Essays on Labor Market and Volatility Changes

By

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THESIS

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With all my heartfelt and deepest appreciation to all.

Abstract

This thesis investigates both the time-varing volatility in the labour market after World War II in the United States and the effect of volatility changes of labour productivity on the movements in labour market in a framework of frictional labour search model.

The first chapter documents the volatility changes in the U.S. labour market from 1951Q1 to 2007Q1. The time-varying volatility of unemployment, vacancy, job finding rate, job separation rate, and other key variables are presented in a series of stochastic volatility models estimated following a Bayesian approach. It is shown that the volatility of the U.S. labour market experienced a notable moderation after the mid-1980s. The estimated stochastic volatility of labour productivity is used as the driving process when studying an extended version of the model in Chapter 2.

Following the findings in Chapter 1, the effect of volatility shocks in labour productivity in the U.S. labour market is investigated in two benchmark models in the second chapter. We first consider a standard labour search model following the calibration approach introduced in Hagedorn and Manovskii (2008). After that, a risk-sharing labour search model with full-commitment contracts following Rudanko (2009) is also introduced for the analysis. It is found that, in both settings, a mean-preserving volatility shock in labour productivity has no effect on the optimal decision made by agents. A volatility change can only introduce a non neutral effect with the fact that it changes the range of the corresponding levels of productivity that the specific volatility has induced. Furthermore, while both of the two models are able to capture the volatility moderation reasonably well, the simulation of both models still suffers from a small magnitude of volatility in unemployment and other variables.

In the third chapter, the discussion on the effect of volatility changes is furthered with a more analytical perspective. In this chapter, we investigate the effect

of uncertainty shocks on job creation in a simplified one-period economy. In particular, two different scenarios are considered in this chapter: a situation where firms hire multiple workers with a production technology that exhibits decreasing return to scales, and another situation where risk-averse workers and risk-neutral firms signing risk-sharing contracts, a setting similar to Rudanko (2009). It is shown that if the firm's profit function is non-linear in labour productivity, then changes in expected volatility affects the expected value of a filled job vacancy and thus causes firms to create more job vacancies. Models that simply add concavity in the production function via diminishing returns to labour inputs do not work as the profit function is still linear in labour productivity. Instead, a model with sticky wages such as that of Michaillat (2012) is sufficient to introduce non neutral effect of volatility changes.

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Chapter 1

The Great Moderation and the U.S. Labour Market

1.1 Introduction

Volatility changes are of great importance in understanding of the economy. In macroeconomics, the "Great Moderation" in the U.S. economy since the mid-1980's has been well documented by the work of Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Blanchard and Simon (2001) and Stock and Watson (2002). Aside from the empirical findings consolidating the observation in smoothing volatility changes post-1980s, many works motivated by the time-varying volatility tried to explain the underlying factors that drives the moderation (McConnell and Perez-Quiros, 2000; Dynan et al., 2006).

More recently, as macroeconomists' interest in aggregate volatility changes keeps growing (Bloom, 2009; Fernández-Villaverde and Rubio-Ramirez, 2010; Fernández-Villaverde et al., 2011; Bloom et al., 2012), there are still few papers trying to investigate the business cycle properties of time series in U.S. labour market and the effects of volatility changes on the dynamics of the labour market. Among the existing works on related topics, Galí and Gambetti (2009) investigated the volatility changes of labour market time series with a focus on working hours and labour productivity in an SVAR framework. Faberman (2008) discussed the moderation in job flows in recent recessions, and the relation between jobless recoveries and the Great Moderation. Stiroh (2009) did a "volatility accounting" of output into capital, labour and TFP but with no focus on the dynamic changes in labour market.

However, as the Diamond-Mortenscn-Pissarides (DMP, Diamond 1982; Mortensen 1982; Pissarides 1985) model of labour market search is currently the workhorse model of studying the labour market [1], in order to fully understand the observed fluctuations in the U.S. labour market and sources behind its time-varying volatility, it is important that we investigate the effect of volatility changes on the labour market in it.

In the baseline DMP model, labour productivity is taken as an exogenous variable. Fluctuations in labour productivity affects the revenues generated by employing workers from the firms' perspective. These fluctuations in the value of employing workers then determines the number of job vacancies created by firms. Much recent work has focused on understanding how fluctuations in exogenous labour productivity can generate realistic fluctuations in equilibrium unemployment and job creation in the DMP model (Shimer, 2005; Hall, 2005; Hagedorn and Manovskii, 2008). However, little work has tried to understand how well the model captures the reduction of volatility in unemployment and job creation relative to that observed in the U.S. economy. This thesis takes a first step in filling this gap in the literature.

As the DMP model takes fluctuations in labour productivity as exogenously determined, this chapter estimates a stochastic volatility model of labour produc-

[1] See Pissarides textbook "Equilibrium Unemployment Theory" (Pissarides, 2001) for a good overview of the DMP model.

tivity that is used as the driving process when studying an extended version of the DMP model in Chapter 2. Additionally, this chapter documents how some simple statistics capturing volatility in the unemployment rate and job creation rates changed before and after 1984. These simple statistics are used as reference points in evaluating the extended DMP model of Chapter 2.

Specifically, we estimate the volatility changes in time series of the labour market by using a stochastic volatility model with a Bayesian approach. With a comparison to the well established "Great Moderation" observation in the aggregate economy, it is shown that the volatility of those key variables has experienced a notable moderation after the mid-1980s as well, and the time-varying volatility is an important feature in time series data of the U.S. labour market.

The rest part of the chapter is organized as follows: Section 1.2 discusses the data we are going to investigate, Section 1.3 estimates stochastic volatility of U.S. labour maker time series, a discussion on the estimated results follows in 1.4, and Section 1.5 concludes.

1.2 Data

This section introduces the main time series that we are going to investigate in this chapter with regard to the volatility changes in the U.S. labour market after World War II. Specifically, the main times series of U.S. labour market between from 1951Q1 and 2007Q1 are examined. We ignore the recent recession in our analysis so that data after 2007Q1 are not included in the scope of our discussion in this chapter for that the vigorous changes in pattern of the data during the latest crisis might complicate our analysis. In addition, these time series are sectioned into to two parts, i.e., those prior to and post 1984 (1951Q1-1983Q4 ;1984Q1-2007Q1), based on McConnell and Perez-Quiros (2000)'s dating of the Great Moderation.

To capture main characteristics of the labour market dynamics and make it consistent with the literature based on the prevailing DMP model of labour market search, we include a relatively wide variety of time series, which are respectively the average labour productivity, the unemployment rate, the job finding rate, the job separation rate, and market tightness in the U.S. labour market. Besides, in order to check the comovements of the stochastic elements of the data with respect to the aggregate economy, we include real GDP in our estimation as well.

The quarterly real GDP data is issued by the Bureau of Economic Analysis (BEA), while quarterly labour productivity is constructed by the Bureau of labour Statistics (BLS). Unemployment data are available from the Current Population Survey (CPS) by BLS. Robert Shimer published his calculation of quarterly series of the job finding rate and the job separation rate based on the monthly raw data from the CPS (Shimer, 2005, 2012)[2], with a correction for the adjustment in the survey introduced in 1994 by the CPS. We use Shimer's calculation of data for both the job finding and separation rates[3].

As for the job vacancy series, we use a composite Help-Wanted Advertisement Index for the estimation. The original Help-Wanted Index (HWI) was issued by the Conference Board since 1951, which measured the Help-wanted advertisements printed on 51 major newspapers all over the U.S. each month. In 2005, a new version of the index, i.e., the Help-Wanted Index Online, was published to ensure a broader coverage of job openings given the widely use of the Internet for advertisements for job vacancies since the early 1990s. To assure the completeness of our data exploration, a composite HWI after December 2000 is constructed following the method introduced by Barnichon (2010). We then construct a series of labour market tightness with both the composite HWI data and the unemployment data

[2]Data are available for download from: `https://sites.google.com/site/robertshimer/research/flows`.

[3]More detailed discussion about the correction of the data could be seen in Shimer (2012) and Fujita and Ramey (2009).

from the CPS. All the raw data except for job vacancy , job finding rate, job separation rate and labour market tightness are available for download from FRED Economic Data[4].

The estimation of stochastic volatility in the time series is implemented at the quarterly frequency. Quarterly averages are taken for the monthly time series. All the data are detrended via the HP filter with a smoothing parameter equal to 1600, which makes it convenient to compare our results with many other works in the line of the relevant em8pirical literature.

Table 1.1 presents the descriptive statistics of the data used in our analysis. As been seen, the standard deviations for all the variables decreased after mid-1980s. As a preliminary exploration before we go to further analysis using more sophisticated Bayesian methods, in Figure 1.1, 3-year rolling standard deviations of quarterly time series in the U.S. labour market are displayed. As real GDP and labour productivity showed a dramatic drop in post 1984, other variables, while displaying a decreasing trend with the magnitude, experienced a different pattern. For example, although the standard deviation of the job separation rate dropped by a large magnitude in late 1950s, the changes were relatively smoother afterwards. More robust and detailed observation with the stochastic volatility estimation will be shown in the following section.

1.3 Estimating the stochastic volatility

In this section, we show the presence of volatility moderation in the U.S. labour market with more details from a perspective of its stochastic volatility changes. In the literature, there are several tools available for estimating volatility changes in time series, including a class of GARCH methods, the stochastic volatility model

[4]Website for FRED: `http://research.stlouisfed.org/fred2/`.

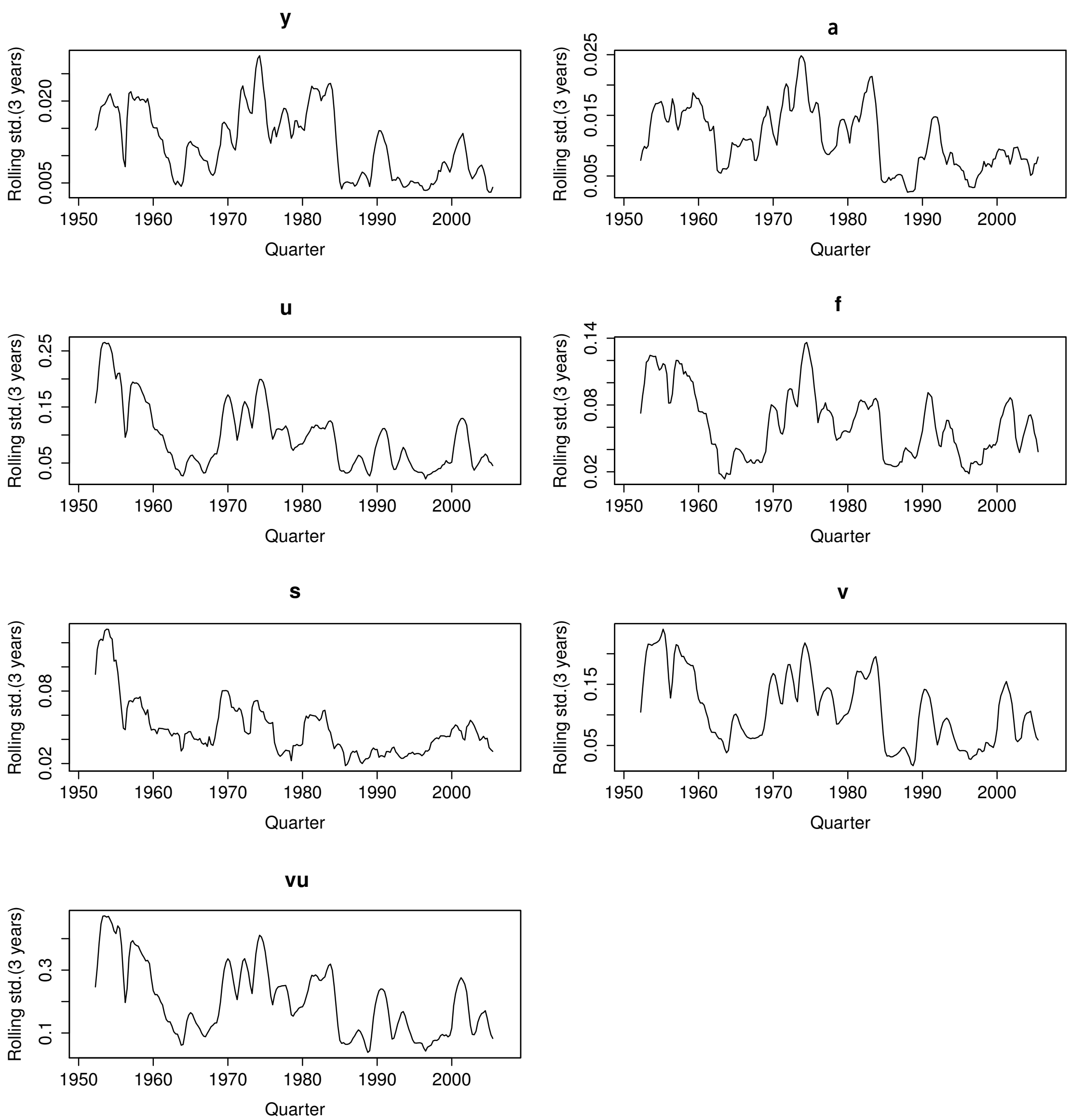

Figure 1.1: 3-year rolling standard deviation of U.S. labour market variables.

Table 1.1: Summary statistics of the labour market variables

	y	a	u	v	vu	f	s
pre 1984							
σ_{pre}	0.0185	0.0154	0.1466	0.1597	0.2982	0.0862	0.0637
ρ_{pre}	0.8313	0.7615	0.8559	0.9007	0.8859	0.8062	0.5348
post 1984							
σ_{post}	0.0092	0.0086	0.0824	0.1007	0.1761	0.0633	0.0362
ρ_{post}	0.8711	0.7298	0.9299	0.9167	0.9359	0.8297	0.3231
volatility change since 1984							
$\sigma_{post}/\sigma_{pre}$	0.4973	0.5584	0.5621	0.6306	0.5905	0.7343	0.4710

Note: The real GDP, y, is constructed by the Bureau of Economic Analysis (BEA). labour productivity, a, and unemployment rate, u, is issued by the Bureau of labour Statistics (BLS). Quarterly series of job finding rate, f, and job separation rate, s, are calculated by Shimer(2005; 2012) based on the monthly raw data from the Current Population Survey (CPS). Job vacancy in the labour market, v, is represented by a composite Help-Wanted Index (HWI) constructed based on the data issued by the Conference Board following Barnichon (2010). *All statistics are calculated as log-deviations from their H-P filtered trends (*$\lambda = 1600$*) at a quarterly frequency, 1951Q1-2007Q1.* σ_{pre} *and* ρ_{pre} *are used to denote the standard deviation and correlation of the corresponding variables pre 1984 while* σ_{post} *and* ρ_{post} *are those for post 1984 periods.*

estimation and the Markov chain regime switching estimation. More detailed discussion about the advantages and disadvantages of these methods and their applications in macroeconomic empirical analysis can be found in Fernández-Villaverde and Rubio-Ramirez (2010). Generally speaking, GARCH is one of the most applied methods in time series analysis. However, the application of ARCH/GARCH estimation in macroeconomics is limited due to its inability to separate the shocks on the volatility and that on the level itself. On the other hand, compared with the Markov chain regime switching estimation, a benefit of the stochastic volatility model is that it is convenient to apply perturbation methods to track the equilibrium dynamics in a structural economic model. While keeping the estimation in this chapter easier to implement, the stochastic volatility estimation fits the purpose of our analysis well in that it provides enough flexibility for further numerical discussion in the following chapter and possible future work, where the search and matching mechanism in the labour market together with volatility shocks is investigated in a more detailed manner.

1.3.1 The model

We now lay out the stochastic volatility model to be estimated for all the time series. It is assumed that the labour market time series is such that each of them follows an AR(1) process of the following form such that for a specific time series p_t,

$$a_t = \rho a_{t-1} + e^{\theta_t} u_t, \tag{1.1}$$

where u_t follows a standard normal distribution, $u_t \backsim N(0,1)$, and e^{θ_t} acts as the standard deviation of innovation in the simple process. In addition, it is also assumed that θ_t in Equation (1.1) follows an AR(1) process as well,

$$\theta_t = \rho' \theta_{t-1} + (1-\rho')\eta' + v_t, \tag{1.2}$$

where v_t is normally distributed $v_t \backsim N(0, \frac{1}{\tau'})$, in which τ' is defined as the precision of the normal distribution, i.e $\tau' = \frac{1}{\sigma'^2}$ with σ' as the standard deviation of volatility shocks in the model. In this way, the process generates a stochastic volatility for a_t. Our task following is then to estimate the parameters ρ, η', ρ' and τ' in the stochastic volatility model, i.e., equations (1.1) and (1.2), for each time series.

1.3.2 Prior distributions

We estimate the parameters of the stochastic volatility models with the Bayesian inference. For simplicity, same prior distributions are picked for all the time series in the estimation, while parameters of these distributions are calculated separately. Specifically, we follow an approach similar to that in Fernández-Villaverde et al.

(2011) to determine the parameters of the priors for the Bayesian analysis.

For the priors of ρ and ρ', we take a *Beta* distribution whose parameters are tuned such that the persistence is of a mild magnitude (0.83) and a relatively broad standard deviation (0.085) for our quarterly data. As for η', we pick a normal distribution as the prior. The mean of the prior distribution is picked to match the standard deviation of the data when there is no stochastic volatility shocks. Also, the variance of the prior normal distribution is set relatively large (1, in particular) to allow more flexibility for the posteriors.

As for the estimation of τ', since there is not much prior information directly available about the magnitude of shocks to the stochastic volatility, we first estimate an AR-GARCH model to get the conditional standard deviation, which itself does not separate the shocks to the level and those to the volatility. Then, we employ a truncated normal distribution as the prior for τ'.

We also estimated the volatility with alternative prior distributions. Specifically, we set the shape and scale parameters of the Gamma distribution for τ' such that its mean and standard deviation are both equal to the conditional standard deviation from the GARCH estimation to allow relatively high flexibility for the posterior. Though, the standard deviations of posterior estimates with the alternative prior distributions turn out to be greater than the ones we have with a truncated normal distribution we are using here, the main results remain the same. In addition, we also tried a uniform distribution for ρ and ρ' in estimation. The results are robust to the alternative setting. Table 1.2 presents details of the prior distributions used in the estimation.

We adopt the posterior means as our point estimates for the corresponding parameters. Posteriors for the quarterly estimation are presented in Table 1.3, where *S.D* is the standard deviation of the posterior distribution and *MC Error* indicates to what extent the MCMC algorithm has been converged. These two statistics

Table 1.2: The priors for estimating the stochastic volatility model

Quarterly estimation: Priors

	ρ	ρ'	η'	τ'
y	$\mathcal{B}(15,3)$	$\mathcal{B}(15,3)$	$\mathcal{N}(-4.1744,1)$	$\mathcal{N}^{+}(8.5377,8.5377)$
a	$\mathcal{B}(15,3)$	$\mathcal{B}(15,3)$	$\mathcal{N}(-4.3256,1)$	$\mathcal{N}^{+}(19.1559,19.1559)$
u	$\mathcal{B}(15,3)$	$\mathcal{B}(15,3)$	$\mathcal{N}(-2.0873,1)$	$\mathcal{N}^{+}(4.5415,4.5415)$
f	$\mathcal{B}(15,3)$	$\mathcal{B}(15,3)$	$\mathcal{N}(-2.5579,1)$	$\mathcal{N}^{+}(28.1713,28.1713)$
s	$\mathcal{B}(15,3)$	$\mathcal{B}(15,3)$	$\mathcal{N}(-2.9195,1)$	$\mathcal{N}^{+}(13.5691,13.5691)$
v	$\mathcal{B}(15,3)$	$\mathcal{B}(15,3)$	$\mathcal{N}(-1.9778,1)$	$\mathcal{N}^{+}(7.1566,7.1566)$
vu	$\mathcal{B}(15,3)$	$\mathcal{B}(15,3)$	$\mathcal{N}(-1.3671,1)$	$\mathcal{N}^{+}(6.5756,6.5756)$

Note: Priors for (1.1) and (1.2), where $\mathcal{B}$ and $\mathcal{G}$ represent Beta *and* Gamma *distributions respectively.*

confirm that the estimations perform fairly well with respect to the convergence of the Markov chains. The comparison of prior and posterior distributions of the estimation can be found in the Appendix at the end of this chapter.

1.4 Estimation results

The results of stochastic volatility estimation are presented in this section.

1.4.1 Stochastic volatility

Table 1.3 shows the estimated coefficients for the model discussed in the previous section. Means of the posterior distributions, as well as their standard deviations and MC errors, are listed. The estimated stochastic volatility model for the U.S. labour productivity therefore could be written as

$$a_t = 0.7636 \cdot a_{t-1} + \exp(\theta_t)u_t \tag{1.3}$$

$$\theta_t = 0.8264 \cdot \theta_{t-1} - 0.8450 + v_t. \tag{1.4}$$

Figure 1.2 illustrates the estimated volatility changes in the U.S. labour market

Table 1.3: Parameter estimation of the stochastic volatility model

Quarterly estimation: Posteriors

		y	a	u	f	s	v	vu
ρ	Mean	0.8301	0.7636	0.9135	0.8264	0.4855	0.9109	0.9284
	s.d	0.0349	0.0446	0.0298	0.0395	0.0611	0.0262	0.0243
	MC s.e	0.0016	0.0020	0.0013	0.0018	0.0027	0.0012	0.0011
ρ'	Mean	0.8549	0.8264	0.8326	0.8810	0.7844	-3.0876	0.8413
	s.d	0.0630	0.1260	0.0607	0.0691	0.1555	0.1523	0.0577
	MC s.e	0.0028	0.0056	0.0027	0.0031	0.0070	0.0068	0.0026
η'	Mean	-5.0144	-4.8674	-3.1645	-3.2538	-3.2019	0.7794	-2.5336
	s.d	0.1831	0.1341	0.1863	0.1582	0.1385	0.0818	0.1865
	MC s.e	0.0082	0.0060	0.0083	0.0071	0.0062	0.0037	0.0083
τ'	Mean	13.6983	25.9733	8.4020	33.7133	19.8274	8.0755	8.9485
	s.d	3.4722	7.5008	1.6467	11.0611	5.8966	2.4066	2.2831
	MC s.e	0.1553	0.3354	0.0736	0.4947	0.2637	0.1076	0.1021

Note: SD stands for Standard deviation, and MC SE for Markov Chain standard error.

over the time period investigated. It can be clearly seen that the volatility of the U.S. labour productivity before mid-1980's was much higher that those afterwards. This characteristic of the movement in the stochastic volatility in productivity is consistent with the usual perception of the so-called "Great Moderation" in the U.S. economy well documented in various literature.

As has already been noted by Faberman (2008), it can also been seen in Figure 1.2 that there is a difference between the patterns of the moderation in the aggregate economy and that in the labour market. Even though it is still noticeable that the volatility of the U.S. labour market declined at around mid-1980's but unlike a dramatic drop in aggregate output, the moderation of labour maker volatility generally can be traced back to periods around late 1950s. To verify the moderation we also shorten the window for the time series that we are investigating and focus on the that are more "recent", the results still hold. One can also notice that the dynamics of job separation, s, is relatively smaller and more stable, which is consistent with many findings in previous works that job creation plays an more important role in explaining the in and out flows in the U.S. labour market (Fujita and Ramey, 2009).

1.4.2 Co-movements

Table 1.4 shows the cyclicality and the correlations between the volatility changes in the U.S. labour market time series.

It can be seen in the table that the cyclical component of real GDP (y) has a negative correlation with the volatility changes in all variables, that is, volatility changes in both real GDP and labour market variables are counter cyclical. Specifically, real GDP itself has a relatively low negative correlation (-0.1285) with its stochastic volatility. In addition, we can also see that time-varing volatility changes in the labour market are also acyclic with a relatively low magnitude. Among the variables we investigated, the volatilities in the job opening (v), labour market tightness (vu), and unemployment (u) have relatively higher negative correlation with the cyclical components in real GDP, which are -0.2848, -0.2751 and -0.2248 respectively. In the mean time, the volatility in the job finding rate (f), the job separation rate (s) and that in labour productivity (PRS85006163) have the lowest correlation with the business cycle, which are -0.1139, -0.0558 and -0.0990 respectively.

The counter cyclicality of the volatility indicates that uncertainty in the aggregate economy and labour market is more likely to relate to time periods that when the economy is in a recession rather than time periods when it is booming. This is in some sense consistent to the observation of how average people react to the ups and downs in the economy, i.e., people tend to be more confident when the economy is booming but tend to worry more when the economy is in a recession. However, the relationship between the react of agents and the volatility changes observed in the data here is subject to further examination with a more sophisticated micro foundation.

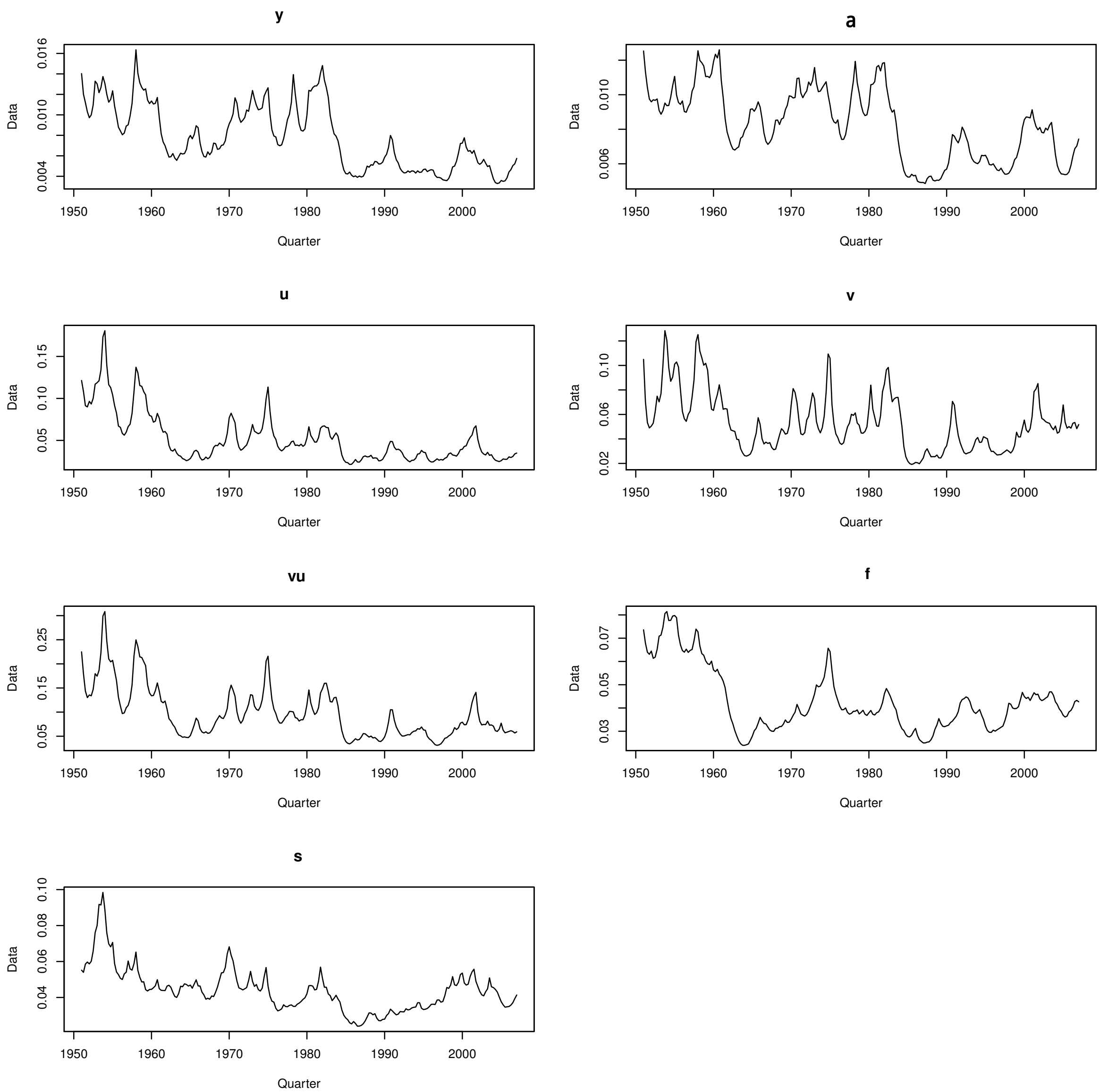

Figure 1.2: Stochastic volatility of U.S. labour market

Note: quarterly data, log-deviation from H-P filtered trend ($\lambda = 1600$*), period: 1951Q1-2007Q1.*

Table 1.4: Correlation of the volatility changes in U.S. labour market

		cyc	volatility						
		y	y	a	u	f	s	u	vu
cyc	y	1	-0.1142	-0.09890	-0.2248	-0.1139	-0.0558	-0.2848	-0.2751
volatility	y	-0.1142	1	0.9084	0.7930	0.6422	0.6122	0.7738	0.8334
	a	-0.0990	0.9084	1	0.6654	0.5935	0.5739	0.6985	0.7229
	u	-0.2248	0.7930	0.6654	1	0.8604	0.7856	0.8585	0.9708
	f	-0.1139	0.6422	0.5935	0.8604	1	0.7126	0.7774	0.8363
	s	-0.0558	0.6122	0.5739	0.7856	0.7126	1	0.6723	0.7505
	v	-0.2848	0.7738	0.6985	0.8585	0.77741	0.6723	1	0.9401
	vu	-0.2751	0.8334	0.7229	0.9708	0.8363	0.7505	0.9401	1

Note: Quarterly data; the cyclical component of real GDP, y, is measured as log-deviation from H-P filtered trend ($\lambda = 1600$), period: 1951Q1-2007Q1.

Table 1.4 also documents that the correlations between the stochastic volatility of variables of the labour market we are investigating and that of real GDP are positive and relatively high in magnitude, that is, the uncertainty of the movements in the labour market is highly correlated with that of the aggregate economy. It is also worth noticing in the table that there are positive correlations among stochastic volatilities of variables of the labour market. These observations of high co-movements in the volatility changes imply that a potential exploration of the labour market should provide a relatively wide propagation channel of the dynamics underlying. As the DMP model is currently the workhorse of labour market search, it is therefore necessary that we examine its performance in explaining the labour market dynamics with an incorporation of volatility shocks, which is part of the objective in our next two chapters.

1.5 Conclusion

This chapter examines the volatility changes in the U.S. labour market post-World War II. The volatility of unemployment, job finding rate, job separation rate, and other key variables of the labour market has been recorded via a stochastic volatility model estimated following a Bayesian approach. It is demonstrated that the volatility of these key variables in the U.S. labour market experienced a distinct moderation since the mid-1980s. The movement in the stochastic volatility in the U.S. labour market is consistent with the so-called "Great Moderation" in the aggregate economy, which has been well documented in various literature.

Furthermore, an exploration to the co-movements of the labour market variables and that with the aggregate economy shows that volatility changes in both real GDP and labour market variables are counter cyclical. Besides, the empirical exercise in this chapter also shows that there are the relatively high correlations between the stochastic volatility of variables of the labour market we are investigating and that of real GDP, which indicates that in explaining the dynamics underlying in the U.S. labour market it is necessary that we consider a relatively widespread propagation mechanism that could incorporate the volatility changes observed in the data. Following this idea, the effect of volatility changes as a driving force in the workhorse model of equilibrium unemployment theory is investigated in following chapters.

Appendix

Density plots of the Bayesian estimations

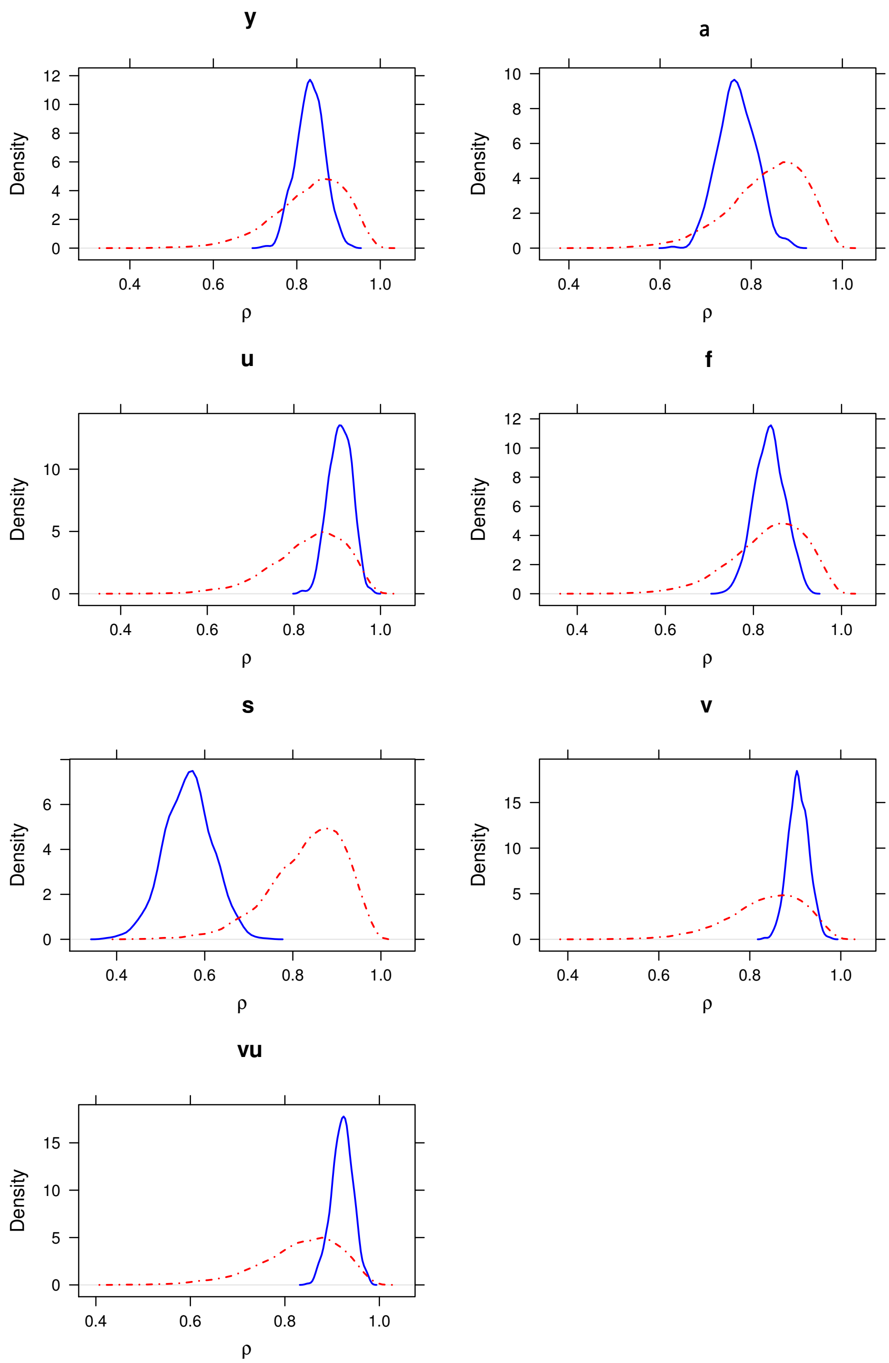

Figure 1.3: Prior and posterior distributions of ρ.

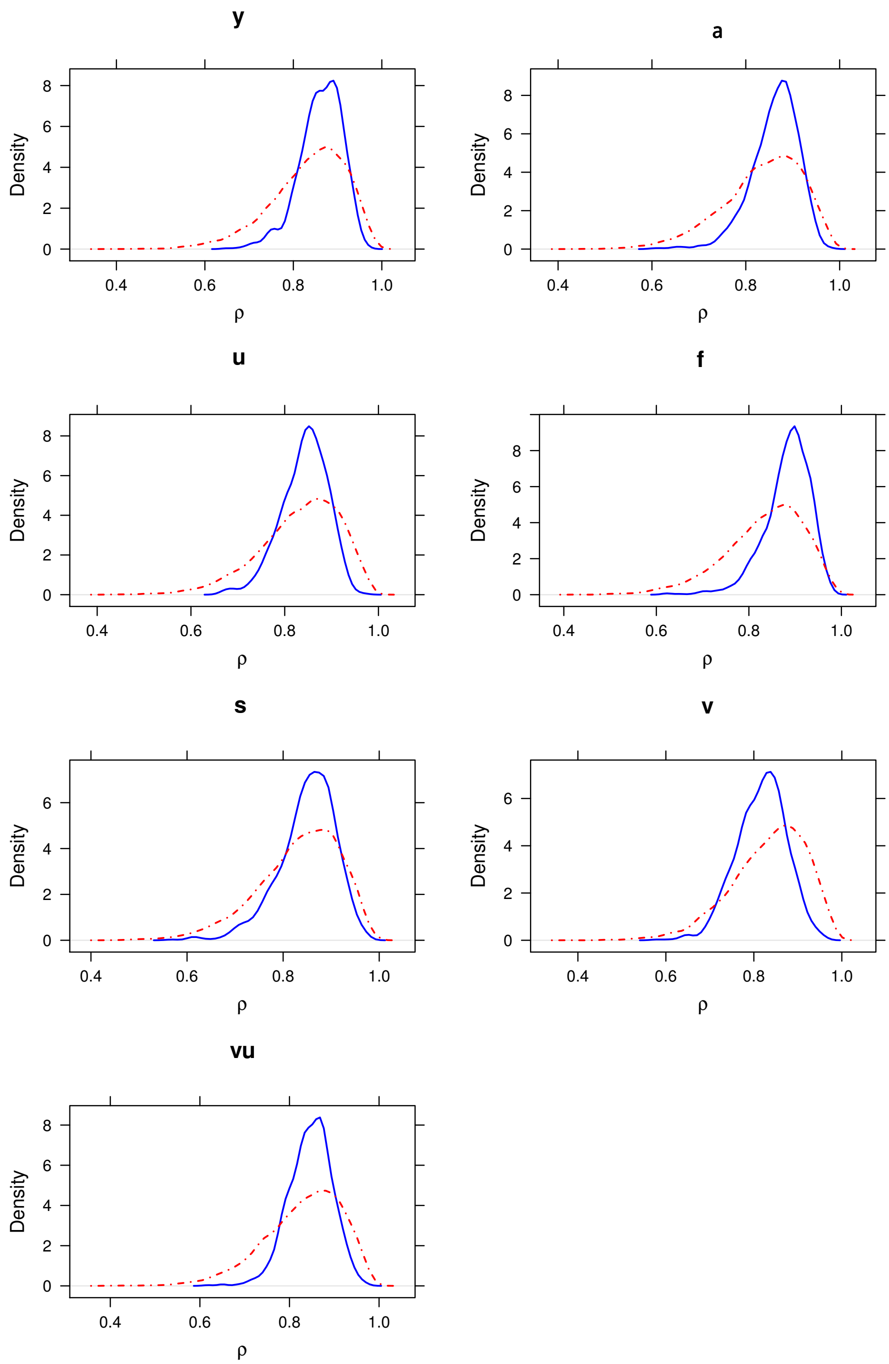

Figure 1.4: Prior and posterior distributions of ρ'.

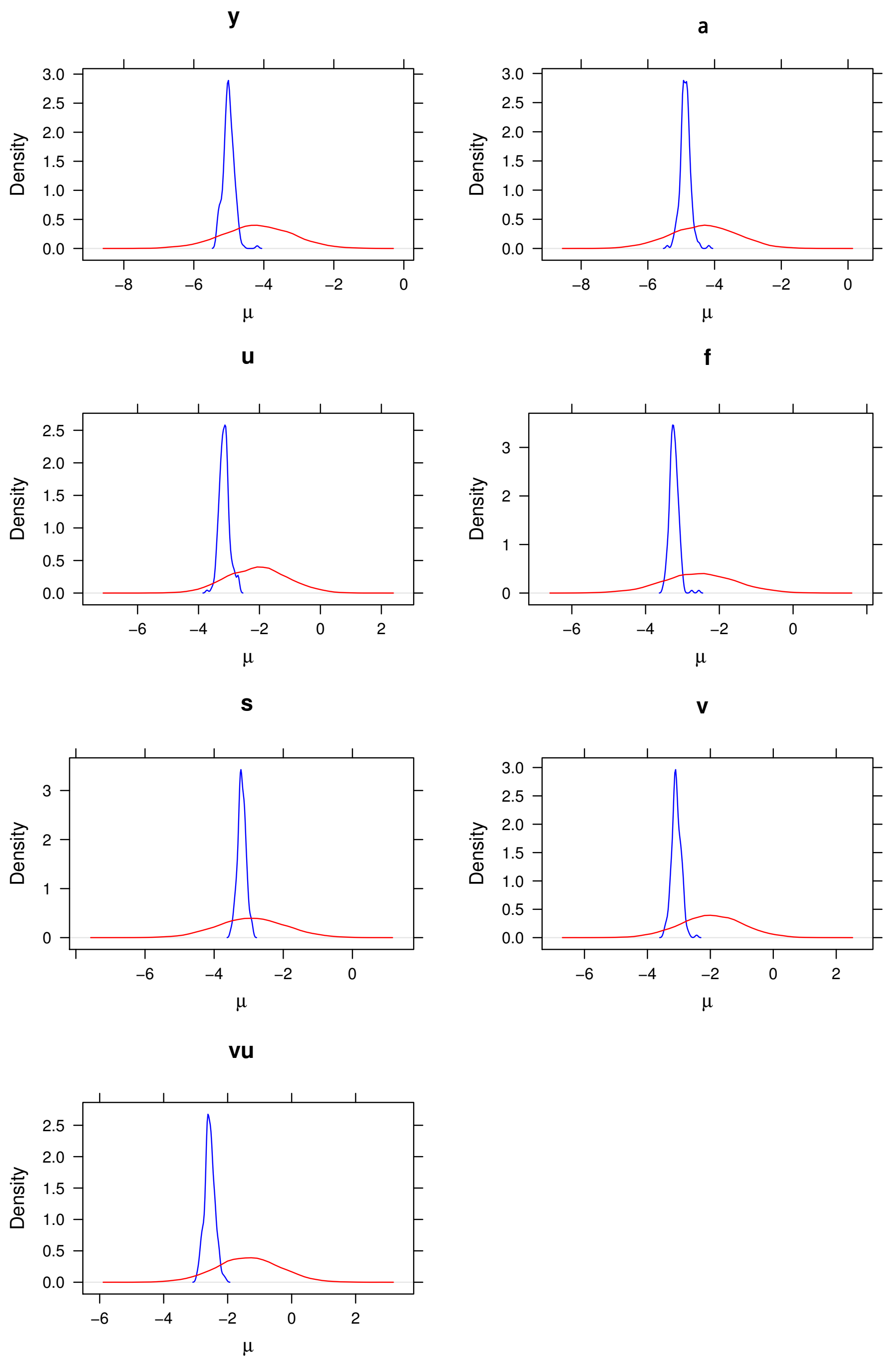

Figure 1.5: Prior and posterior distributions of μ.

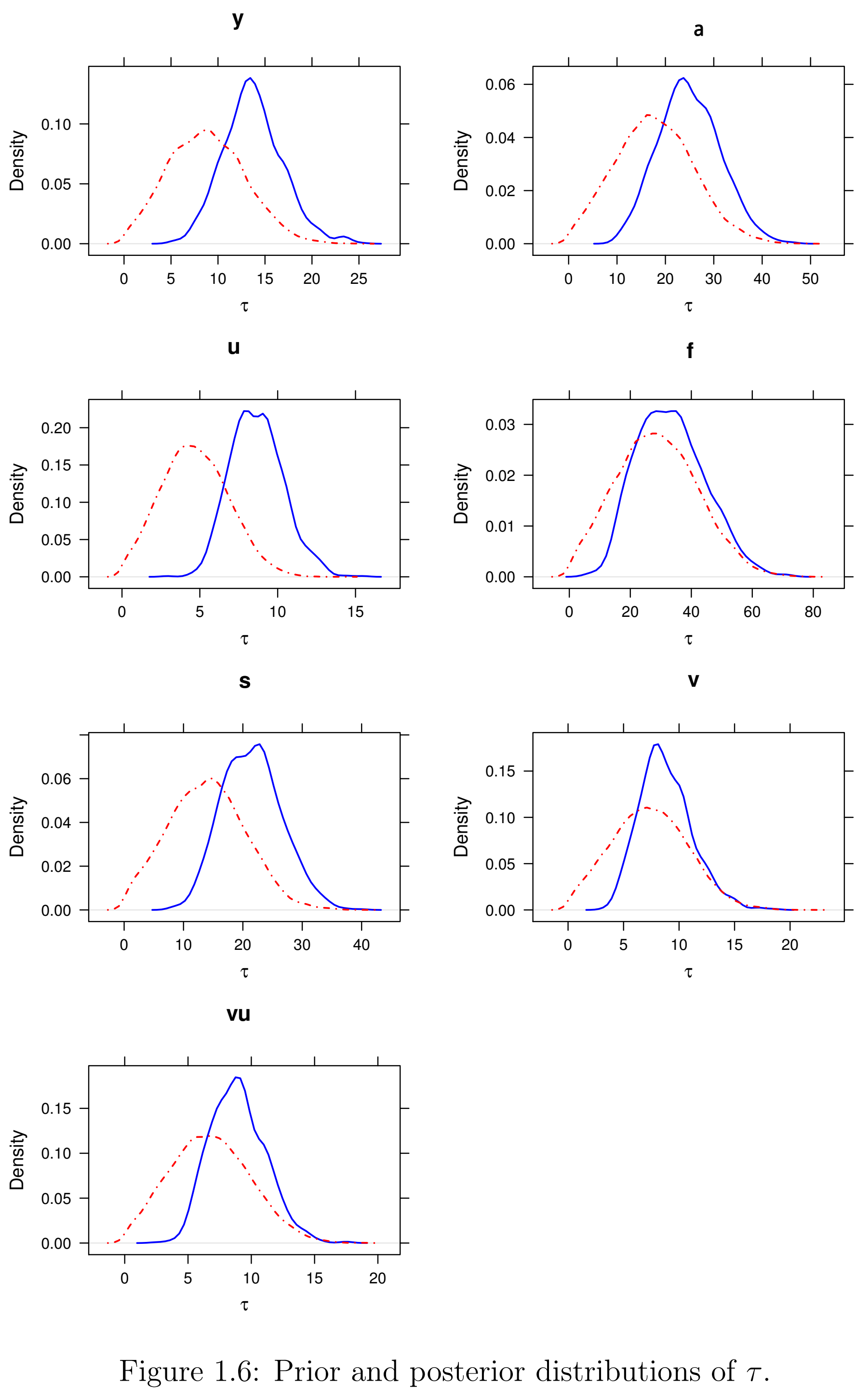

Figure 1.6: Prior and posterior distributions of τ.

Chapter 2

Labour Market and Stochastic Volatility Shocks

2.1 Introduction

This chapter examines how volatility changes in exogenous labour productivity affects the equilibrium dynamics in the Diamond-Mortensen-Pissarides (DMP, Diamond 1982; Mortensen 1982; Pissarides 1985) model, the baseline model of frictional labour markets.

In the previous chapter, it is observed that there was a moderation in volatility in the U.S. labour market after mid-1980s. Especially, one prominent characteristic we can see in the data is that there is a distinct drop in its volatility level in the U.S. labour productivity during the period. Other variables of the labor market also appeared to be less volatile in the past several decades since then. While the "Great Moderation" in the business cycle literature has been relatively well explored (Davis and Kahn, 2008; Galí and Gambetti, 2009), it is of natural interest that we investigate the implication of a similar observation in the labour market. Specifically in this chapter, we want to find out whether the effect of a stochastic volatility in labour productivity could help us understand the changing dynamics in the U.S. labour market including the pattern changes in volatility we have seen in the previous chapter.

On the other hand, uncertainty shocks coming in some different forms are taken in recent literature as a potential driving force in investigating the dynamics of the macro economy (Baker and Bloom, 2011; Bloom, 2009; Schaal, 2011; Fernández-Villaverde and Rubio-Ramirez, 2010). While many authors have introduced uncertainty into a business cycle framework in their analysis, there is still little work dedicated to highlighting the role of volatility changes in the labour market.

Following Shimer (2005) and Hall (2005), much of the literature has focused on understanding how plausible volatility in exogenous labour productivity can result realistic fluctuations in the equilibrium unemployment rate. This chapter takes the arguments of Hall (2005) and Hagedorn and Manovskii (2008) as given so that small changes in labour productivity can generate large changes in the unemployment rate. Instead, this chapter asks whether a plausible reduction in the volatility of labour productivity with similar magnetite as observed in the U.S. economy during the Great Moderation can result in a quantitatively significant decrease in the volatility of the unemployment rate within the confines of the DMP model.

Specifically, in this chapter, we investigate the effect of volatility changes in labour productivity on the U.S. labour market dynamics in two sets of models. We first start our discussion with a standard random search model as a benchmark and calibrate it following the approach introduced in Hagedorn and Manovskii (2008), with the estimated stochastic volatility changes incorporated. After that, following the framework presented in Rudanko (2009), we extend our analysis to a competitive search model with risk averse workers and risk neutral firms where the two sides sign full-commitment contracts when the matches are established. We introduce the risk-sharing framework into our discussion in the hope that the risk aversion can provide an additional mechanism through which uncertainty shocks can have an effect on the labour market.

It is found in the numerical exercise in this chapter that, in both of the two settings we investigate, changes in the volatility of labour productivity do not result in realistic changes in the volatility of the unemployment rate nor the vacancy-to-unemployment rate. Further analysis in this chapter shows that, holding the level of labour productivity constant, a change in the expected volatility of labour productivity has no effect on equilibrium job creation. In other words, the effects that arise from changes in the volatility of labour productivity come from the realized values of labour productivity, not uncertainty from expected volatility in labour productivity.

Furthermore, in the model simulation, it is found that with stochastic volatility changes introduced in the model economy, both of the two models are able to capture the volatility moderation reasonably well while both of them still suffer from a smaller magnitude of volatility in unemployment and other variables compared with those observed in the data. It is worth noticing that in the risk-sharing case, when incorporated with the estimated stochastic volatility in labour productivity, the model performance gets improved in capturing a relatively higher volatility of unemployment without setting an unrealistically high unemployment benefit, which in some sense provides another potential approach to address the Shimer puzzle. In addition, in order to investigate the effect of a volatility change, the elasticity of labour market dynamics to volatility changes in labour productivity are calculated and it turns out that in both models they fit the data pretty well.

The rest of this chapter is organized as follows: Section 2.2 lays out a baseline DMP model incorporating volatility changes in labour productivity, where the effect of volatility shocks are investigated with an Hagedorn and Manovskii (2008) style calibration. Section 2.3 then furthers the discussion on the effect of volatility changes in an extended competitive search model featuring risk sharing contracts between risk averse workers and risk neutral firms. Section 2.4 concludes.

2.2 Labour market search with volatility shocks

As we have observed in previous chapter, after the mid-1980's there is a notable moderation in the volatility in the U.S. labour market. Given that the frictional labour search model is of pivot importance in understanding the dynamics of the labour market, in this section, we investigate the effectiveness of a baseline labour search model in catching the moderation facts we have observed. Specifically, the main question we want to address in this section is to what extent it could replicate the moderation of the U.S. labour market via changes in the volatility in labour productivity as a driving force in the model. We start with laying out the a baseline search model, and then impose volatility changes into the labour productivity.

2.2.1 The model

Workers and firms in the model economy are assumed to be identical and risk neutral. A linear period utility function is assumed for the identical workers in the market: $u(c) = c$. Each firm can open only one vacancy in each period at a cost of z. Also, in each period, a units of output are produced by a matched pair of one vacancy and one worker: hence a is in fact the labour productivity. Following Hagedorn and Manovskii (2008), the cost of opening a vacancy, or equivalently the hiring cost is explicitly defined with the cost of capital and labour involved. Specifically, given the labour productivity equals a, opening cost equals $z_a = z_K a + z_L * a^{\epsilon_{w,a}}$, where z_K is the capital cost of hiring, z_w is the flow labour cost of hiring, and $\epsilon_{w,a}$ is the elasticity of wage to the labour productivity.

It is also assumed in the model that workers and firms are matched one-to-one via a "channel" matching function (Haan et al., 2000): $m(u,v) = \frac{uv}{(u^{\alpha}+v^{\alpha})^{\frac{1}{\alpha}}}$, where u is the measure of unemployed workers and v the measure of opened vacancies in the economy. We denote the job filling probability of a vacancy with $q(\theta) = \frac{m(u,v)}{v} = (1+\theta^{\alpha})^{-\frac{1}{\alpha}}$, where $\theta = \frac{v}{u}$ is the measure of the labour market tightness. Given that, the job finding probability for a worker can be defined by $\mu(\theta) = \frac{m(u,v)}{u} = (1+\theta^{-\alpha})^{-\frac{1}{\alpha}} = \theta q(\theta)$.

In the model economy, matched pairs of workers and vacancies are assumed to break up with an exogenous probability δ each period. Unemployed workers take b units of goods for their period consumption. It is also assumed that wage w of each match is determined via a Nash bargaining between the two sides, in which workers have an exogenous bargaining power of β. Firms with its vacancy filled therefore keeps $a - w$ as its period profit.

Additionally, period productivity a of the representative firm is set to follow an AR(1) process and the standard deviation of its innovation is denoted by sa. As an extension to the baseline model, we assume that sa itself follows an AR(1) process. Note that we here impose volatility shocks as part of the productivity process itself in the model simulation for investigating the performance of the DMP model in capturing observed moderation in data, instead of directly shifting volatilities between high and low levels. The motivation behind is straightforward: imposed volatility changes in the expectation structure of the agents in the model. Besides, by doing so, we can investigate not only the overall performance of the baseline model in capturing the facts observed in the real economy, but also the effects of stochastic volatility shocks.

The brief time line of the model economy goes as follows: at the beginning of each period, the economy starts with u unemployed workers; the agents observe the realization of the volatility of the period productivity sa and a specific productivity

a based on the volatility that has just been observed; firms make decisions of whether to open a job vacancy, unemployed workers search randomly on the labour market and new worker-vacancy matches are established; a units of products are produced by the every match existing at the beginning of this period while newly formed matches do not participate in production in this period; wage decided via a Nash bargaining are paid to the employed workers; after finishing the production, a δ fraction of matches resolves, workers of which now enter into unemployment again.

Given the setting of the baseline model described above, the value function of a job for an employed worker at the beginning of period can be written as

$$\begin{aligned} W(sa,a) &= u(w)+\rho(1-\delta)\sum_{sa\in SA}\pi(sa',sa)\sum_{a'\in A|sa'}\pi(a',a)W(sa',a') \\ &\quad +\rho\delta\sum_{sa\in SA}\pi(sa',sa)\sum_{a'\in A'|sa'}\pi(a',a)U(sa',a'), \end{aligned} \tag{2.1}$$

where SA is the set of all possible volatility levels for the labour productivity, and $A'|sa'$ is the set of all possible states of labour productivity conditional on the realization of volatility sa'. It says that given the current state of the economy (sa,a), an employed worker enjoys utility $u(w)$ from consuming the period wage w. There is a probability of δ that the worker and the firm get separated, in which case the newly unemployed worker will be left with a value of $U(sa',a')$, at the beginning of the next period given an updated state (sa',a). On the other hand, there is also a probability of $(1-\delta)$ that the match survives. The fact that the worker continues being employed in the next period with the state (sa',a') has a value of $W(sa',a')$.

Similarly, the value function of a worker who is unemployed at the beginning of period can be written as

$$
\begin{aligned}
U(sa,a) &= u(b)+\rho\theta q(\theta)\sum_{sa\in SA}\pi(sa',sa)\sum_{a'\in A'|sa'}\pi(a',a)W(sa',a') \qquad (2.2)\\
&\quad +\rho(1-\theta q(\theta))\sum_{sa\in SA}\pi(sa',sa)\sum_{a'\in A'|sa'}\pi(a',a)U(sa',a').
\end{aligned}
$$

Given the current state of the economy (sa, a), an unemployed worker enjoys utility $u(b)$ from consuming unemployment benefit b. There is a probability of $\theta q(\theta)$ that the worker will find a job in the next period, in which case the newly employed worker will enjoy a value of $W(sa', a')$ in the updated state (sa', a). However, there is also a probability of $(1 - \theta q(\theta))$ that the worker fails to find a job. In the case when the worker continues being unemployed in the next period with the state (sa', a'), it leaves a value of $U(sa', a')$.

Also, the value function of to a firm that has a filled vacancy at the beginning of period can be written as

$$
\begin{aligned}
J(sa,a) &= a-w+\rho(1-\delta)\sum_{sa\in SA}\pi(sa',sa)\sum_{a'\in A'|sa'}\pi(a',a)J(sa',a') \qquad (2.3)\\
&\quad +\rho\delta\sum_{sa\in SA}\pi(sa',sa)\sum_{a'\in A'|sa'}\pi(a',a)V(sa',a').
\end{aligned}
$$

The period value of having a filled vacancy to the firm is $a-w$. There is a probability of δ that a match of a firm and a worker dissolves, in which case the firm is left with a value for possessing an unfilled vacancy at the beginning of the next period with the updated state (sa', a'). There is also a probability of $(1 - \rho)$ that the match survives. In this case, the from then enjoys a value of $J(sa', a')$.

Finally, the value function of a unfilled vacancy to a firm at the beginning of period can be written as

$$V(sa,a) = -z + \rho q(\theta) \sum_{sa \in SA} \pi(sa', sa) \sum_{a' \in A'|sa'} \pi(a', a) J(sa', a') \qquad (2.4)$$
$$+\rho(1-q(\theta)) \sum_{sa \in SA} \pi(sa', sa) \sum_{a' \in A'|sa'} \pi(a', a) V(sa', a').$$

The cost of opening an unfilled vacancy for the firm is z for each period. There is a probability $q(\theta)$ that the firm gets matched with a worker in the next period. If this is the case, the value to the firm is then $J(sa', a')$. Otherwise, with a probability $(1 - q(\theta))$, the firm will be left with an unfilled vacancy at the beginning of next period, the value of which will be $V(sa', a')$ given the updated state (sa', a').

In addition, each period, wage is determined by the Nash bargaining solution,

$$w = \arg\max(W(sa,a) - U(sa,a))^{\beta}(J(sa,a) - V(sa,a))^{1-\beta},$$

and, the transition equation for the unemployment rate comes as $u' = u + \delta(1 - u) - \theta q(\theta)u$.

Note that the model is rather standard except that we introduce into it one extra aspect of the volatility change. With all these settings above, the dynamics of the model can be determined with the following equations,

$$\begin{aligned}
u' &= u + \delta(1-u) - \theta q(\theta) u \\
z &= q(\theta)\rho(1-\beta) \sum_{sa \in SA} \pi(sa', sa) \sum_{a' \in A|sa'} \pi(a', a) S(sa', a') \\
S(sa', a) &= a - w - u(b) + u(w) + \rho(1 - \delta - \beta\theta q(\theta)) \sum_{sa \in SA} \pi(sa', sa) \sum_{a' \in A|sa'} \pi(a', a) S(sa', a'),
\end{aligned}$$

where $S(a, \delta) = J(sa, a) + W(sa, a) - U(sa, a)$.

2.2.2 Weekly estimation of stochastic volatility

Before starting our discussion about calibrating the baseline model, we proceed our analysis with estimating a weekly AR(1) stochastic volatility process for the U.S. labour productivity. There are two reasons that could justify our weekly estimation of the labour productivity process.

One reason for conducting a weekly calibration is to address the potential time aggregation problem in the data collected at different frequencies as pointed out by Ebell (2008). Take unemployment and labour productivity for example. When doing the data collection, the Bureau of labour Survey (BLS) takes unemployment observations in one week each month and makes a reference out of it for that month. Hence, the unemployment data in essence is a weekly observation, even though they are issued at a monthly frequency. On the contrary, the labour productivity data are available at the quarterly frequency, which are obtained by taking an average of monthly values. The correlation between these two might be averaged out at a quarterly frequency. Actually, even if the data are of the same frequency, higher frequency data are more preferred since either in empirical exploration or model calibration and simulation, dynamics and co-movements of interested variables with lower frequency data are prone to be averaged out.

Another advantage of taking a weekly estimation of labour productivity volatility is that the model we employ here closely follows that in Hagedorn and Manovskii (2008). Calibrating the model with weekly frequency, it would be convenient for us to follow their strategy of calibration and it is also possible for us to compare the results with previous findings in a more direct way.

The U.S. labour productivity data are reported at the quarterly frequency by the BLS, which dates back to 1947Q1. As we did in Chapter 1, we narrow our empirical exploration down to the time period from 1951Q1 to 2007Q1. To estimate the weekly volatility dynamics in the U.S. labour productivity, we adopt the same Bayesian method as that we used to estimate the stochastic volatility models in the previous chapter. By exploiting the algorithm and estimation strategy we are going to discuss, it is possible for us to retrieve information we needed behind the quarterly data at the weekly frequency

We began our exercise of the estimation with an assumption that we have a latent weekly stochastic volatility process whose parameters are still undetermined and going to be estimated. Specifically, it is assumed that the a weekly data generating process is of the following process:

$$\begin{aligned} a_{w,t} &= \ell_w a_{w,t-1} + \exp(\kappa_{w,t})\epsilon_{w,t} && (2.5) \\ \kappa_{w,t} &= \ell'_w \kappa_{w,t-1} + (1-\ell'_w)\eta'_w + \zeta_{w,t}, && (2.6) \end{aligned}$$

which has the same setting as we have had for the quarterly model in previous section. A subscript w is used to indicate the parameters for a weekly frequency. In this assumed model process, we have five parameters to estimate, i.e.,ℓ_w, ℓ'_w, η'_w and the s.d of ζ_t which we denote as σ_κ.

Based on the assumed AR(1) stochastic volatility model for the weekly data, check the first quarter at first, we have a series of equations (one for each week) in that quarter as follows,

$$\begin{aligned} a_{w,1} &= \ell_w a_{w,0} + \exp(\kappa_{w,1})\epsilon_1 \\ a_{w,2} &= \ell_w a_{w,1} + \exp(\kappa_{w,2})\epsilon_2 \\ a_{w,3} &= \ell_w a_{w,2} + \exp(\kappa_{w,3})\epsilon_3 \\ &\dots \\ a_{w,12} &= \ell_w a_{w,11} + \exp(\kappa_{w,12})\epsilon_{12}. \end{aligned}$$

It can been seen that for the nth week the equation could be written as a function of $a_{w,0}$ and

$$a_{w,n} = \ell_w^n a_{w0} + \sum_{i=0}^{n-1} \ell_w^i \exp\left(\kappa_{w,n-i}\right) \epsilon_{n-i}.$$

In our estimation, the quarterly data are assumed to be the mean values of the corresponding weekly data. As we have mentioned at the beginning of this section, the BLS issues the U.S. labour productivity data only at a quarterly frequency, which are actually the average of their monthly observations. However, the goal of the our estimation strategy is neither to re-obtain the monthly observation nor to retrieve the latent "true" weekly data, both of which are in essence impossible to achieve. Instead, the objective of our exercises is to provide a possible estimation of underlying weekly process, which does not necessarily claim to be a replication of the true data collection procedure. In this sense, the assumptions adopted here should not matter for its own purpose.

Therefore, for the quarterly observations, we have $a_{q,1} = \frac{\sum_{i=1}^{12} a_{w,i}}{12}$, $a_{q,2} = \frac{\sum_{i=13}^{24} a_{w,i}}{12}, \ldots$. Specifically, for $a_{q,1}$,

$$\begin{aligned} a_{q,1} &= \frac{\sum_{i=1}^{12} a_{w,i}}{12} \\ &= \frac{1}{12} \left(\sum_{i=1}^{12} \ell_w^i a_{w0} + \sum_{i=1}^{12} \sum_{j=0}^{i-1} \ell_w^i \exp\left(\kappa_{w,i-j}\right) \epsilon_{i-j} \right), \end{aligned}$$

and for $a_{q,2}$,

$$\begin{aligned} a_{q,2} &= \frac{\sum_{i=13}^{24} a_{w,i}}{12} \\ &= \frac{1}{12} \left(\sum_{i=1}^{12} \ell_w^i a_{w12} + \sum_{i=1}^{12} \sum_{j=0}^{i-1} \ell_w^i \exp\left(\kappa_{w,12+i-j}\right) \epsilon_{12+i-j} \right). \end{aligned}$$

With some algebra, it can be shown that

$$
\begin{aligned}
a_{q,2} \;=\; & \ell_w^{12} a_{q,1} \\
& -\frac{1}{12} \sum_{i=1}^{12} \sum_{j=0}^{i-1} \ell_w^j \exp(\kappa_{i-j}) \epsilon_{i-j} \\
& +\frac{1}{12} \sum_{i=1}^{12} \ell_w^i \sum_{i=0}^{12-1} \ell_w^i \exp\left(\kappa_{12-i}\right) \epsilon_{12-i} \\
& +\frac{1}{12} \sum_{i=1}^{12} \sum_{j=0}^{i-1} \ell_w^j \exp\left(\kappa_{12+i-j}\right) \epsilon_{12+i-j},
\end{aligned}
$$

Note that the above equation consists of a first-order lagged term $\ell_w^{12} a_{q,1}$ and three terms aggregating the weekly innovations, which as whole are set to be captured by the quarterly process of stochastic volatility as we did in Chapter 1.

Two parameters in equation (2.5) and (2.6) are easy to pin down. In the first chapter, we have already estimated the persistence of the quarterly process of labour productivity, $\ell_q = 0.7636$, therefore the weekly persistence of (2.5) could be easily calculated such that $\ell_w = \ell_q^{\frac{1}{12}} = 0.9786$. With these two parameters determined, we still have three to estimate. By exploiting the specific structure of the quarterly and weekly process and the convenient flexibility of the JAGS programming language, we are able to adopt an estimation strategy that estimates the rest three parameters with the Bayesian inference. The modeling of the Bayesian estimation is explained in a more detailed manner in the Appendix of this chapter.

In our weekly estimation, a strategy similar to the one we have adopted in the previous quarterly estimation is taken here to decide priors for the undecided parameters. Specifically, due to the lack of information about persistence of the stochastic volatility process, ℓ'_w, we choose a uniform distribution so that we could have a wider range as a start for the Markov chain convergence. A normal distribution is used as the prior distribution for the mean value of the weekly stochastic volatility process η'_w. Specifically, the normal distribution has a mean value that

Table 2.1: Priors for estimating the weekly stochastic volatility model

Weekly estimation: Priors

ℓ'_w	η'_w	τ^2_w
$U(0,1)$	$\mathcal{N}(-4.3017, 8.6033)$	$\mathcal{N}^+(12.7391, 25.4782)$

Note: Priors for (2.5) and (2.6), where U represents the uniform distribution, and N^+ represents the Normal distribution.

Table 2.2: Posterior parameter estimation of the weekly stochastic volatility model

Weekly estimation: Posteriors

	ℓ'_w	η'_w	τ^2_w
Mean	0.93877	-5.7667	43.2227
S.D	0.0378	0.0935	18.3739
MC Error	0.0012	0.0030	0.5810

Note: SD stands for Standard deviation, and MC SE for Markov Chain standard error.

is equal to the logged standard deviation of labour productivity, and a standard deviation that is twice the magnitude of its mean.

As for the prior for the precision of the volatility shock distribution, a truncated normal distribution is set to have a mean value of quarterly standard deviation volatility changes in labour productivity. In order to allow more flexibility for our prior when there is no much other reference available , we set the standard deviation of the truncated normal distribution to be twice as the mean value. To sum up, Table 2.1 lists all the prior distributions adopted for the Bayesian estimation.

Table 2.2 presents the estimated posteriors for the three coefficients in 2.6. The estimated stochastic volatility model for U.S. labour productivity at a weekly frequency therefore could be laid out as

$$a_{w,t} = 0.9786 \cdot a_{w,t-1} + \exp(\kappa_{w,t})\epsilon_t \tag{2.7}$$

$$\kappa_{w,t} = 0.9388 \cdot \kappa_{w,t-1} - 0.3531 + \zeta_t, \tag{2.8}$$

where $\epsilon_t \backsim N(0,1)$ and $\zeta_t \backsim N(0, \frac{1}{\tau_w^2})$ and $\tau_w^2 = 43.2227$. Given the estimated model, the average level of estimated weekly stochastic s.d is 0.0031, which will be used in solving and simulating the model economy. One should note that the standard deviation of τ^2 is relatively large compared with the rest two estimated coefficients. However, since τ^2 is the precision of the normal distribution, once we take the reverse of it and calculate the square root, the range of difference becomes a minor in magnitude and should not change the results in this chapter[1].

Figure 2.1 demonstrates the estimated stochastic volatility in the U.S. labour productivity from 1951Q1 to 2007Q1. It can be seen that estimated weekly volatilities in the first few years in 1950's are to some extent lower when compared with following periods of time. One might concern that the initial value together with our specific estimation strategy may have cast unwanted effect on the shape of that part. To address these concerns, we did two additional checks: using data of the full range of time period (1947Q1-2007Q1), and try higher initial points. The result is actually robust to those alternative settings.

Another observation worth noticing is that when compared with the quarterly estimation obtained in the previous chapter (Figure 1.2), there is a difference in scales between the quarterly and weekly estimation of the volatility. Note that in our algorithm we specify our weekly process to replicate the quarterly observations. In other words, the target of our estimation strategy is not to fit the quarterly volatilities themselves. More fundamentally, compared to a relatively longer period of time (1 week vs. 12 weeks), the states of the economy tend not to be changed in short periods of time, which turns out to be that there should more volatilities in quarterly data as is exactly captured in our estimation.

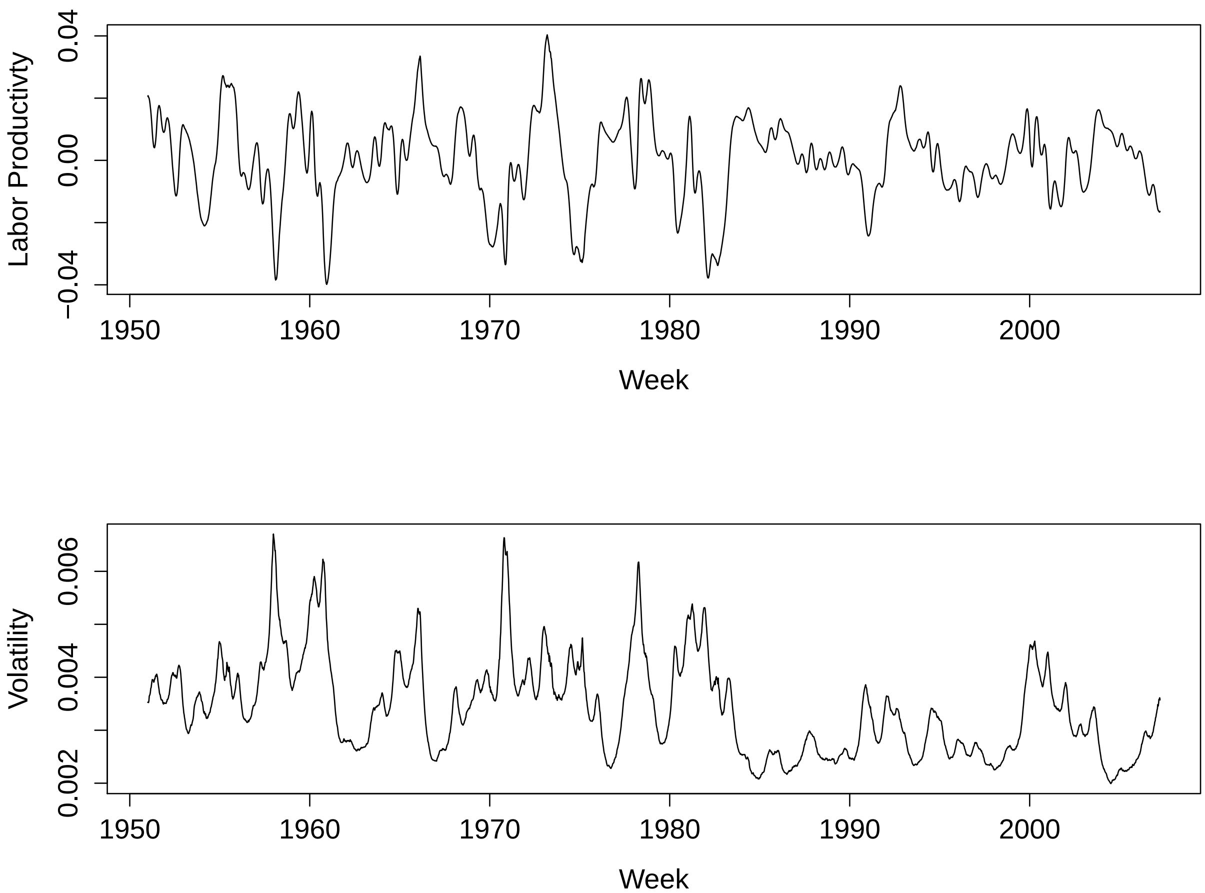

Figure 2.1: Estimated weekly stochastic volatility of U.S. labour productivity (1951Q1-2007Q1).

/mnt/Windows/Users/Jiang Lin/My Study/Projects/

2.2.3 Calibration

We closely follow the approach of Hagedorn and Manovskii (2008) (HM for short) to calibrate and solve the model.

As the baseline Diamond-Mortensen-Pissarides search and matching model has become the work horse for the theory of equilibrium unemployment, there is still some debate about whether the search and matching model fails to capture some dynamic properties of the labour market. In particular the dynamics of unemployment and vacancy (Andolfatto, 1996; Costain and Reiter, 2003; Shimer, 2005), the volatility of which in the data are documented to be much higher than that in the model. In responding to the gap between the data and model, among several other approaches, Hagedorn and Manovskii (2008) proposed a new strategy to the model calibration, which highlights the importance of hiring cost and the elasticity of wage to the labour productivity as well on the firm's side.

While critiques to Hagedorn and Manovskii's strategy of parameter calibration mainly focus on the fact that the calibrated unemployment benefit turns out to be "unrealistically" high and the bargaining power for the workers on the contrary pretty low compared to that in a standard calibration, it is still arguable, as the authors declare, that as the baseline Motensen-Pissarides model is an approximation to the labour market, it is proper to include the value of home production and leisure as part of the an extended conception of "unemployment benefits". Given that, the HM approach of calibration still shows a rightful channel that allows the baseline model successfully replicates the relatively high volatile unemployment and vacancy. Furthermore, since the main task of this chapter does not restrict itself to this so-called "Shimer puzzle", following this approach to calibrate our model

[1]More specifically, $\sqrt{\frac{1}{43.2227-18.3739}} - \sqrt{\frac{1}{43.2227+18.3739}} \approx 0.07319$.

provides a necessary starting point to compare the results of this extended model to its original version.

Table 2.3: Parameter calibration, model with risk neutral workers and volatility changes

Parameters values in model calibration

Parameter	Description	Value
d	discounting factor	0.98
δ	job separation rate	0.0083
z_k	capital hiring cost for the firm	0.474
z_w	labour hiring cost for the firm	0.110
b	value of being unemployment	0.9364
β	worker's bargaining power	0.0859
α	parameter for the matching function	0.4037
ρ	persistence of weekly labour productivity	0.9774
ρ'	persistence of weekly stochastic volatility	0.9911
σ_{se}	standard deviation of stochastic volatility	0.0031

We implement Hagedorn and Manovskii's calibration following a simulated moment approach with Nelder–Mead method: the parameter for the matching function (α), value of unemployment (z), and worker's bargaining power (β), are tuned to make sure the simulated model matches well with the data. The matching function parameter, α, is calibrated to match the weekly job finding rate $\mu = 0.139$. The rest two variables, β and z are of great importance in the calibration strategy. As illustrated in Hagedorn and Manovskii (2008), by calibrating z targeting at firm's vacancy opening cost while β jointly being fixed by targeting it to the elasticity of wage to labour productivity, the model is able to generate higher volatility of the elasticity of market tightness to productivity.

Specifically, in calibrating the value of β and z, given that the values for capital and labour cost of hiring, c_k and c_w, do not depend directly on statistics updated along time, we follows those adopted in Hagedorn and Manovskii (2008). Aggregated quarterly time series used in calibration are HP filtered with a smoothing parameter of 1600. Details of the calibrated parameters in the model are listed in

Table 2.3.

When calibrating and simulating the model, we incorporate volatility shocks of labour productivity via an exogenous stochastic volatility process, parameters of which have already been estimated in the previous section. The two stochastic processes are approximated discretely with Markov chains using the method described in Tauchen (1986). Compared with Tauchen and Hussey (1991), Tauchen (1986) in recent research is found to be more accurate on average when the target process is highly persistent(Floden, 2008). Even though due to the difficulty in the high dimension computation involved in approximating the volatility changes, following a rule of thumb for the approximation of a highly persistent process, we set the finite grid numbers equal to 9 and 15 for the volatility process and productivity process, respectively (Tauchen, 1986). More specifically, we first build a grid of 9 points with each point of which represents a specific volatility level, then build up another grid of 15 points which represents the range of productivity generated based on the given specific volatility. By doing this, a 9 by 15 matrix could be constructed for the labour productivity, $A_{9\times 15}$. The transitional matrices for each process are denoted by π_{seA} and π_A .

For a benchmark comparison, Table 2.4 and Table 2.5 demonstrate both the statistics of labour market variables in the U.S. data and those from the model simulation. It is worth noticing that while the extended version of the model successfully captures the relatively high volatility of unemployment and vacancy in the labour market when compared with those from Shimer (2005), the simulated results fall short in magnitude when compared with the one obtained in Hagedorn and Manovskii (2008). One possible reason is that this is a side result of our specific approach of modeling stochastic volatility and the discrete approximation method. When approximating the stochastic volatility process, the volatility shocks (v_t in Equation 2.6) and shocks to the productivity itself (u_t in Equation 2.5) are set to be independent from each other by assumption in our stochastic volatility model, while it is not uncommon in real economy that we observe some correlation between these two shocks. While it is ideal to take the correlation into account when modeling the stochastic volatility, it is technically challenging and makes calibrating and solving the model computationally difficult.

2.2.4 Results

2.2.4.1 Policy functions

Figure 2.2 illustrates a set of wage functions of the identical firms in the model economy. As we can see in the figure, given that in our model simulation there are 9 states for the volatility changes in labour productivity, we therefore have 9 curves in the plot such that each curve represents the wage function conditional on one specific realization of the volatility. Specifically, shorter lines on the plot represent wage functions under lower volatilities. One should note that due to the specific approach of modeling the stochastic processes we adopted in the simulation of the model, a productivity grid generated in the approximation for a lower volatility is shorter than that for a higher volatility, even though the number of points on the volatility grid for them are the same. Therefore, we can only compare the portion of points where both the lower volatility grid and the higher volatility grid that overlap each other.

It could be found that when the realization of labour productivity is higher than its median level ($a = 1$), the optimal constant contract wage in the labour market under a lower volatility is higher than that under a higher volatility. On the other hand, when the realization of labour productivity is lower than its median level, the optimal wage instead is lower than that under a higher volatility.

Table 2.4: Parameter statistics in the U.S. labour market (1951Q1 - 2007Q1)

U.S. labour market statistics (1951Q1 - 2007Q1)

	u	v	θ	f	a
std.	0.1240	0.1384	0.2548	0.0775	0.0132
corr.	u	v	θ	f	a
u	1	-0.9153	-0.9764	-0.9179	-0.2660
v	-	1	0.9802	0.9058	0.4179
θ	-	-	1	0.9311	0.3520
f	-	-	-	1	0.2624
a	-	-	-	-	1

U.S. labour market statistics (1951Q1 - 1983Q4)

	u	v	θ	f	a
std.	0.1466	0.1597	0.2982	0.0862	0.0154
corr.	u	v	θ	f	a
u	1	-0.9258	-0.9805	-0.9276	-0.3566
v	-	1	0.9815	0.9315	0.4972
θ	-	-	1	0.9472	0.4400
f	-	-	-	1	0.4119
a	-	-	-	-	1

U.S. labour market statistics (1984Q1 - 2007Q1)

	u	v	θ	f	a
std.	0.0824	0.1007	0.1761	0.0633	0.0086
corr.	u	v	θ	f	a
u	1	-0.8783	-0.9601	-0.9092	0.1544
v	-	1	0.9765	0.8344	0.1173
θ	-	-	1	0.9311	-0.0020
f	-	-	-	1	-0.2074
a	-	-	-	-	1

Note: Statistics are calculated based on data at a quarterly frequency. Unemployment u is constructed based on data from BLS from the Current Population Survey (CPS). Vacancy v is constructed based on Help-wanted data issued by the Conference Board; data after December, 2000 are constructed following the method introduced by Barnichon (2010). Job finding rate f is constructed based on Shimer (2005). Aggregated quarterly data are detrended by the HP filter with a smooth parameter $\eta = 1600$, presented as log deviation from its trend.

Table 2.5: Variable statistics with model simulation, model with risk neutral workers and volatility changes

Model simulation: variable statistics

	u	v	θ	f	a
std.	0.0723	0.0852	0.1454	0.0886	0.0126

corr.	u	v	θ	f	a
u	1	-0.7030	-0.9088	-0.90816	-0.8678
v	-	1	0.9356	0.9357	0.9020
θ	-	-	1	0.9997	0.9598
f	-	-	-	1	0.9582
a	-	-	-	-	1

Note: The model is calibrated and simulated at weekly frequency. Simulated weekly time series are aggregated to quarterly by taking the quarterly average. Aggregated quarterly data are detrended by the HP filter with a smooth parameter $\eta = 1600$*, presented as log deviation from its trend.*

Taking a look at the value of non-market activity, or the value of being unemployment U, for an unemployed worker might be more intuitive. It is straightforward that when the productivity is high, opening a vacancy is more profitable to the firm and therefore wage offers to new employment in the market are relatively higher, and the value of non-market activity is higher if the volatility is lower since given a lower volatility, the economy is going to stay at a high level for some time, and unemployed workers can still expect a good wage payment for the next period.

On the contrary, when the productivity is low, opening a vacancy is less profitable to the firms and the wage offers turn out to be lower. The value of non-market activities to a worker is lower if the volatility is lower because given a lower volatility, the economy is going to stay at a low level for a longer time, the unemployed workers would prefer a higher volatility that would bring the economy to a better state. The effect of volatility is higher if the state of productivity is at an extreme level. The reason behind is that the transition probability matrix is sparse at extreme states. Specifically, it depends on the persistence of the AR(1) process and the volatility of the shocks. Given these two forces over the higher and lower productivity levels, we could expect that value functions with different levels of volatility would cross at the median level.

2.2.4.2 The effect of volatility shocks

It is worth noticing that the property of the value functions crossing at the same point bears a very important implication: the volatility itself (the median state in the plot, where productivity is normalized to equal 1) has no effect on the optimal decision making of the agents, which is a natural result of the model setting given that all agents are risk-neutral.

It is also worth noticing that even though in the figures all the policy functions for different levels of volatility appear to cross at the same point when the level of productivity is at the median state (steady state, $a = 1$), numerically, there are still some subtle shifts that hold them from crossing at the some point. This can also been seen in the impulse response of a one-time volatility shock in Figure 2.4, where we switch the volatility level from its lowest level to the highest for 100 periods.

To explain the tiny shifts in the policy functions and impulse response plots, note that in approximating the processes of the labour productivity, we take $exp(\cdot)$ for each nodes to make sure all of them are positive. Given a random variable following a log-normal distribution, $a_t \sim LN(\mu, \sigma^2)$,

$$E(a_t) = E(e^{\log(a_t)}) = exp\left\{E\left(\log(a_t)\right) + \frac{1}{2} var\left(\log(a_t)\right)\right\}.$$

It is clear that the expectation of a_t is subject to the variance change of $\log(a_t)$. Specifically, by changing the volatility of $\log(a_t)$ while keeping $E\left(\log(y_t)\right)$ constant as we did in our simulation. Imposing a positive volatility shock would in fact increase the productivity level by a tiny amount in our simulation, as we have seen in Figure 2.4. The shift is due to a constraint we have with the approximation

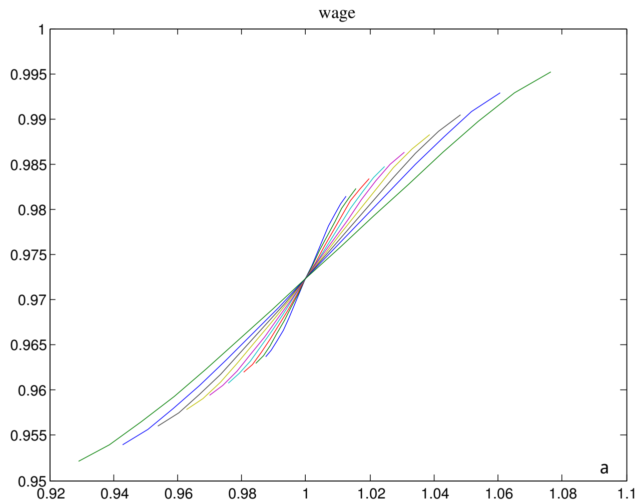

Figure 2.2: Wage function, with risk neutral workers and volatility changes

method and numerically so small that it does not change the conclusion for the neutral effect of a volatility shock at the median state.

However, the implication we have above does not mean a change in productivity volatility will have no effect in the labour market volatility. On the contrary, the effect of a volatility shock comes into effect only with the fact that it changes the range of the levels of productivity that the specific volatility has induced. In fact, this can be seen in the plots of the policy functions. Once we move away from the median state of the labour productivity, differences in the optimal decisions across different levels of the volatility emerge. A model simulation on the effect of volatility change will be presented in the following section.

2.2.4.3 Volatility change and labour market dynamics

In this section, in order to investigate to what extent the observed moderation in the U.S. labour market in previous chapter could be captured by the volatility changes in labour productivity and as well as to determine the effect of volatility change on the labour market dynamics, we conduct a simulation of the model with two separated sections characterized by labour productivity movement with high

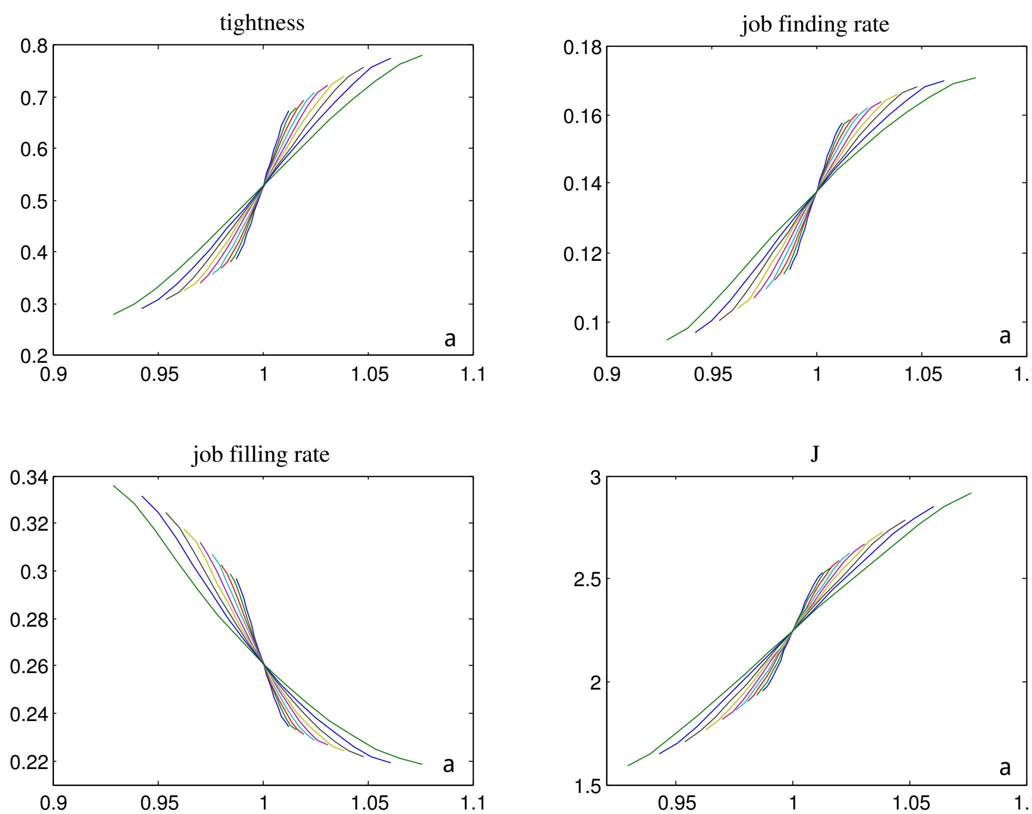

Figure 2.3: Policy and value functions, with risk neutral workers and volatility changes

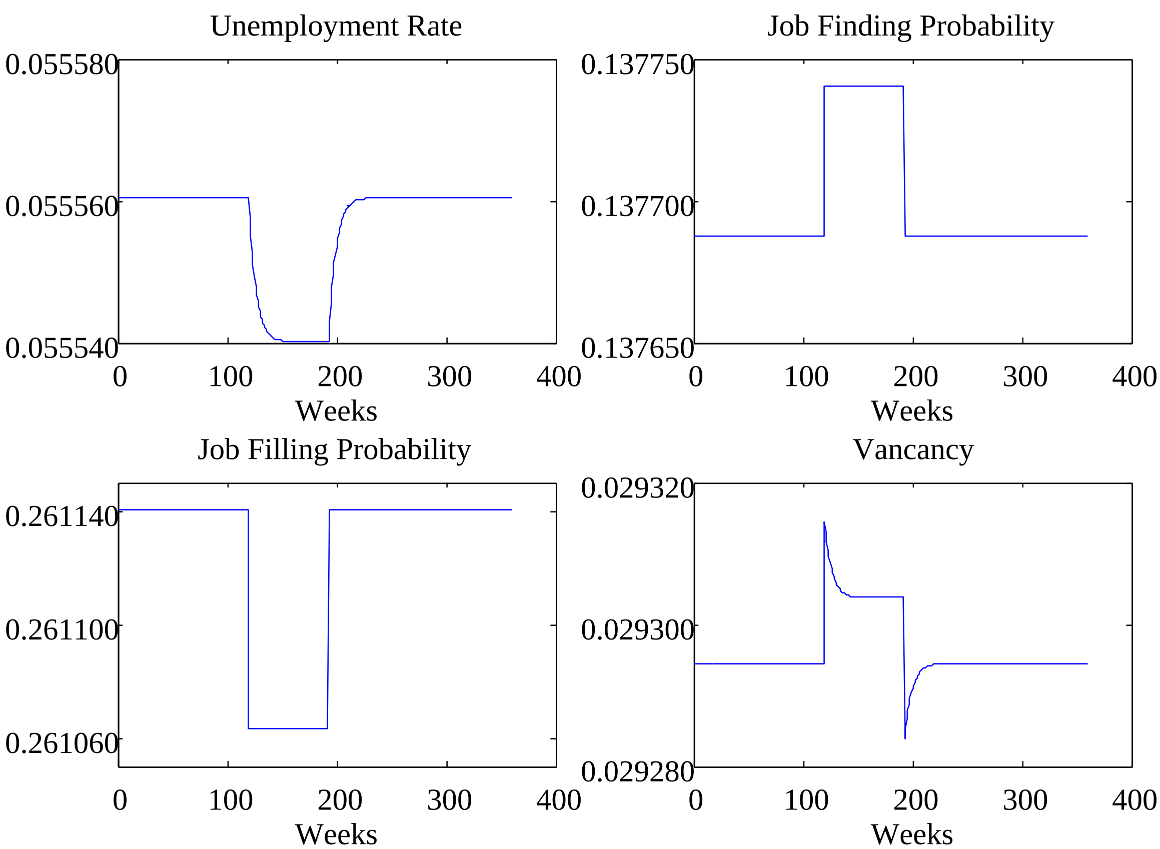

Figure 2.4: Impulse response functions, with a one-time volatility shock.

and low volatility respectively.

In the simulation in this section, each of the two separated sections consists of 200 × 12 periods are kept after 200 × 12 burn-in periods are dropped. The volatility levels are picked so that they are equal to their real data counterparts. Periods in the first section are simulated with high volatility ($se_a = 0.0039$) and the rest 200 × 12 kept periods are those with low volatility ($se_a = 0.0028$)[2]. Table 2.6 presents the statistics of the simulated periods as a whole and those for each individual section.

It can be seen from both the figures and statistics that, while the correlations among the variables do not change much, the economy with a relatively high volatility change is much lower than the one with lower volatility. However, compared with the data, the extended model falls short in capturing the volatility change in the labour market. For example, in the data, the percentage change in volatility in unemployment pre and post 1984 is about - 43.76%, while in the model simulation, -29.44% .

Aside from the brief observation above, it is of more interest that we have a quantitative comparison between the data and model that is more focused on the effect of a volatility shock. As we have mentioned in the previous subsection, though our approach of approximate the stochastic volatility process is convenient for us to calibrate and solve the model, it imposes an important restriction on the way we do an impulse response to the model economy. To further illustrate the effect of volatility shocks, we calculate the elasticity of the volatility of the labour market to the changes in the volatility of the labour productivity. Specifically, for the data, since we have already estimated quarterly stochastic volatility of labour

[2] According to the estimation we have done in the previous section, the mean value of volatility pre-1984 is about 0.0039 and post-1984 is 0.0028.

Table 2.6: Model simulation: high volatility vs low volatility

Model simulation (High volatility) : variable statistics

	u	v	θ	f	a
std.	0.0890	0.1048	0.1789	0.1028	0.013
	(0.0042)	(0.0036)	(0.0075)	(0.0044)	(0.0006)
corr.	u	v	θ	f	a
u	1	-0.7017	-0.9082	-0.9075	-0.9033
		(0.0277)	(0.0119)	(0.0120)	(0.0115)
v	-	1	0.9353	0.9353	0.9267
			(0.0040)	(0.0039)	(0.0048)
θ	-	-	1	0.9997	0.9924
				(0.000)	(0.0023)
f	-	-	-	1	0.9896
					(0.0011)
a	-	-	-	-	1

(a) Model simulation: high volatility periods.

Model simulation (Low volatility): variable statistics

	u	v	θ	f	a
std.	0.0628	0.0736	0.1263	0.0721	0.0093
	(0.0053)	(0.00476)	(0.0096)	(0.0056)	(0.0007)
corr.	u	v	θ	f	a
u	1	-0.7103	-0.9113	-09109	-0.9080
		(0.0293)	(0.0124)	(0.0125)	(0.0122)
v	-	1	0.9371	0.9371	0.9322
			(0.0042)	(0.0042)	(0.0048)
θ	-	-	1	0.9999	0.9954
				(0.000)	(0.0012)
f	-	-	-	1	0.9940
					(0.0008)
a	-	-	-	-	1

(b) Model simulation: low volatility periods.

Note: Aggregated quarterly data are detrended by the HP filter with a smooth parameter $\eta = 1600$, *presented as log deviation from its trend. The standard deviation across 400 model simulation are reported in parentheses.*

productivity, the elasticity of the standard deviation of a certain variable to the volatility change could be calculated as follows,

$$E^{data}_{\cdot sv} = \left[\frac{std(\cdot)_{lw} - std(\cdot)_{hi}}{std(\cdot)_{hi}}\right] / \left[\frac{mean(sv)_{lw} - mean(sv)_{hi}}{mean(sv)_{hi}}\right],$$

For the elasticity in the model, since we fixed the levels of volatility for each section of time periods,

$$E^{model}_{\cdot sv} = \left[\frac{std(\cdot)_{lw} - std(\cdot)_{hi}}{std(\cdot)_{hi}}\right] / \left[\frac{sv_{lw} - sv_{hi}}{sv_{hi}}\right].$$

As we can see in Table 2.7, in a DMP model of labor search model, the effect of a volatility change in the labor productivity fits the data reasonably well to that we observed in the data: the model simulation accounts about 75% of its real data counterpart in the U.S labor market. This observation further justifies that labor productivity could be taken as the driving force of the labor market dynamics in a labor search model.

Table 2.7: Elasticity of labour market dynamics to volatility change in labor productivity

Elasticity of labour market dynamics to volatility change

	u	v	θ	f	a
Data	1.2512	1.0556	1.1709	0.7597	1.1486
Model	0.9712	0.9794	0.9736	0.9711	0.9873

2.2.5 Section summary

In this section, we simulated the dynamic movements pre and post the Great

Moderation in a Diamond-Motensen-Pissarides labour search model characterized with a Hagedorn-Manovskii type of calibration. We take this as a benchmark since the model setting itself is still rather standard, even though the HM calibration in some sense saves the standard DMP model in explaining high volatility in variables of the labour market. It is found in the results in this section that a volatility shock itself has no effect in explaining the labour market dynamics and non-neutral effects only come through the fact that it changes the corresponding levels of actual labour productivity. Moreover, in our model simulation, it is illustrated that the model lacks its ability to take full account of the volatility moderation after mid-1980s.

However in our specific baseline model, the neutral effect of the volatility is actually not so surprising in that all agents are in fact risk-neutral. The result is a natural outcome when the effect of volatility changes in the baseline model is limited due to a lack of a transmission channel. Keeping this consideration in mind, we will try to extend the baseline model by introducing risk aversion into agent's preference so that we can further investigate the effect of volatility shock to the labour market dynamics in a specific perspective.

2.3 Competitive search with risk-sharing contracts

In this section, we investigate the effect of volatility change in a competitive search model with long-term wage contracts. An extension of the model calibration to incorporate volatility shocks is introduced after we present the model. We finish by investigating the numerical implications.

2.3.1 The model

The main framework of the model is a replicate of Rudanko (2009), which itself is an extension of the DMP model. The model is featured firstly by the assumption that in the labour market workers are risk averse while firms are risk neutral. Furthermore, identical workers are assumed to face an incomplete asset market, so they cannot smooth their consumption over time on their own. Therefore, in the model, it is the workers that facing a trade-off between risk and possible lower paid wage contract. These two features developed in Rudanko (2009), following a flavor that could be traced back to Azariadis (1975) and Baily (1974), are based on real life observations that entrepreneurs as they are in their positions are likely less risk averse and usually are wealthier than workers and have better access to asset markets.

Specifically, it is assumed in the model economy that, there is a unit measure of workers with a risk-averse utility function, $E_t \sum_{t=0}^{\infty} \beta^t U(C_{t+1})$, as well as a unit mass of risk neutral firms, each of which can hire only one worker per period. New matches are established through a competitive search framework(Moen, 1997), where active firms post vacancies at the beginning of each period. The establishment of the match between the two sides is characterized by a channel function $M(u, v) = \frac{uv}{(u^\alpha + v^\alpha)^{\frac{1}{\alpha}}}$, in which v is the measure of posted vacancies in the labour market while u is the measure of unemployed workers searching for jobs, and α is the matching function parameter. With this matching function, we define the labour market tightness as $\theta = \frac{v}{u}$, then the job fining probability for the worker could be denoted with $\mu(\theta) = \frac{M}{u} = (1 + \theta^{-\alpha})^{-\frac{1}{\alpha}}$, and the probability of a firm to fill the vacancy therefore is $\eta(\theta) = \frac{M}{v} = (1 + \theta^{\alpha})^{-\frac{1}{\alpha}}$.

The output of firm production in each period is $y = a$ once the post vacancy is filled, $y = 0$ if not, where labour productivity, a, is determined exogenously by a stochastic process. At the time when a match is established, a state-contingent long-term wage contract is signed. Contracts are assumed to be conditional on the aggregate productivity at the time of contracting and specify wages for all continuation histories of a after that, for as long as the match lasts. Each match separates with probability δ at the end of every period.

Productivity and its volatility in the model economy follow the stochastic process described and estimated in Section 2.2.2. Suppose a job match is established at period t and it separates T periods after. Then we will define one specific history of productivity before the separation of the match by $a_t^T = (a_t, a_{t+1}, ..., a_{t+T})$. A wage contract offered by the firm to a worker is a set of functions defined on the realized productivity (a_t) in each period $t < t+T-1$,

$$w(a_t) = \left\{\omega_t(a_t^T) \text{ for all } a_t^T, T = 0, ...\right\},$$

where a_t is the state of the productivity when the contract was offered by the firm.

The value of being employed for a worker offered with a wage contract w is denoted by V therefore is defined as follows,

$$V(w(a_t), a_t) = u(\omega(a_t)) + E_a \sum_{T=1}^{\infty} \beta^T (1-\delta)^{T-1} \left[(1-\rho)u(\omega(a^{t+T})) + \delta V^u(a_{t+T})\right], \quad (2.9)$$

where $(1-\delta)^{T-1}$ is the probability of the match to survive till period T, and $V^u(a_{t+T})$ is the value of unemployment if the match was separated at period $t+T$. The worker enjoys utility $u(\omega(a_t))$ given the period wage payment $\omega(a_t)$. There is a probability δ that the worker and the firm separated by the beginning of the next period, leaving the worker a utility of being unemployed of $V^u(a_{t+T})$. If the match continues at the probability of $(1-\delta)$, the period utility then is given by $u(\omega(a^{t+T})$, which is defined in the wage function $\omega(\cdot)$ given the history of the realized productivity.

Similarly, we can also define the utility value of being unemployed with the state a,

$$V^u(a_t) = u(b) + \beta E_a \left[\mu(\theta(a_{t+1})V(w(a_{t+1}), a_{t+1}) + (1-\mu(\theta(a_{t+1})))V^u(a_{t+1})\right]. \quad (2.10)$$

The unemployed worker enjoys utility of consuming the unemployment benefit b. At the mean time, the probability that the worker finds a job is $\mu(\theta(a_{t+1})$ in the next period. The value of offered contract is then $V(w(a_{t+1}), a_{t+1})$ with the updated state a_{t+1}. There is also a probability of $(1 - \mu(\theta(a_{t+1})))$ that the worker will find a job in the next period, leaving him a value of being unemployed $V^u(a_{t+1})$.

For the firm, the present value of profits from a filled vacancy is as follows,

$$F(w(a_t), a_t) = a - \omega(a_t) + \beta E_a(1 - \delta)F(\omega(a_{t+1}), a_{t+1}), \tag{2.11}$$

which indicates the return to the firm offering a contract $w(\cdot)$ for a filled vacancy is $a - \omega(a_t)$ and there is a probability $(1 - \delta)$ that the match will continue in the next period.

Given that vacancies are opened at a fixed cost c for the firm, and the free entry of the firms drives the value of filling a vacancy down to zero, we have the free entry condition in the equilibrium such that

$$\eta(\theta(a_t))F(w(a_t), a_t) = c. \tag{2.12}$$

Rudanko (2009) discusses the competitive search labour market with three different cases: full-commitment, 1-sided limited commitment, and 2-sided limited commitment. To make our analysis simple, especially to keep the model computation easier to implement, we assume that both sides make full commitments when a wage contract is reached, i.e., matches can only break up with exogenous separation shocks and neither side could leave the match on their own decisions.

When there are full commitments from both sides of matched pairs of workers and firms, given $V \geq V^u$, the efficient contract specifies a constant wage for all periods that the match exists (Rudanko, 2009). The intuition is straightforward: given that the workers are risk averse and firms are risk neutral, it must be a constant wage in the Pareto efficient contract so that the workers can smooth their consumption over periods. In other words, the contract wage is independent from the state of the productivity in the duration of the existing match; it is the state-of-the-world when the contract is established that matters in determine the optimal wage. Therefore, we can solve the optimal contract problem for the firm with the market tightness θ and constant period wage ω as the control variables,

$$\max_{\{\omega,\theta\}} \eta(\theta)F(\omega, a),$$

s.t

$$\mu(\theta)V(\omega, a) + (1 - \mu(\theta))V^u(a) = R(a).$$

And, we have the following first order conditions for solving the optimal contract problem,

$$\begin{aligned} \eta'(\theta)F(\omega, a) - \lambda_t \left[\mu'(\theta)V(\omega, a) - \mu'(\theta)V^u(a)\right] &= 0 && (2.13) \\ \eta(\theta)F'(\omega, a) - \lambda_t\mu(\theta)V'(\omega, a) &= 0, && (2.14) \end{aligned}$$

where λ_t is the Lagrange multiplier. Rearranging these two equations yields

$$\frac{\mu(\theta)\eta'(\theta)F(\omega, a)}{\mu'(\theta)\eta(\theta)F'(\omega, a)} = \frac{[V(\omega, a) - V^u(a)]}{V'(\omega, a)}.$$

That is,

$$V(a, V^u) - V^u(a) = -\frac{\eta'(\theta)\mu(\theta)}{\eta(\theta)\mu'(\theta)}F(w(a), a) \cdot u(w). \quad (2.15)$$

2.3.2 An extension with volatility changes

So far we have not introduced the specific form for volatility shocks in the model economy yet. When compared with the case with fixed volatility, since the element incorporated here is exogenous to the previous baseline model, even though it changes agent's structure of expectation when making their optimal decisions, the framework of the extended model holds unchanged. Nevertheless, numerically, the system now becomes considerably more complicated to solve, which also causes some difficulties in the numeral analysis as we will discuss in the following section.

By introducing volatility changes in labour productivity, the state variables of the extend model economy now consist of two elements: the observed labour productivity level a and the stochastic volatility of behind a, denoted by sa. Same as the approach adopted in the Section 2.2.3, we use the Markov chain method described in Tauchen (1986) to approximate the stochastic volatility process. Denote the number of volatility states with m and the number of productivity states with n. We have a $m \times m$ transitional matrix, π_{sa}, for the process of realization of volatility shock as described in Equation (2.6), and given a specific volatility level, we approximate the process of the labour productivity based on that level of volatility, and the transitional probability for that approximated process is denoted by a $n \times n$ matrix π_a. Note that since these two shocks are assumed to be independent, transitional matrices for each labour productivity process based on one certain level of stochastic volatility are identical.

Given the setup of the model described in previous section, we can now impose the volatility changes into the model with notations defined above. Equation (2.11) now can be written in the form of vectors as:

$$\begin{aligned}
F(\omega(a_t, sa_t), a_t, sa_t) &= a_t - \omega(a_t, sa_t) + \beta E(1-\delta)F(\omega(a_{t+1}, sa_{t+1}), a_{t+1}, sa_{t+1}) \\
&\Longleftrightarrow \\
F &= a - w + \beta E(1-\delta)F \\
F &= a - w + \beta(1-\delta)\pi_{sa} F \pi_a', \qquad (2.16)
\end{aligned}$$

where π'_a is the transpose matrix of π_a. Given that we now have two state variables in the extended model, F is a three dimension matrix $(n \times n \times m)$, which is the same case for other value functions.

In order to solve F in Equation (2.16), we see it as a Lyapunov Equation, which could be solved such that

$$vec(F) = (I \otimes A' + B' \otimes I)^{-1} vec(C), \tag{2.17}$$

where $A = \pi_{sa}^{-1}$, $B = -\beta(1-\delta)\pi_a$, $C = {\pi_{sa}}^{-1}(a - w)$, operator $vec(\cdot)$ stacks all the columns of the matrix in it[3], $\otimes$ is the Kronecker product and I is the identity matrix.

The above induction also applies to Equation (2.9). So similarly we have

$$V^u = u(b) + \beta\left[(1-\mu(\theta))\pi_{sa} V^u \pi'_a + \mu(\theta)\pi_{sa} V \pi'_a\right]$$

and,

$$\begin{aligned} V &= u(\omega) + \beta(1-\delta)\pi_{sa} V \pi'_a + \beta\delta\pi_{sa} V^u \pi'_a, & (2.18) \\ vec(V) &= (I \otimes K' + L' \otimes I)^{-1} vec(M), & (2.19) \end{aligned}$$

where $K = \pi_{sa}^{-1}$, $L = -\beta(1-\delta)\pi_{sa}$ and $M = \pi_{sa}^{-1}(u(w) + \beta\delta\pi_{sa} V^u \pi_a)$.

To sum up the work above, the dynamics of the extended model could be rep-

[3]In particular, in our cases, the $vec(\cdot)$ operator produces an $(n * n * m) \times 1$ matrix.

resented by the following system of equations:

$$\begin{aligned}
vec(F) &= (I \otimes A' + B' \otimes I)^{-1} vec(C) \\
vec(V) &= (I \otimes K' + L' \otimes I)^{-1} vec(M) \\
V^u &= u(b) + \beta \left[(1 - \mu(\theta)) \pi_{sa} V^u \pi_a' + \mu(\theta) \pi_{sa} V \pi_a' \right] \\
V(a, V^u) - V^u(a) &= -\frac{\eta'(\theta)\mu(\theta)}{\eta(\theta)\mu'(\theta)} F(w(a), a) \cdot u(w) \\
c &= \eta(\theta) F(w(a), a)
\end{aligned}$$

2.3.3 Calibration

To introduce risk averseness into our analysis, we use a CRRA utility function for the workers. Specifically, it follows $U(C) = \frac{C^{1-\gamma}}{1-\gamma}$, where C is the period wage ω if the worker is employed or it is the unemployment benefit b if he/she is unemployed, and γ is the utility function parameter. Following many in the literature, we set $\gamma = 2$.

Same as we had for the extended Hadgedorn-Manovskii model in Section 2.2.3, the model in this section is also calibrated at the weekly frequency. Unlike the HM calibration, model parameters are set following a standard approach. Unemployment benefit for the unemployed is set equal to 0.4, which is also used in Shimer (2005). Job separation rate is set to be equal to 0.0083 based on the data. Posting cost c and matching parameter α are calibrated to match the probability of the firms to fill the job vacancies η and labour market tightness θ. Table 2.8 shows all

Table 2.8: Parameter calibration, model with risk averse workers and volatility changes

Parameters values in model calibration

Parameter	Description	Value
δ	job separation rate	0.0083
b	value of being unemployment	0.4
β	discount factor	0.9987
γ	utility function parameter	2
ρ	persistence of weekly labour productivity	0.9774
ρ'	persistence of weekly stochastic volatility	0.9911
σ_{se}	standard deviation of stochastic volatility	0.0031

Table 2.9: Variable statistics with model simulation, model with risk averse workers and volatility changes

Model simulation: variable statistics

	u	v	θ	f	a
std.	0.0445	0.0516	0.0878	0.0522	0.0126
corr.	u	v	θ	f	a
u	1	-0.6664	-0.8985	-0.8983	-0.8567
v	-	1	0.9260	0.9260	0.8948
θ	-	-	1	0.9999	0.9605
w	-	-	-	1	0.9600
a	-	-	-	-	1

Note: The model is calibrated and simulated at weekly frequency. Simulated weekly time series are aggregated to quarterly by taking the quarterly average. Aggregated quarterly data are detrended by the HP filter with a smooth parameter $\eta = 1600$*, presented as log deviation from its trend.*

the model parameters calibrated.

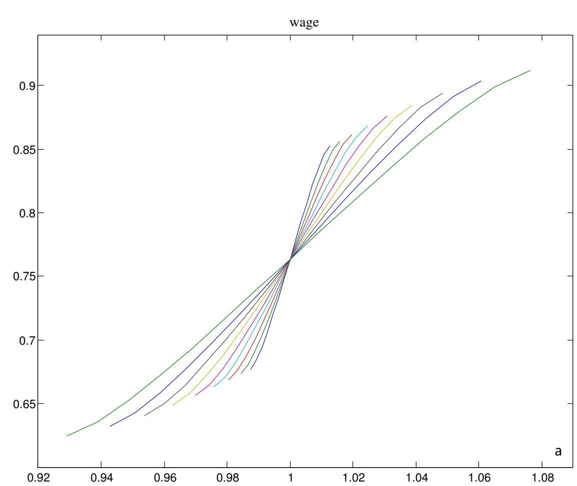

Figure 2.5: Wage function with risk-averse workers and volatility changes

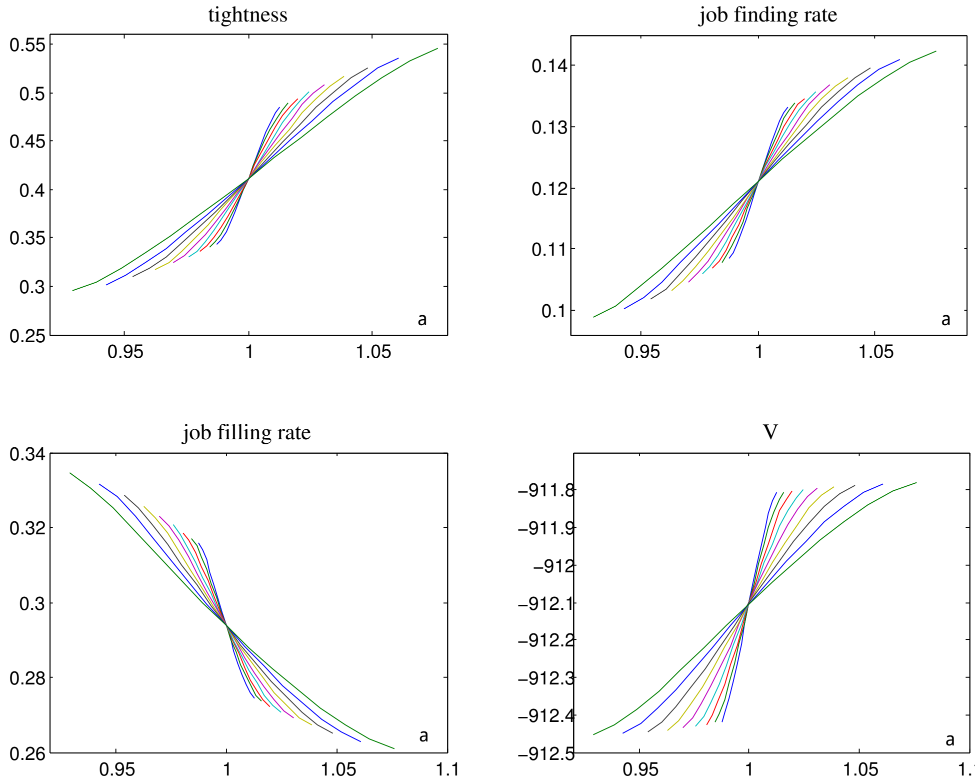

Figure 2.6: Value functions with risk averse workers and volatility changes

2.3.4 Results

As we can see from Figure 2.5, 2.6 and 2.7, the shape of the policy functions and impulse response plots are very similar to those we have in Section 2.2.4.3. Though there are some differences in the slops of the functions due to different model structures and calibrations used, the intuition behind this finding is the same as we have discussed in previous sections. Specifically, it could been seen that a median-preserving volatility shock to the labour productivity in the model simulation has no effect, though there are some tiny numerical shifts given the approximation method applied. Aside from the similarities, we could also see some differences in the numerical performance of the two models.

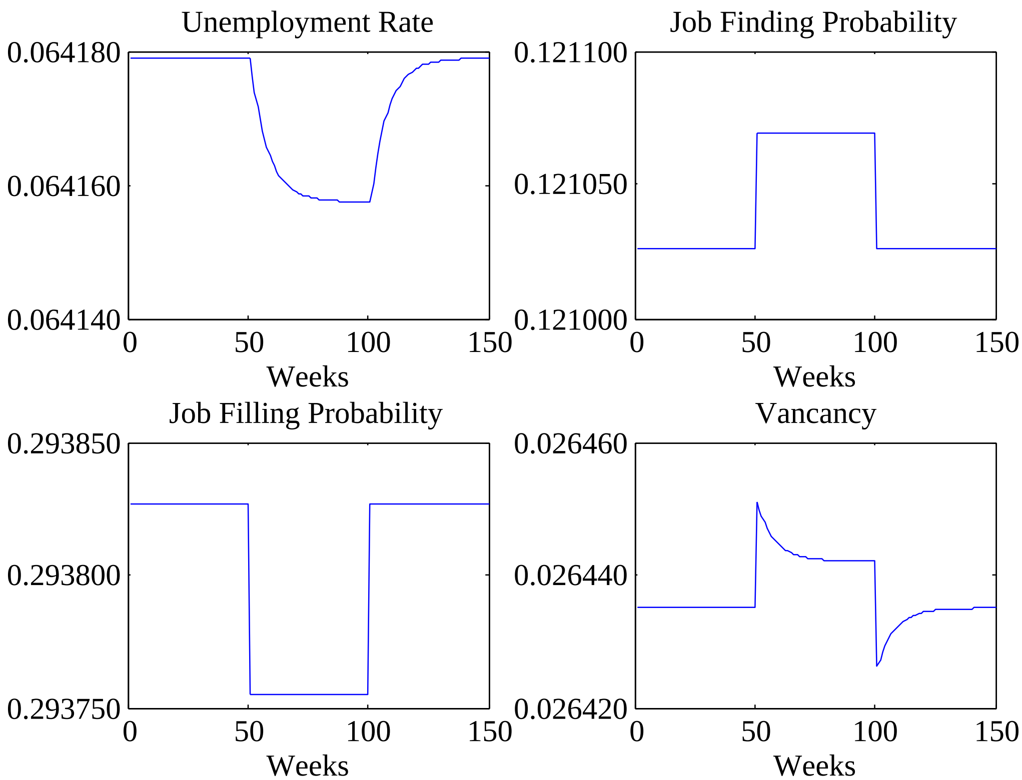

Figure 2.7: Impulse response functions, with a one-time volatility shock.

2.3.4.1 Volatility change and labour market dynamics

Similar as we have done with the extended Hagedorn and Manovskii model in Section 2.2.4.3, we here run a simulation of the model with period sections of both high and low stochastic volatility. We also calculate the elasticity of the labour market dynamics to the changes in the volatility based on the simulation. Table 2.10 presents the simulation results. Compared to the results with the Hagedorn and Manovskii setting, the estimated elasticity of labour market dynamics to the volatility change in labour productivity is similar in its magnitude. In both cases, the elasticity of a change in volatility of labour productivity is close to what we observed in the data.

2.3.4.2 Relative volatility

Table 2.9 presents the variable statistics with the model simulation. It is notable that compared to the results either from Shimer (2005) or Rudanko (2009), volatility of the model incorporating volatility changes is higher. Specifically, the relative volatility of unemployment to that of labour productivity ($\frac{\mathrm{std}(u)}{\mathrm{std}(p)}$) is now 3.5212, while in Shimer (2005), the volatility of unemployment is only about 10% of that of labour productivity, and it is achieved when the benefit of unemployment (b) is about as high as 0.8 while in Rudanko (2009) it is only set to equal to 0.4. Therefore, from the numerical simulation above, we could see a better performance of the extended version of Rudanko (2009) in capturing the volatility changes in the labour market.

2.3.4.3 Discussion

As we have seen in Section 2.2, it is not so surprising that volatility changes introduces only a limited effect while it is in a model where all agents are risk neutral, even though the expectation structure itself has been changed. However, the analysis in this section presents some results contrary to a "convenient" expectation that an introduction of risk-averse agents would provide a potential channel for the volatility change such that uncertainty matters in the model economy. The reason why these two extended models generate quite similar results might be a little bit more subtle than that. Intuitively, it is true that we introduce risk-averse workers in our extended model in this section, however it is also assumed in the economy that risk-averse workers can fully insured by freely entering risk-neutral firms. In fact, it is the risk-sharing setting of the Rudanko (2009) framework that matters here. Given that, it is necessary we turn to examine further whether some characteristics of the firm's side could explain the neutral effects of volatility changes we have seen here. Chapter 3 will discuss the mechanism analytically with more details in a simplified one-period model.

2.4 Conclusion

This chapter investigates how exogenous volatility changes in labour productivity affect labour market dynamics in frictional labour search models. We start with a baseline model following a Hagedorn and Manovskii (2008) type of calibration. After that, we further our analysis with a risk-sharing competitive search model with long-term full-commitment contracting following Rudanko (2009), where workers are risk averse and firm are risk neutral.

It is found that while these two models inherit and in some aspects improve the relatively good performance of the Diamond-Mortensen-Pissarides models in capturing basic characteristics of the labour market with the respect to the moderation in its dynamics, the incorporation of stochastic volatility shocks to the labour productivity shows quantitatively limited effect on the movements of the key variables in the model. Specifically, the two models fall short in capturing realistic volatility in unemployment and market tightness, which is consistent with the previous findings in the literature. However, for the risk-sharing model, introducing the stochastic volatility provides a potential approach to produce more realistic volatility changes in the dynamics of the U.S. labor market.

Besides, it is also found with both of the extended models that a mean-preserving volatility shock in labour productivity introduces no effect on the optimal decision made by agents. A volatility change can only have non-neutral effects in the labour market in the sense that it changes the range of the corresponding levels of productivity that the specific volatility has induced.

Overall, the result that the volatility changes introduces no non-neutral effect in

Table 2.10: Model simulation: high volatility vs low volatility

Model simulation (High volatility) : variable statistics

	u	v	θ	f	a
std.	0.0486	0.0563	0.0961	0.0568	0.0130
	(0.0041)	(0.0034)	(0.0072)	(0.0043)	(0.0010)
corr.	u	v	θ	f	a
u	1	-0.6876	-0.9051	-0.9050	-0.9037
		(0.0295)	(0.0128)	(0.0128)	(0.0126)
v	-	1	0.9309	0.9309	0.9294
			(0.0041)	(0.0041)	(0.0043)
θ	-	-	1	0.9999	0.9983
				(0.0000)	(0.005)
f	-	-	-	1	0.9981
					(0.0006)
a	-	-	-	-	1

(a) Model simulation: high volatility periods.

Model simulation (Low volatility): variable statistics

	u	v	θ	f	a
std.	0.0331	0.0384	0.0658	0.0384	0.0093
	(0.0026)	(0.0047)	(0.0027)	(0.0007)	(0.0019)
corr.	u	v	θ	f	a
u	1	-0.6876	-0.9051	-0.9050	-0.9037
		(0.0295)	(0.0128)	(0.0128)	(0.0126)
v	-	1	0.9309	0.9309	0.9294
			(0.0041)	(0.0041)	(0.0043)
θ	-	-	1	0.9999	0.9983
				(0.0000)	(0.005)
f	-	-	-	1	0.9981
					(0.0006)
a	-	-	-	-	1

(b) Model simulation: low volatility periods

Note: Aggregated quarterly data are detrended by the HP filter with a smooth parameter $\eta = 1600$, *presented as log deviation from its trend. The standard deviation across 400 model simulation are reported in parentheses.*

Table 2.11: Elasticity of labour market dynamics to volatility change in labor productivity

Elasticity to volatility change

	u	v	θ	f	a
Data	1.2512	1.0556	1.1709	0.7597	1.1486
Model	1.1103	1.1155	1.1025	1.1313	0.9961

the two models we investigated where all agents are risk neutral is not surprising, though the case with the risk averse setting is more subtle. Intuitively, it is the risk-sharing setting of the Rudanko (2009) framework that smooths out the introduced volatility. The results presented in this chapter shows that it is necessary we turn to examine further how some characteristics of the firm's side might provide potential valid explanation.

Appendix

Methodology for weekly estimation

We estimate the weekly parameters exploiting the specific parameters and the weekly and quarterly processes assumed.

After priors for relevant parameters to be estimated are settled, following steps are taken in the algorithm:

1. First, pick the estimated quarterly volatility in the first quarter as the initial point $\kappa_{w,0}$. The first quarterly observation of the detrended log deviation of the labour productivity is taken as the starting point for estimating the "underlying" weekly process, $a_{w,0}$. Alternative initial values are used and dose not affect the results to a sensible extent;

2. Given the specified weekly process, weekly data are generated by the prior distributions with undetermined parameters;

3. Quarterly estimations of the data then are constructed with the weekly data. It is assumed the quarterly estimations follows a normal distribution with a precision of $\frac{1}{\tau^2}$.

4. Undetermined parameters are pinned down by fitting the quarterly date generated to real data with the precision parameter $\tau^2 = 1e + 7$.

Chapter 3

Uncertainty and Job Creation

3.1 Introduction

In Chapter 2, we have investigated two settings where the incorporation of volatility shocks to the labour productivity has only limited effect to the dynamics of the labour market with a focus on the numerical aspect. This chapter provides a deeper understanding in the baseline model of equilibrium unemployment theory about the effect that changes in the volatility of labour productivity has on the job creation margin.

It was shown in the previous chapter that changes in the volatility of labour productivity can affect the volatility of the unemployment rate and labour market tightness. However, it was also shown that it is not uncertainty about future labour productivity that drives this result. The expected volatility change had virtually no effect on job creation. Rather it is the increase volatility in realized labour productivity that mattered for job creation.

In this chapter, we show further why this result is obtained and explore how changes to the baseline Pissarides model can cause increased expected volatility to affect job creation. Specifically, we first present a simple one-period model featuring with multiple workers in both a baseline Stole and Zwiebel (1996) bargaining game and an ad hoc sticky wage setting that follows Michaillat (2012). After that, we also investigate a situation similar to Rudanko (2009) with risk-averse workers signing risk-sharing contracts with risk-neutral firms.

It is shown in the result that if the firm's profit function is non-linear in labour productivity then changes in expected volatility affect the expected value of a filled job vacancy and this causes firms to create more job vacancies. Models that simply add concavity in the production function via diminishing returns to labour inputs do not work as the profit function is still linear in labour productivity. In both the standard multi-worker firm model with intrafirm wage bargaining following Stole and Zwiebel (1996) and models such as that of Rudanko (2009) in which workers are risk-averse such that firms offer risk-sharing contracts, their profit functions of the firms turn out to be linear in labour productivity. Instead, theoretically, a model with sticky wages such as that of Michaillat (2012) is sufficient to obtain the desired result.

The rest of the chapter rolls out as follows, in Section 3.2 a random matching model where firms hire multiple workers is considered. After that, in Section 3.3 we investigate the case where the workers are risk-averse and firms are risk-neutral in a framework of risk-sharing contracts. Section 3.4 concludes.

3.2 Random matching with multi-worker firms

In this section, we present a simple one-period model featuring random matching and firms hiring multiple workers in two scenarios: a baseline model with Stole and Zwiebel (1996) bargaining game for wage determination and an alternative with a sticky wage setting following Michaillat (2012). Within the baseline model economy with diminishing returns to labour input, it is shown that in the Stole and Zwiebel

(1996) bargaining setting, a volatility change in labour productivity may introduce no effect on job creation given that the profit function of the firms are linear, while with the Michaillat (2012) ad hoc sticky wage setting, it could have a non-neutral effect.

3.2.1 Intrafirm wage bargaining

We start our analysis with a simple baseline model where firms could hire multiple workers and wages are set via a Stole-Zwiebel bargaining game (Stole and Zwiebel, 1996). This section lays out the baseline model and show how the wage is determined within it.

In the model economy, there are a unit mass of identical workers searching for jobs. Firms are identical and enter freely. It is also assumed that each firm could post multiple job vacancies, v_i, and hire workers after observing the realization of the total factor productivity, a.

The value function of hiring to the firms could be denoted by

$$J(n_i) = E\left[f\left(n_i, a\right) - w\left(n_i, a\right) n_i\right] - \kappa v_i,$$

where v_i is the number of vacancies posted, n_i is the total number of workers hired, and κ is the fixed cost of posting one vacancy in each period. Since all firms are identical, firm's employment is related to the aggregate measure of unemployment, u, and vacancy, v, via $n_i = v_i\mu(\theta)$, where $\theta = \frac{v}{u}$ is the labour market tightness.

Taking the derivative of $J(n_i)$, we have the firm's optional hiring policy such that

$$\kappa = \mu\left(\theta\right) E\left[f_n(n_i, a) - w_n(n_i, a)n_i - w(n_i, a)\right]. \tag{3.1}$$

Now we assume that wages are determined via a Stole-Zwiebel bargaining game. We define $\tilde{J}(n_i, a, \epsilon)$ as the value of employing n_i workers at the Stole-Zwiebel bargaining wage and an additional ϵ workers at a wage $\bar{w}$,

$$\tilde{J}(n_i, a, \epsilon) = f(n_i + \epsilon_i, a) - w(n_i + \epsilon, a)(n_i + \epsilon) - w(n_i + \epsilon, a) - \bar{w}\epsilon.$$

Define the derivative $\tilde{J}'(n_i, a)$ as

$$\begin{aligned} \tilde{J}'(n_i, a) &= \lim_{\epsilon \to 0} \left[\frac{\tilde{J}(n_i, a, \epsilon) - \tilde{J}(n_i, a)}{n_i + \epsilon - n_i} \right] \\ &= f'(n_i, a) - w_n(n_i, a) - \bar{w}. \end{aligned} \tag{3.2}$$

Given the Stole-Zwiebel bargaining, it is required that the wage is obtained such that

$$\max_{\bar{w}} (\bar{w} - b)^{\xi} (f'(n_i, a) - w_n(n_i, a) - \bar{w})^{1-\xi},$$

where $\xi \in (0, 1)$ is the bargaining power of the worker. Solving the wage function gives the outcome of the bargaining,

$$w(n_i, a) - b = \left(\frac{\xi}{1 - \xi} \right) \tilde{J}'(n_i, a).$$

We can rewrite the the above equation by inserting Equation (3.2) so that,

$$w_n(n_i, a) + \left(\frac{1}{\xi n_i} \right) w(n_i, a) = \frac{1}{n_i} \left[f'(n_i, a) + \left(\frac{1 - \xi}{\xi} \right) b \right].$$

Solving the differential equation above with $w_i(0, a) = 0$, we have the wage determination function as follows,

$$w(n_i a) = \left(\frac{1}{n_i} \right)^{\frac{1}{\xi}} \left\{ \int_0^{n_i} \bar{n}^{\frac{1-\xi}{\xi}} \left[f'(n_i, a) + \left(\frac{1 - \xi}{\xi} \right) b \right] d\bar{n} \right\}. \tag{3.3}$$

3.2.2 Sticky wages

As a comparison of the baseline model in which wage is determined via an intrafirm bargaining, we also introduce an alternative setting of the wage function following (Michaillat, 2012). It is assumed that a sticky wage function is given by $w(n_i, a) = \omega a^{\phi}$, with a $\phi \in (0, 1)$ and $\omega \in (0, +\infty)$. Given that $\phi < 1$, the wage of employed workers can only partially adjust to the movement of productivity. Inserting this assumed wage function into Equation (3.1), the optimal hiring policy then is given by

$$\kappa = \mu(\theta) E \left[f_n(n_i, a) - \omega a^{\phi} \right]. \tag{3.4}$$

We go back to this setting in next section where we investigate the effect of uncertainty shocks in both the intrafirm bargaining setting and sticky wage setting, and see how these two differ from each in inducing the movement of job creation when driven by volatility changes in productivity.

3.2.3 The effect of uncertainty shocks

We continue our analysis by laying out the model for the production side of the economy. Also following Michaillat (2012), consider a special case for the production function of the firms such that $f(n_i, a) = a n_i^{\alpha}$. According to this setting, firms in the model economy produce outputs with diminishing returns to the labour input. In Michaillat (2012), the setting with both rigid wage and diminishing returns is employed to model the job rationing in the labor market.

Therefore in the Stole and Zwiebel (1996) bargaining setting, the wage determination function, Equation (3.3), simply turns out to be

$$w(n_i, a) = \left[\frac{\alpha \xi}{1 - (1 - \alpha)\xi} \right] a n_i^{\alpha - 1} + (1 - \xi) b,$$

which could be rearranged as follows,

$$w_n(n_i, a) = \left[\frac{\alpha(\alpha-1)\xi}{1-(1-\alpha)\xi}\right] a n_i^{\alpha-2}. \tag{3.5}$$

Insert the wage function above into firm's optional hiring policy function, Equation (3.4), we have

$$\kappa = \mu(\theta) E\left[\left(\frac{(1-\xi)\alpha}{1-(1-\alpha)\xi}\right) a n_i^{\alpha-1}\right] - \mu(\theta)(1-\xi)b. \tag{3.6}$$

With the above equation, we can solve the optimal hiring number of workers, n_i.

With all the preparation work above, we can now have a look at how an uncertainty shock can affect the job creation in both the baseline and sticky wage models. To make our analysis simpler, we assume the productivity fall in a set of three states: the high, low and median state, denoted by $a \in \{a_l, \bar{a}, a_h\}$, where $\bar{a} - a_l = a_h - \bar{a} = \Delta$. Let π_l, π_h and π be the probability of being in the low, high and media states, respectively. Assume that $\pi_l = \pi_h = \frac{1-\pi}{2}$. Therefore, the expectation of the productivity is equal to its median level, $\bar{a}$:

$$E(a) = \frac{1-\pi}{2} a_l + \pi\bar{a} + \frac{1-\pi}{2} a_h = \frac{1-\pi}{2}\left[\Delta + \bar{a} + \bar{a} - \Delta\right] + \pi\bar{a} = \bar{a}$$

and the standard deviation of productivity could be denoted by $\sigma_a^2 = E\left[(a - E(a))^2\right] = (1-\pi)\Delta^2$. Note that the standard deviation σ_a^2 can be increased by decreasing π or by increasing Δ^2 while holding $E(a)$ constant.

Given that in the time line of the model economy, n is set before the firms observe the realization of a. In the baseline model, the job creation condition of the baseline model is given by the equation within the symmetric equilibrium:

$$\kappa = \mu(\theta) E(a) \left[\left(\frac{(1-\xi)\alpha}{1-(1-\alpha)\xi}\right) n^{\alpha-1}\right] - \mu(\theta)(1-\xi)b.$$

As the above condition shows, the determination of the optimal hiring policy of the firm depends only on the median state of the technology a. Therefore, it could be concluded that in the baseline model with intrafirm bargaining wage, changing the variance of a while holding its mean constant has no effect on job creation.

On the contrary, with a sticky wage setting following Michaillat (2012), after inserting the production function into Equation (3.4), the job creation condition in a symmetric equilibrium is

$$\kappa = \mu(\theta)E(a)\alpha n_i^{\alpha-1} - \mu(\theta)\omega E(a^{\phi}),$$

where

$$\begin{aligned} E(a^{\phi}) &= \frac{1-\pi}{2}a_l^{\phi} + \pi\bar{a}^{\phi} + \frac{1-\pi}{2}a_h^{\phi} \\ &= \frac{1-\pi}{2}(\bar{a}-\Delta)^{\phi} + \pi\bar{a}^{\phi} + \frac{1-\pi}{2}(\bar{a}+\Delta)^{\phi}. \end{aligned}$$

Therefore, $E(a^{\phi})$ is a function of both π and Δ. Note that the volatility of a, σ_a^2, is defined as $\sigma_a^2 = E\left[(a - E(a))^2\right] = (1-\pi)\,\Delta^2$. A change in the volatility of a could affect $E(a^{\phi})$ and thereafter the job creation via changes in either π or Δ.

It can be shown that

$$\frac{dE(a^{\phi})}{d\pi} = \bar{a}^{\phi} - \frac{(\bar{a}-\Delta)^{\phi} + (\bar{a}+\Delta)^{\phi}}{2}.$$

It is clear when $\phi = 1$, $\frac{dE(a^{\phi})}{d\pi} = 0$. So given that in sticky wage function $\phi \in (0,1)$ and $\bar{a} - a_l = a_h - \bar{a}$, we have $\bar{a}^{\phi} > \frac{1}{2}(\bar{a}-\Delta)^{\phi} + (\bar{a}+\Delta)^{\phi}$. It implies that $\frac{dE(a^{\phi})}{d\pi} > 0$. As for the case of $\frac{dE(a^{\phi})}{d\Delta}$,

$$\frac{dE(a^{\phi})}{d\Delta} = \frac{1-\pi}{2}\phi[(\bar{a}-\Delta)^{\phi-1} + (\bar{a}+\Delta)^{\phi-1}],$$

the relation is not as straightforward as with that of π. For $\phi < 1$, with economically valid values of Δ, that is $\Delta < \bar{a}$, $\frac{dE(a^{\phi})}{d\Delta} > 0$.

Therefore, in either cases, within the Michaillat (2012) setting of wage stickiness and diminishing returns to labor that induces a nonlinear profit function with respect to labor productivity, a change in volatility of productivity could have a non-neutral effect on the determination of the job creation.

3.3 A model with risk-sharing contract

We now move forward our analysis in previous section to a different situation where the workers are risk-averse and firms are risk-neutral in a framework of risk-sharing contracts among them, which is also the essential framework in Rudanko (2009) we adopted in the previous chapter.

Same as in the previous section, for simplicity, the model economy is assumed to last for only one period. There is a unit measure of identical workers. Identical firms enter into the labour market freely. All workers are unemployed at the bargaining of the period. The two sides are matched via a competitive search. The matching technology is defined by $m(u, v)$, where u and v are respectively the measures of unemployment and vacancies. The tightness of the labour market is denoted as $\theta = \frac{v}{u}$.

In the model economy, each firm can only hire one worker by posting a contract for the vacancy for each possible level of productivity, a. When a firm and a worker are matched, a units of output occur. It is also assumed that firms post their contracts at a cost of κ and the unmatched workers receive unemployment benefits of b.

3.3.1 Optimal wage contract

We define V as the promised utility of a contract to a worker. The representative firm determines its optimal wage by solving the following problem:

$$\max_{w(a)} \left\{ \int (a - w(a))\, dF(a) \right\}$$

subject to the constraint

$$\int u(w(a)) dF(a) = V.$$

The Lagrangian for solving the optimal problem therefore comes as follows

$$\mathcal{L}(w) = \int \int (a - w(a))\, dF(a) + \lambda \left[V - \int u(w(a)) dF(a) \right], \tag{3.7}$$

where λ is the multiplier on the constraint.

Imposing a perturbation of an increment α in direction h into the Lagrangian, we can rewrite Equation (3.7) in the following form

$$\mathcal{L}(w, h, \alpha) = \int (a - (w(a) + \alpha h(a)))\, dF(a) + \lambda \left[V - \int u(w(a) + \alpha h(a)) dF(a) \right].$$

To solve the problem, we have the first order condition with respect to $w(a)$ such that

$$\int \left[-1 - \lambda u'(w(a)) \right] h(a) dF(a) = 0. \tag{3.8}$$

For this optimality condition to hold for any feasible function $h(a)$, it must be that $\lambda = -\frac{1}{u'(w(a))}$ for all possible value of a. This implies $w(a) = \bar{w}$, i.e., contract wage w is some constant $\bar{w}$ but not a function of a, and thus the firm fully insures its worker in the contract. The value of the contract to the worker therefore could be denoted by

$$V = \int u(\bar{w}) dF(a) = u(\bar{w}), \tag{3.9}$$

which is only decided by the level of the constant contract wage.

Let $J(V)$ be the value of a contract V to the firms. It is obvious that

$$J(V) = \int (a - w(V))dF(a),$$

where $w(V)$ is the contract wage decided according to Equation (3.9), which is the expected surplus of the output subtracted by the contract wage.

Denote U^* as the market utility offered to the worker in the equilibrium. The optimal problem for the firms drops as

$$\max_{V,\theta} \left\{ \mu(\theta) \int (a - w(V))\, dF(a) \right\},$$

subject to the constraint

$$\eta(\theta)u(w(V)) + (1 - \eta(\theta))u(b) = U^*. \tag{3.10}$$

As the optimal wage being set within the contract by $\bar{w} = u^{-1}(V)$, we could rewrite the constraint to yield

$$\bar{w} = u^{-1}\left(\frac{U^* - (1 - \eta(\theta))u(b)}{\eta(\theta)}\right). \tag{3.11}$$

Imposing the above equation and $\eta(\theta) = \theta\mu(\theta)$ into the objective function, the problem of the firm could then be rewritten as

$$\max_{\theta} \left\{ \frac{\eta(\theta)}{\theta} \int (a - \bar{w}(\theta))dF(a) \right\},$$

with $\bar{w}(\theta)$ defined by Equation (3.11). Derive the first-order condition of the problem, and we have

$$\frac{\theta\eta'(\theta) - \eta(\theta)}{\theta^2} \int (a - \bar{w}(\theta))dF(a) - \frac{\eta(\theta)}{\theta} \int \frac{d\bar{w}(\theta)}{d\theta} dF(a) = 0.$$

Given that $\bar{w}(\theta)$ is not a function of a, the equation above could then be rewritten as

$$\bar{w}(\theta) + \frac{\theta\eta(\theta)}{\theta\eta'(\theta) - \eta(\theta)} \frac{d\bar{w}(\theta)}{d\theta} = E(a). \tag{3.12}$$

With the optimal contract wage $\bar{w}$ and labour market tightness θ defined as above, the market utility U^* could then be defined as

$$U^\star = \sup_{\bar{w} \in W} \left\{ \eta(\theta)u(\bar{w}) + (1 - \eta(\theta))u(b) \right\}, \tag{3.13}$$

where W is the set of all contracts offered by firms in the equilibrium.

Also notice that since firms enters freely, the zero-expected profit condition requires in equilibrium that

$$\kappa = \mu\left(\theta\left(\bar{w}\right)\right) \int \left(a - \bar{w}\right) dF(a). \tag{3.14}$$

Equilibrium then could be defined with a optimal level of market tightness θ in Equation (3.12), the optimal contract wage $\bar{w}$ in Equation (3.11) and the market utility U^* in Equation (3.13) as well. The first-order condition, Equation(3.12), along with Equation (3.13) and (3.14) determine the equilibrium variables of interest.

We then rewrite the free-entry condition, Equation (3.14) of the firms as follows:

$$\kappa = \mu\left(\theta\left(\bar{w}\right)\right)\left(E(a) - \bar{w}\right). \tag{3.15}$$

Notice that in equilibrium, the free entry condition and the firm's first-order condition Equation (3.12) are the only equations that feature labour productivity, a. This implies that under optimal contracts, the firm fully insures the worker given any level of uncertainty in the economy. Thus, increasing uncertainty would only have equilibrium effects if a rise (or fall) in uncertainty changes the way firms choose their optimal contracts to offer in equilibrium.

However, in the one period model, as firms are risk-neutral, changing higher moments around the mean labour productivity has no effect on the them. This can be seen from Equation (3.12) which describes the firm's preferred level of market tightness. Given the free-entry condition in the equilibrium Equation (3.15) and Equation (3.9), the level of market utility offered to workers will also remain unchanged with an increase or decrease in the variance of a while holding the value of $E(a)$ constant. Therefore, changes in uncertainty do not affect the firm if uncertainty is delivered through changes in the higher-order moments in a keeping the level of $E(a)$ unchanged.

3.4 Conclusion

Following a numerical analysis in the previous chapter of the effect of volatility changes in labour productivity in the labour market, this chapter investigate the effects analytically in different settings. We first took a look at a baseline labour search model where firms could hire multiple workers and wage are determined by a bargaining following Stole and Zwiebel (1996). In the baseline model, volatility changes with the median status of the technology unchanged only have neutral effect to the job creation, while a sticky wage setting following a special case in Michaillat (2012) could introduce non-neutral effects.

We also checked the effect in a one-period model within a risk-sharing framework similar to that of Rudanko (2009), where optimal wage contracts are reached between risk-averse workers and risk-neutral firms. Similar to the case with the baseline Stole and Zwiebel (1996) wage setting, given that the profit function of the firms in the model is linear, a volatility change has only neutral effect in the framework where risk-averse workers are fully insured by risk-neutral firms with the volatility change keeping the median level unchanged.

It can been learned in our discussion in this chapter that, in order to introduce some non-neutral effects of uncertainty shocks in the labour market, it is the non linearity of the profit function of the firms that matters. When the expected profit function of the firm is linear in labour productivity then any mean-preserving increase in volatility of labour productivity does not affect the firm's incentive to create job openings.

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