (SODA) geometries which allow unambiguous AOA estimation to be performed in a computationally efficient manner. In the context of interferometry, the so-called “SODA interferometer” will be shown to synthesise the equivalent output of a smaller virtual aperture to allow unambiguous AOA estimation to be performed at the expense of a coarser estimation performance compared to the physical aperture of the array. It will also be shown that the coarse SODA AOA estimate can be used to cue the conventional ambiguity resolution algorithms to provide higher accuracy in a computationally efficient manner. This thesis will also show that the creation of virtual arrays from SODA geometries can be generalised to a larger number of antennas to allow conventional array processing techniques to perform unambiguous AOA estimation in a computationally fast manner.

The AOA estimation performance of each algorithm is compared through simulations and also verified using experimental data. This thesis will show that the SODA interferometer, SODA-cued ambiguity resolution algorithms and so-called “second-order array processors” can be used to obtain high accuracy AOA estimates in a more computationally efficient manner than the conventional algorithms.



# Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968.

I also give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library Search and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

.....  
Peter Quoc Cuong Ly  
3 December 2013



# Acknowledgements

I would like to thank my university supervisor, Professor Doug Gray, for encouraging me to enter the field of signal processing. It was under your mentorship during my final years as a undergraduate student that sparked my interest in a career in signal processing research. Thank you for all of your time, effort and patience over the last ten years. Without your support and encouragement, the completion of this thesis would not have been possible.

I would also like to thank my DSTO supervisor, Dr Stephen Elton, for bringing this PhD to fruition. This thesis would not have been possible without your tireless efforts in negotiating the agreement between the university and DSTO. Your friendship and unwavering support both academically and personally during this candidacy has been an invaluable source of encouragement.

I would also like to thank my co-supervisor, Professor Bevan Bates, for helping me navigate through the necessary administration at the university and DSTO and also for allowing me to join in on a number of experiments to collect experimental data to support this PhD research.

I am grateful for the support of my DSTO superiors, Dr Len Sciacca, Dr Jackie Craig, Dr Warren Marwood, Dr Peter Gerhardy and Brian Reid, for providing me with the opportunity to undertake this PhD research as part of my DSTO working arrangements. I am also grateful for the support from my DSTO colleagues, including but not limited to Simon Herfurth, John Quin, Phil Wandel, Jarrad Shiosaki, Andrew Evans and Songsri Sirianunpiboon for their assistance with the hardware development and experimental trials.

I would like to express my gratitude to my family for their unconditional love and support. I would like to thank my parents for instilling in me the appreciation of academic excellence and the value of persistence. I would also like to thank my sister for grounding me in reality.

Finally, I thank my wife, Joy, for being my rock during the last seven years. Not only do you deserve credit for helping me academically, but your unwavering support, patience and faith in me helped me get through some of my darkest times. You are my light.





# Publications

Some parts of this thesis have been published for presentation at conferences. The following list is the bibliographic information pertaining to these preliminary presentations.

- Ly, P. Q. C., Elton, S. D., Li, J. & Gray, D. A., “Computationally Fast AOA Estimation Using Sparse Large Aperture Arrays for Electronic Surveillance,” in *Proceedings of the IEEE International Conference on Radar (RADAR 2013)*, 2013. Accepted.
- Ly, P. Q. C., Elton, S. D., Gray, D. A., & Li, J., “Unambiguous AOA Estimation Using SODA Interferometry for Electronic Surveillance,” in *Proceedings of the IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM 2012)*, pp. 277–280, 2012.
- Ly, P. Q. C., Gray, D. A., Elton, S. D., & Bates, B. D., “A Digital Interferometric Testbed for ES/ELINT Research,” in *Proceedings of the Seventh Direction Finding and Geolocation Symposium*, 2008.



# Acronyms

Acronym	Description
ADC	Analogue to Digital Converter
AOA	Angle of Arrival
CAS	Common Angle Search
CBF	Conventional Phaseshift Beamforming
COS	Correlative Interferometry using the Cosine Cost Function
CRLB	Cramér-Rao Lower Bound
CW	Continuous Wave
DOA	Direction of Arrival
DF	Direction Finding
DFT	Discrete Fourier Transform
DSTO	Defence Science and Technology Organisation
ELINT	Electronic Intelligence
ES	Electronic Support
ESM	Electronic Support Measures
EW	Electronic Warfare
FFT	Fast Fourier Transform
FMCW	Frequency Modulated Continuous Wave
FOV	Field of View
FPGA	Field Programmable Gate Array
GSPS	Giga-Samples Per Second, $1 \times 10^9$ Samples Per Second
LPI	Low Probability of Intercept
LS	Correlative Interferometry using the Least Squares Cost Function
MSPS	Mega-Samples Per Second, $1 \times 10^6$ Samples Per Second
MLE	Maximum Likelihood Estimator
MSE	Mean Square Error
MUSIC	Multiple Signal Classification
POI	Probability of Intercept
PRI	Pulse Repetition Interval
RMS	Root Mean Square
RMSE	Root Mean Square Error
RWR	Radar Warning Receiver
SBI	SODA-Based Inference
SNR	Signal to Noise Ratio
SODA	Second Order Difference Array
TDC	Time to Digital Converter
TDOA	Time Difference of Arrival
TOA	Time of Arrival
TOB	Time of Burst



# Notation

## Symbols

The following symbols will be used throughout this thesis:

Symbol	Description
$\eta$	Signal to Noise Ratio
$A$	Peak Amplitude of the Signal
$f$	Carrier Frequency of the Signal
$\lambda$	Wavelength of the Signal's Carrier Frequency
$\varphi$	Initial Phase of the Signal
$\theta$	Azimuth Component of the Signal's Angle of Arrival
$\phi$	Elevation Component of the Signal's Angle of Arrival
$t_s$	Sample Interval
$f_s$	Sampling Rate (or Sampling Frequency)
$\tau$	Propagation Time Delay Between Two Antennas
$\psi$	Propagation Phase Delay Between Two Antennas
$d$	Distance Between Two Antennas (or Interferometer Baseline)
$t$	Time Instance
$s(t)$	Continuous Time-Varying Signal Without Noise
$x(t)$	Continuous Time-Varying Signal With Noise
$\epsilon(t)$	Continuous Time Additive Gaussian Noise
$s[n]$	Discrete Time-Varying Signal Without Noise
$x[n]$	Discrete Time-Varying Signal With Noise
$\epsilon[n]$	Discrete Time Additive Gaussian Noise
$\beta(\theta)$	Phase Error due to Channel Imbalance
$N$	Number of Samples
$K$	Number of Antennas
$M$	Number of Interferometer Baselines
$D$	Set of Interferometer Baselines
$\Upsilon$	Set of Potential Ambiguity Numbers for a Single Baseline
$\Omega$	Set of Ambiguity Number Combinations from all Baselines
$\sigma$	Standard Deviation of Noise
$v(\theta)$	Propagation Delay for a Single Baseline
$\mathbf{v}(\theta)$	Propagation Delay Vector for all Baselines
$\Delta\theta$	Azimuth Grid Search Resolution
$BW_{NN}$	Null-to-Null Beamwidth

## Scripts and Accents

Scripts and accents will be used to confer additional meaning to the above symbols.

- A single letter subscript specifies that the associated parameter belongs to a specific hardware channel. For example,  $A_k$  and  $f_k$  refers to the peak amplitude and carrier frequency of the  $k$ -th receiver channel. A single letter subscript can also specify that the parameter belongs to a particular interferometric baseline. For example,  $\psi_m$  and  $d_m$  refers to the phase delay and baseline of the  $m$ -th interferometer baseline. When only a single letter subscript is used, it is generally implied that the specified parameter refers to an arbitrary interferometer baseline.
- A double letter subscript specifies the parameter of a particular channel with respect to another channel. For example,  $\psi_{kl}$  and  $d_{kl}$  refers to the phase delay and interferometer baseline of the  $k$ -th antenna relative to the  $l$ -th antenna. When a double letter subscript is used, it is generally implied that the specified parameter refers to a specific interferometer baseline.
- The subscript  $_s$  specifies a steered parameter that is under the control of the radar intercept receiver. For example,  $\theta_s$  refers to the steered AOA of a grid search algorithm for AOA estimation.
- The superscript  $^u$  specifies an uncalibrated parameter that is subject to channel imbalances.
- The superscript  $^c$  specifies a calibrated parameter free of channel imbalances.
- The tilde accent  $\sim$  specifies a measured parameter. In particular, when specifying the phase delay measurement,  $\psi$  refers to the unwrapped, unambiguous phase delay, however,  $\tilde{\psi}$  refers to the measured, ambiguous phase delay that is constrained to the interval  $[-\pi, \pi]$ .
- The hat accent  $\hat{\phantom{a}}$  specifies an estimated parameter.
- Plain typeface symbols, e.g.  $v(\theta)$ , are used to denote scalar variables.
- Bold typeface symbols in lower case characters, e.g.  $\mathbf{v}(\theta)$ , are used to denote vector variables.
- Bold typeface symbols in upper case characters, e.g.  $\mathbf{R}$ , are used to denote matrix variables.

## Mathematical Operators

Operator	Description
$\text{round}[a]$	Rounds the scalar $a$ to the nearest integer.
$\lceil a \rceil$	Rounds the scalar $a$ upwards to the next integer.
$\lfloor a \rfloor$	Rounds the scalar $a$ downwards to the next integer.
$\odot$	Cartesian product.
$\mathbf{a}^T$ or $\mathbf{A}^T$	Transpose of the vector, $\mathbf{a}$ , or matrix, $\mathbf{A}$ .
$\mathbf{a}^H$ or $\mathbf{A}^H$	Hermitian (complex conjugate) transpose of the vector, $\mathbf{a}$ , or matrix, $\mathbf{A}$ .

## Units

All parameters are assumed to adhere to the International System of Units (or SI units).

The units of phase-related values, such as the phase delay and angle-of-arrival, are assumed to be expressed in radians for all mathematical expressions. However, for readability, the phase-related values will often be expressed in degrees in the text and figures.