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Economic Integration and Endogenous Growth Revisited: Pro-Competitive Gains from Trade in Goods and the Long Run Benefits to the Exchange of Ideas

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Abstract: This paper re-examines the Romer [1990] "knowledge driven" endogenous growth model in an open economy setting. As an alternative to Rivera-Batiz and Romer [1991], we consider trade between two absolutely identical countries that are characterized by imperfect competition in one of the trade goods. Contrary to Rivera-Batiz and Romer [1991], we find that trade in goods without trade in ideas is detrimental to long run growth while trade in goods in conjunction with trade in ideas is good for long run growth. We further demonstrate that the pro-competitive gains from trade in goods is analogous to the analysis of imperfect competition by standard international trade theory.

Keywords: Knowledge Driven, Endogenous Growth, International Trade, Imperfect Competition

JEL classification: F12, F15, F43

I. Introduction

Endogenous growth theory has enjoyed enormous attention over the last several years. New growth theory, as it is sometimes referred, considers technological change, growth, and welfare in the context of a neoclassical representative agent model. Amongst the abundant literature, papers that explicitly consider the nature of technological change include Romer [1990] with "knowledge driven growth", Grossman and Helpman [1991] with "quality ladders," and Aghion and Howitt [1992] with "creative destruction." Each of these papers has received wide acclaim to the effect that they now rank among the seminal works in the New Growth Theory literature. Consequently, these papers provide the frameworks for subsequent research extensions.

One such extension is the paper by Rivera-Batiz and Romer [1991]. They attempt to analyse the Romer [1990] model in an open economy setting. Their results are now part of the standard fare of many graduate macroeconomics courses and the textbooks that they use.[1]

This paper re-examines the Romer [1990] "knowledge driven" endogenous growth model in an open economy setting. We present an alternative specification to that which is found Rivera-Batiz and Romer [1991]. They consider two countries that are identical only up until the point in which trade opens, after which, by assumption they cease to be identical. They assume that once open, each country may produce unique intermediate goods, avoid redundancy, and thereby earn monopoly rents worldwide. Therefore, each firm may exploit its monopoly across both countries until a competitor, who must necessarily be foreign, comes along with a better intermediate good. As a result, the two countries take turns introducing innovations. Furthermore, there is no change in output, work effort, or growth from autarky to trade because, with trade, the intermediate goods producer is effectively faced with twice the market for half the time. They conclude that "free trade in goods (without trade in ideas)… does not affect log run growth rates" [Rivera-Batiz and Romer, 1991, p.544].

We assume that the two countries are absolutely identical before *and* after trade. In autarky, each country produces its own version of each new innovation. With trade, each country simultaneously continues to produce a version of each new intermediate good. The home and foreign versions of each new intermediate good are perfect substitutes for one another such that pro-competitive gains from trade may result.

This paper shows, in the context of the Romer [1990] model, that trade in goods without trade in ideas is detrimental to long run growth while trade in goods in conjunction with trade in ideas is good for long run growth. Furthermore, we demonstrate that the nature of trade in goods is analogous to the standard pro-competitive gains from trade result from the international trade literature on imperfect competition.

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The Romer [1990] model considers technological change to be a result of existing technology combined with human capital. The greater the stock of technology at any given time, the greater is the potential for even greater technological advances at that time, given some human capital expenditure. The representative agent allocates his human capital competitively between the final goods sector and R&D sector. Final goods are produced from human capital, labour, and a continuum of intermediate goods. Intermediate goods, imperfect substitutes for one another, are produced trivially from final goods.

Trade between identical countries under these circumstances is intuitively identical to those results from imperfect competition in the trade literature. In other words, imperfect competition is an effective determinant of trade that results in pro-competitive gains.[2] The move from autarky to free trade in goods effectively changes each intermediate good producer's market structure from monopoly to duopoly. Consistent with standard trade theory, cross country competition between rival monopolists results in each intermediate firm producing more output to sell at a lower price. Although there is no actual trade, existence of a rival creates pro-competitive gains from trade, which implies increased production of intermediate goods and a fall in its price. With greater intermediate goods to work with, the marginal products of labour and human capital both increase in the production of final goods. Furthermore, lower monopoly profits today implies lower profits tomorrow. In other words, pro-competitive gains from trade also implies a lower marginal product of research. The agent responds accordingly by devoting more human capital towards final production and less toward research. Since the growth rate of technology is a function of the human capital devoted to research, trade in goods without trade in ideas hurts long run growth.

Once trade in ideas is also allowed, the wealth effect of doubling the size of the market for new ideas overwhelms the substitution effect from the change in the relative price of human capital. Agents respond by devoting more human capital to research relative to the case of trade in only goods. Since the growth rate of technology is now a function of the world stock of ideas, although the agent still devotes less human capital to research relative to autarky, the growth rate of technology with trade in ideas is higher.

The paper is organized as follows. Section II presents a synopsis of the methodology used by Romer [1990]. Section III explains the difference between Rivera-Batiz and Romer [1991] and our alternative specification. Section IV discusses the amended results. Section V contains concluding remarks.

II. Summary of Romer [1990]

The Romer [1990] model considers an infinitely lived representative agent who is endowed with labor ($\bar{L}$) and human capital ($\bar{H}$) and consumes only final goods that are competitively produced from labor, human capital (H_Y), and a continuum of intermediate goods (x_i).

(1) $$Y = H_Y^{\alpha} L^{\beta} \int_{i=0}^{A} x_i^{1-\alpha-\beta} di$$

Technological change ($\dot{A}$) is the result of human capital (H_A) and the stock of technology (A).

(2) $$\dot{A} = \delta H_A A \qquad \delta > 0$$

The market for human capital is competitive.

(3) $$\bar{H} = H_A + H_Y$$

Each intermediate producer is a monopolist facing with an inverse demand for its variety of input that is exactly equal to its marginal product in the production of Y.

(4) $$P_i = (1-\alpha-\beta) H_Y^{\alpha} L^{\beta} x_i^{-\alpha-\beta} = r$$

The profit maximizing price and output of the representative monopolist is defined as follows.

(5) $$\bar{P}_i = r \qquad \& \qquad \bar{x}_i = \left[\frac{r H_Y^{-\alpha} L^{-\beta}}{1-\alpha-\beta}\right]^{\frac{1}{-\alpha-\beta}}$$

The market for ideas is competitive and therefore $P_A = MC_A = MR_A$. The marginal revenue of a new idea is derived from the discounted future profits to the R&D firm once it has exploited the monopoly rents in subsequent periods.

(6) $$\int_t^{\infty} e^{-\int_t^{\tau} r(s) ds} \pi_{\tau} d\tau = P_{At}$$

While the firm is still in the R&D phase, it faces a competitive market. Therefore, it is the zero profit condition faced by the representative R&D firm that determines the wage for human capital devoted to R&D.[3]

(7) $$w_H = P_A \delta A$$

The wage for human capital devoted to final goods (H_Y) is determined from its marginal product in terms of final goods.

(8) $$w_Y = \alpha H_Y^{\alpha-1} L^{\beta} A \bar{x}^{1-\alpha-\beta}$$

A competitive human capital market implies that $w_H = w_A$. This may be used to solve for the optimal allocation of human capital between final production and R&D as well as the price of technology (P_A).

(9) $$H_Y{}^* = \frac{r}{\delta} \cdot \frac{\alpha}{(1-\alpha-\beta)(\alpha+\beta)} = \frac{r\Lambda}{\delta}$$

(10) $$H_A{}^* = \bar{H} - \frac{r\Lambda}{\delta}$$

(11) $$P_A{}^* = \frac{\alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta}}{\delta}$$

The growth rate of technology is therefore given by

(12) $$g^* = \frac{\dot{A}}{A} = \delta H_A{}^* = \frac{\delta \bar{H} - \rho\Lambda}{\Lambda\theta + 1}$$

III. Analysis of Rivera-Batiz and Romer [1991]

Rivera-Batiz and Romer [1991] consider trade between two identical economies as described by Romer [1990]. The economies are identical only up until trade is opened.[4] In so doing, they first consider trade in intermediate goods without trade in ideas. Regarding the price of a new idea, they state the following:

"For the research sector, opening of trade implies that the market for any newly designed good is twice as large as it was in the absence of trade. This doubles the price of the patents and raises the return to investing human capital in research from $P_A \delta A$ to $2P_A \delta A$." [Rivera-Batiz and Romer, 1991, p. 543-4]

This statement rests on the assumption that new intermediate goods produced in the open economy are not redundant. In Rivera-Batiz and Romer [1991], there is no foreign alternative intermediate good available to the final goods producer. On the other hand, we assume that each country produces its own version of each new intermediate good. The return to investing in human capital in research still increases by a factor of 2 but in a slightly different manner.

The return to research, w_H, is determined from the zero profit condition of the individual researcher, which does not change.

(13) $$P_A \cdot \delta H_{Aj} A - w_A H_{Aj} = \pi_j = 0$$

The output of individual R&D firm *j* equals $A_j = \delta H_{Aj} A$. Rivera-Batiz and Romer [1991] assume that R&D firm *j* doubles its output of *A* with trade. This is strictly true only when new innovations are not redundant. If each country can produce its own version of every new innovation, then firm *j* may still double its output of *A*, but only as a response by firm *j* to a change in the competitive price, P_A. Therefore, consider the model similar to that presented by Rivera-Batiz and Romer [1991] except that firm *j* produces output A_j in response to price, P_A.

The model is specified as follows. There are two absolutely identical countries that may trade in intermediate goods but not in ideas. Final goods production is therefore defined as follows.

(14) $$Y = H_Y^{\alpha} L^{\beta} \int_{i=0}^{A+A^F} x_i^{1-\alpha-\beta} di$$

Model symmetry implies that $A=A^F$. Since there is no trade in ideas, the change in technology is solely a function of domestic stocks of A.

(15) $$\dot{A} = \delta H_A A \qquad \delta > 0$$

The return on human capital in R&D, w_A, is derived from the zero profit condition, equation (13), and the return on human capital in final goods production, w_Y, is derived from its marginal product.

(16) $$w_A = P_A \delta A$$

(17) $$w_Y = \frac{\partial Y}{\partial H_Y} = \alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta} 2A$$

A competitive human capital market implies that $w_H = w_A$ which further determines P_A as well as the optimal choice of human capital allocation.

(18) $$P_A{}^* = \frac{2\alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta}}{\delta}$$

(19)

$$H_Y{}^* = \frac{2r}{\delta} \cdot \left[\frac{\alpha}{(\alpha+\beta)(1-\alpha-\beta)} \right] = \frac{2r\Lambda}{\delta}$$

(20) $$H_A{}^* = \bar{H} - \frac{2r\Lambda}{\delta}$$

The growth rate of technology is therefore given by

(21) $$g* = \frac{\delta H - 2\rho\Lambda}{2\theta\Lambda + 1}$$

Notice that P_A under trade, equation (18), versus P_A under autarky, equation (11) differs by a factor of 2. In other words, trade has doubled the relative price of patents, which has raised the return to investing in human capital, just as Rivera-Batiz and Romer [1991] predict. The difference between them and us manifests itself in the analytic solution for the growth rate of technology, g. Compare equations (21) and (12). Rivera-Batiz and Romer's [1991] solution for g as a result of trade in goods is analytically identical to the solution for g under autarky (equation (12)) such that they conclude that trade in goods has no growth effects.

Note the intuitive difference. In Rivera-Batiz and Romer [1991], upon opening trade in goods, one country, say H, goes first by introducing an innovation. In exactly half the time its takes H to invent a still newer innovation, F introduces its latest innovation. The two countries now proceed to take turns introducing new goods. The proprietary firm of each new innovation may capitalize on both home and foreign demands but only for half the time that it did so under autarky.

Here, upon opening trade in goods, both countries simultaneously introduce their respective versions of the newest innovation. Since the two versions, foreign and domestic, are perfect substitutes, they must share the market thereby creating a duopoly where the intermediate producers are Cournot-Nash competitors.[5]

Trade in ideas as well as in intermediate goods is specified exactly as above except that the technology constraint, equation (15), and the zero profit condition, equation (13), must be altered to reflect trade in ideas.

(22) $$\dot{A} = \delta H_A \left(A + A^F \right) \qquad \delta > 0$$

(23) $$P_A \cdot \delta H_{Aj} \left(A + A^F \right) - w_A H_{Aj} = \pi_j = 0$$

Trade in ideas implies that the change in technology, $\dot{A}$, is a result of the world stock of technology combined with domestic human capital effort. Output of the individual R&D firm j also must reflect trade in ideas such that $A_j = \delta H_{Aj} \left(A + A^F \right)$. The rest of the model solves to the following equilibrium conditions.

(24) $$P_A* = \frac{\alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta}}{\delta}$$

(25) $$H_Y* = \frac{r}{\delta}\left[\frac{\alpha}{(\alpha+\beta)(1-\alpha-\beta)} \right] = \frac{r\Lambda}{\delta}$$

(26) $$H_A* = H - \frac{r\Lambda}{\delta}$$

(27) $$g* = \frac{\delta H - \rho\Lambda}{\theta\Lambda + \frac{1}{2}}$$

IV. Comparisons

Table 1 presents the results from (1) autarky, (2) trade in intermediate goods only, (3) trade in goods as well as ideas, and (4) Rivera-Batiz and Romer's [1991] trade in goods as well as ideas. Notice that the growth rate of technology, $g*$, is a strictly a function of coefficients and the stock of human capital, H. Given their analytic solutions, it must be that $g*^2 < g*^1$, $g*^1 < g*^3$, $g*^2 < g*^3$, and $g*^3 < g*^4$, which implies that $g*^4 > g*^3 > g*^1 > g*^2$ as well as $r*^4 > r*^3 > r*^1 > r*^2$.[6]

Next consider the output of the intermediate good, x_i, and the price of technology, P_A, across the three cases. Technical Appendix 1 clearly shows that $x_i*^4 < x_i*^3 < x_i*^1 < x_i*^2$ and $P_A*^4 < P_A*^3 < P_A*^1 < P_A*^2$.

Figures I and II present a graphical representation of these results. Notice that from Table I the demand for the intermediate goods, x_i, as well as the price of technology, P_A, depend solely on the human capital in final goods production, H_Y.

Finally consider the human capital allocations in cases 1, 2, 3, and 4. Recall that for cases 1 and 2, $g* = \delta H_A*$. Therefore if $g*^1 > g*^2$, then $H_A*^1 > H_A*^2$ and $H_Y*^1 < H_Y*^2$. Intuitively, pro-competitive gains from trade implies higher production of intermediate goods, x_i, which necessarily raises the marginal product on human capital in final production. Simultaneously, the marginal product of human capital in research falls with the lower expectation of future monopoly profits. Thus it follows that there should be relatively less human capital effort in research (and more in final goods production) with trade in goods than in autarky.

When trade in ideas is allowed in addition to trade in goods, the growth rate of technology increases relative to autarky, i.e. $g*^1 < g*^3$. Notice that from Table 1, the analytic solutions for the optimal work effort, H_A*, H_Y*, and consequently, the interest rate are identical for case 1 and case 3. Therefore, if $r*^1 < r*^3$, then $H_Y*^1 < H_Y*^3$ and $H_A*^1 > H_A*^3$. Technical Appendix 2 shows that $H_A*^2 < H_A*^3$ and $H_Y*^2 > H_Y*^3$ as well as $H_A*^1 < H_A*^4$ and $H_Y*^1 > H_Y*^4$. Summing up the results, we may conclude that $H_A*^4 > H_A*^1 > H_A*^3 > H_A*^2$. Figure III presents a graphical representation of these results.

V. Conclusion

The results herein are complementary to those found in Rivera-Batiz and Romer [1991]. The general results from that paper as well as the limitations placed on those results by the authors still hold here. They are that economic integration, when the change in technology is subject to increasing returns, has a positive long run effect on economic growth. And given the nature of the exponential growth function, policies that affect trade necessarily affect growth and can have large cumulative effects on economic welfare. Furthermore, the two models ultimately characterize different sets of stylised facts that we observe in the world. There certainly does exist the ability to innovate and reap the returns across the entire world (i.e. Microsoft). But there also exists the stylised fact that countries do produce their own versions of goods without the explicit exchange of ideas (i.e. automobile industry).

The two different model specifications each have analytic strengths as well as weaknesses. The main weakness of Rivera-Batiz and Romer [1991] is the stepwise nature of trade where each country takes turns innovating. The main weakness here is that without trade in ideas, each country still comes up independently with identical innovations. The main strength of Rivera-Batiz and Romer [1991] is that worldwide monopoly rents are available to innovators. The main strength here is the pro-competitive gains from trade from imperfect competition result in the dynamic setting.

This paper adds to the literature in four ways. First, it provides an alternative specification to a widely cited piece of literature, Rivera-Batiz and Romer [1991]. Second, it demonstrates the relevance of standard trade theory on imperfect competition in the context of dynamic models of technological change. In so doing, we highlight the pro-competitive gains from trade available as a result of imperfect competition in one of the sectors and show the negative growth effects of disallowing trade in ideas. Third, the paper shows that the growth benefits from increased integration (i.e. trade in ideas) outweigh the negative growth effects of the pro-competitive gains. Forth, it opens an interesting avenue of research into the other parallels that must exist between trade and new growth theory. In other words, one may now consider in the above framework, any number of extensions from differentiated countries to tax effects to the consideration of different manners of technological change.

Technical Appendices

<u>Technical Appendix 1</u> – Comparison of Intermediate Good, x_i and the price of technology, P_A, across cases

- *Show that* $x_i*^1 < x_i*^2$ *: Proof by contradiction*

Assume that $x_i^1 > x_i^2$. From the analytic solution to x_i, the assumption implies that

$\left(H_Y*^1\right)^{1-\alpha} < \dfrac{\left(H_Y*^2\right)^{1-\alpha}}{2}$ or $2^{\frac{\alpha}{1-\alpha}} H_Y*^1 < \dfrac{H_Y*^2}{2}$. Given that $2^{\frac{\alpha}{1-\alpha}} > 1$, it follows that $\dfrac{H_Y{}^2}{2} > H_Y{}^1$.⊗

(note: The analytic solutions to r^{*1} and r^{*2} imply that $r^{*1} = \frac{\delta H_Y^{*1}}{\Lambda} > \frac{\delta H_Y^{*2}}{2\Lambda} = r^{*2}$ or that $H_Y^{*1} > \frac{H_Y^{*2}}{2}$)

$\therefore$ It must be that $x_i^{*1} < x_i^{*2}$ and given the analytic solution to P_A, it must also be that $P_A^{*1} < P_A^{*2}$. **chk**.

- *Show that* $x_i^{*1} > x_i^{*3}$:

The comparison of x_i^{*1} vs. x_i^{*3} may be simplified to H_Y^{*1} vs. H_Y^{*3}, whose solution we already know to be that $H_Y^{*1} < H_Y^{*3}$ which implies that $x_i^{*1} > x_i^{*3}$ and $P_A^{*1} > P_A^{*3}$. **chk**.

- *Show that* $x_i^{*2} > x_i^{*3}$ *: Proof by Contradiction*

Assume that $x_i^2 < x_i^3$. From the analytic solution to x_i, the assumption implies that $\frac{\left(H_Y^{*2}\right)^{1-\alpha}}{2} > \left(H_Y^{*3}\right)^{1-\alpha}$ or $\frac{H_Y^{*2}}{2} > 2^{\frac{\alpha}{1-\alpha}} H_Y^{*3}$. Given that $2^{\frac{\alpha}{1-\alpha}} > 1$, it follows that $\frac{H_Y^{2}}{2} > H_Y^{3}$.$\otimes$

(note: The analytic solutions to r^{*2} and r^{*3} imply that $r^{*3} = \frac{\delta H_Y^{*3}}{\Lambda} > \frac{\delta H_Y^{*2}}{2\Lambda} = r^{*2}$ or that $H_Y^{*3} > \frac{H_Y^{*2}}{2}$)

$\therefore$ It must be that $x_i^{*2} > x_i^{*3}$ and given the analytic solution to P_A, it must also be that $P_A^{*2} > P_A^{*3}$. **chk**.

- *Show that* $x_i^{*4} < x_i^{*3}$ *: Proof by contradiction*

Assume that $x_i^4 > x_i^3$. From the analytic solutions to x_i, the assumption implies that $2\left(H_Y^{*4}\right)^{1-\alpha} < \left(H_Y^{*3}\right)^{1-\alpha}$ or $2^{\frac{\alpha}{1-\alpha}} 2H_Y^{*4} < H_Y^{*3}$, which my be rewritten as $2^{\frac{\alpha}{1-\alpha}} H_Y^{4} < \frac{H_Y^{3}}{2}$. Given that $2^{\frac{\alpha}{1-\alpha}} > 1$, it follows that $H_Y^{4} < \frac{H_Y^{3}}{2}$.$\otimes$

(note: The analytic solutions to r^{*4} and r^{*3} imply that $r^{*4} = \frac{2\delta H_Y^{*4}}{\Lambda} > \frac{\delta H_Y^{*3}}{\Lambda} = r^{*3}$ or that $H_Y^{*4} > \frac{H_Y^{*3}}{2}$.)

$\therefore$ It must be that $x_i^{*4} < x_i^{*3}$ and given the analytic solution to P_A, it must also be that $P_A^{*4} < P_A^{*3}$. **chk**.

Technical Appendix 2 – Comparisons of Human capital across cases

- *Show that* $H_Y{*}^2 > H_Y{*}^3$ *and* $H_A{*}^2 < H_A{*}^3$*: Proof by Contradiction*

Assume that $H_Y{}^2 < H_Y{}^3$ and $H_A{}^2 > H_A{}^3$.

$H_A{}^2 > H_A{}^3$, given that $g^2 = \delta H_A{}^2$ and $g^3 = 2\delta H_A{}^3$, implies that $2g^2 > g^3$.

$2g^2 > g^3$, given that $g^2 = \dfrac{\delta H - 2\rho\Lambda}{2\theta\Lambda + 1}$ and $g^3 = \dfrac{\delta H - \rho\Lambda}{\theta\Lambda + \frac{1}{2}}$, implies that

$$2\frac{\delta H - 2\rho\Lambda}{2\theta\Lambda + 1} > \frac{\delta H - \rho\Lambda}{\theta\Lambda + \frac{1}{2}}$$ or simply that $2\rho\Lambda < \rho\Lambda$.⊗

$\therefore$ It must be that $H_Y{*}^2 > H_Y{*}^3$ and $H_A{*}^2 < H_A{*}^3$. **chk**.

- *Show that* $H_Y{*}^1 > H_Y{*}^4$ *and* $H_A{*}^1 < H_A{*}^4$*: Proof by Contradiction*

Assume that $H_Y{}^1 < H_Y{}^4$ and $H_A{}^1 > H_A{}^4$.

$H_A{}^1 > H_A{}^4$, given that $g^1 = \delta H_A{}^1$ and $g^4 = 2\delta H_A{}^4$, implies that $2g^1 > g^4$.

$2g^1 > g^4$, given $g^1 = \dfrac{\delta H - \rho\Lambda}{\theta\Lambda + 1}$ and $g^4 = \dfrac{2\delta H - \rho\Lambda}{\theta\Lambda + 1}$, implies that

$$2\left(\frac{\delta H - \rho\Lambda}{\theta\Lambda + 1}\right) > \frac{2\delta H - \rho\Lambda}{\theta\Lambda + 1}$$ or simply that $-2 > -1$.⊗

$\therefore$ It must be that $H_Y{*}^1 > H_Y{*}^4$ and $H_A{*}^1 < H_A{*}^4$. **chk**.

Technical Appendix 3 – *Case 1: No Trade*

Consider two identical economies in autarky defined as follows:

Infinitely lived representative agent: $U = \int_{t=0}^{\infty} e^{-\rho t} \dfrac{C_t^{1-\theta} - 1}{1-\theta} dt$

Final goods production: $Y = H_Y^{\alpha} L^{\beta} \int_{i=0}^{A} x_i^{1-\alpha-\beta} di$

Capital Formation: $\dot{K} = Y_t - C_t$

Technological Change: $\dot{A} = \delta H_A A$

Total human capital: $\bar{H} = H_A + H_Y$

Human capital market is competitive➔ $P_A{*} = \dfrac{\alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta}}{\delta}$

$$H_Y^* = \frac{\Lambda r}{\delta}$$

$$H_A^* = H - \frac{\Lambda r}{\delta}$$

where $\Lambda = \dfrac{\alpha}{(1-\alpha-\beta)(\alpha+\beta)}$

The growth rate of technology is therefore given by: $g^* = \dfrac{\dot{A}}{A} = \delta H_A^* = \dfrac{\delta H - \rho\Lambda}{\Lambda\theta + 1}$

The equilibrium quantity of intermediate good *i* is given by:

$$\bar{x}_i^* = \left[\frac{(\alpha+\beta)\delta H_Y^{1-\alpha} L^{-\beta}}{\alpha}\right]^{\frac{1}{-\alpha-\beta}}$$

<u>Technical Appendix 4</u> – *Case 2: Trade Only In Intermediate Goods*

Consider trade only in intermediate goods, x_i ➔ $Y = H_Y^{\alpha} L^{\beta} \int_{i=0}^{A+A^F} x_i^{1-\alpha-\beta} di$

(note that there are *A* different intermediate goods per country)

2 identical countries➔ $A = A^F$

There is NO trade in ideas➔ $\dot{A} = \delta H_A A$

The human capital market in each country is competitive➔ $w_A = w_Y$

$$w_A = P_A \delta A$$
$$w_Y = \frac{\partial Y}{\partial H_Y} = \alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta} 2A$$

(note: The price of A is determined by the horizontal summation of the demand for x_i in each country. The fact that the market for x_i is twice as big in free trade versus autarky is captured endogenously in the price of A)

$$w_A = w_Y$$

$$P_A \delta A = \alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta} 2A$$

$$P_A^* = \frac{2\alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta}}{\delta}$$

$$P_A \delta A = \frac{\pi}{r}\delta A$$
$$= \frac{P_x x(\alpha+\beta)}{r}\delta A$$
$$= (1-\alpha-\beta) H_Y^{\alpha} L^{\beta} x^{1-\alpha-\beta}\left(\frac{\alpha+\beta}{r}\right)\delta A$$

$$\alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta} 2A = (1-\alpha-\beta) H_Y^{\alpha} L^{\beta} x^{1-\alpha-\beta}\left(\frac{\alpha+\beta}{r}\right)\delta A$$
$$\therefore 2\alpha = (1-\alpha-\beta) H_Y \left(\frac{\alpha+\beta}{r}\right)\delta$$

$$H_Y * = \frac{2r}{\delta}\left[\frac{\alpha}{(\alpha+\beta)(1-\alpha-\beta)}\right] = \frac{2r\Lambda}{\delta}$$
$$g* = \frac{\dot{A}}{A} = \delta H_A * = \delta\left(H - H_Y *\right)$$
$$H_A * = H - \frac{2r\Lambda}{\delta}$$
$$g* = \frac{\dot{A}}{A} = \delta H_A * = \frac{1}{\theta}(r-\rho)$$
$$g* = \frac{1}{\theta}\left(\frac{\delta H_Y *}{2\Lambda} - \rho\right)$$
$$= \frac{1}{\theta}\left(\frac{\delta\left(H - H_A *\right)}{2\Lambda} - \rho\right)$$
$$= \frac{1}{\theta}\left(\frac{\delta H - g *}{2\Lambda} - \rho\right)$$
$$= \frac{\delta H - 2\rho\Lambda}{2\theta\Lambda\left(1 + \frac{1}{2\theta\Lambda}\right)}$$

$$g* = \frac{\delta H - 2\rho\Lambda}{2\theta\Lambda + 1}$$

$$\overline{x}_i * = \left(\frac{rH_Y^{-\alpha} L^{-\beta}}{1-\alpha-\beta} \right)^{\frac{1}{-\alpha-\beta}}$$

$$= \left[\frac{\left(\frac{\delta H_Y}{2\Lambda} \right) H_Y^{-\alpha} L^{-\beta}}{1-\alpha-\beta} \right]^{\frac{1}{-\alpha-\beta}}$$

$$= \left[\frac{(\alpha+\beta)\delta H_Y^{1-\alpha} L^{-\beta}}{2\alpha} \right]^{\frac{1}{-\alpha-\beta}}$$

<u>Technical Appendix 5</u> – *Case 3: Trade In Intermediate Goods + Trade In Ideas*

Consider trade only in intermediate goods, x_i ➔ $Y = H_Y^{\alpha} L^{\beta} \int_{i=0}^{A+A^F} x_i^{1-\alpha-\beta} di$

2 identical countries➔ $A = A^F$

Trade in ideas➔ $\dot{A} = \delta H_A \left(A + A^F \right)$

The human capital market in each country is competitive➔ $w_A = w_Y$

$$w_A = P_A \delta \left(A + A^F \right)$$
$$w_Y = \frac{\partial Y}{\partial H_Y} = \alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta} 2A$$

$$w_A = w_Y$$

$$2P_A \delta A = \alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta} 2A$$

$$P_A * = \frac{\alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta}}{\delta}$$

$$2P_A \delta A = \frac{\pi}{r} 2\delta A$$
$$= \frac{P_x x(\alpha+\beta)}{r} 2\delta A$$
$$= (1-\alpha-\beta) H_Y^{\alpha} L^{\beta} x^{1-\alpha-\beta} \left(\frac{\alpha+\beta}{r} \right) 2\delta A$$

$$\alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta} 2A = (1-\alpha-\beta) H_Y^{\alpha} L^{\beta} x^{1-\alpha-\beta} \left(\frac{\alpha+\beta}{r}\right) 2\delta A$$

$$\therefore \alpha = (1-\alpha-\beta) H_Y \left(\frac{\alpha+\beta}{r}\right)\delta$$

$$H_Y * = \frac{r}{\delta}\left[\frac{\alpha}{(\alpha+\beta)(1-\alpha-\beta)}\right] = \frac{r\Lambda}{\delta}$$

$$g* = \frac{\dot{A}}{A} = 2\delta H_A * = 2\delta (H - H_Y *)$$

$$H_A * = H - \frac{r\Lambda}{\delta}$$

$$g* = \frac{\dot{A}}{A} = 2\delta H_A * = \frac{1}{\theta}(r-\rho)$$

$$g* = \frac{1}{\theta}\left(\frac{\delta H_Y *}{\Lambda} - \rho\right)$$

$$= \frac{1}{\theta}\left(\frac{\delta (H - H_A *)}{\Lambda} - \rho\right)$$

$$= \frac{1}{\theta}\left(\frac{\delta H - \frac{g*}{2}}{\Lambda} - \rho\right)$$

$$= \frac{\delta H - \rho\Lambda}{\theta\Lambda\left(1 + \frac{1}{2\theta\Lambda}\right)}$$

$$g* = \frac{\delta H - \rho\Lambda}{\theta\Lambda + \frac{1}{2}}$$

$$\overline{x}_i * = \left(\frac{r H_Y^{-\alpha} L^{-\beta}}{1-\alpha-\beta}\right)^{\frac{1}{-\alpha-\beta}}$$

$$= \left[\frac{\left(\frac{\delta H_Y}{\Lambda}\right) H_Y^{-\alpha} L^{-\beta}}{1-\alpha-\beta}\right]^{\frac{1}{-\alpha-\beta}}$$

$$= \left[\frac{(\alpha+\beta)\delta H_Y^{1-\alpha} L^{-\beta}}{\alpha}\right]^{\frac{1}{-\alpha-\beta}}$$

Table I

Analytic Comparisons
Case 1 vs. 2 vs. 3 vs. 4

1. Autarky = Rivera-Batiz & Romer [1991] Trade in Intermediaries

$$g^{*1} = \frac{\delta H - \rho\Lambda}{\Lambda\theta + 1} = \delta H_A{}^{*1}$$

$$\bar{x}_i{}^{*1} = \left[\frac{(\alpha+\beta)\delta H_Y^{1-\alpha} L^{-\beta}}{\alpha}\right]^{\frac{1}{-\alpha-\beta}}$$

$$P_A{}^{*1} = \frac{\alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta}}{\delta}$$

$$= \left[\frac{\alpha L^{\beta}}{\delta(\alpha+\beta)^{1-\alpha-\beta}}\left(H_Y{}^{*1}\right)^{\alpha-1}\right]^{\frac{1}{\alpha+\beta}}$$

$$H_A{}^{*1} = H - \frac{r\Lambda}{\delta}$$

2. Trade in Intermediaries

$$g^{*2} = \frac{\delta H - 2\rho\Lambda}{2\theta\Lambda + 1} = \delta H_A{}^{*2}$$

$$\bar{x}^{*2} = \left[\frac{(\alpha+\beta)\delta H_Y^{1-\alpha} L^{-\beta}}{2\alpha}\right]^{\frac{1}{-\alpha-\beta}}$$

$$P_A{}^{*2} = \frac{2\alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta}}{\delta}$$

$$= \left[\frac{\alpha L^{\beta}}{\delta(\alpha+\beta)^{1-\alpha-\beta}} 2\left(H_Y{}^{*2}\right)^{\alpha-1}\right]^{\frac{1}{\alpha+\beta}}$$

$$H_A{}^{*2} = H - \frac{2r\Lambda}{\delta}$$

3. Trade in Intermediaries + Trade in Ideas

$$g^{*3} = \frac{\delta H - \rho\Lambda}{\theta\Lambda + \frac{1}{2}} = 2\delta H_A{}^{*3}$$

$$\bar{x}_i{}^{*3} = \left[\frac{(\alpha+\beta)\delta H_Y^{1-\alpha} L^{-\beta}}{\alpha}\right]^{\frac{1}{-\alpha-\beta}}$$

$$P_A{}^{*3} = \frac{\alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta}}{\delta}$$

$$= \left[\frac{\alpha L^{\beta}}{\delta(\alpha+\beta)^{1-\alpha-\beta}}\left(H_Y{}^{*3}\right)^{\alpha-1}\right]^{\frac{1}{\alpha+\beta}}$$

$$H_A{}^{*3} = H - \frac{r\Lambda}{\delta}$$

4. Rivera-Batiz & Romer [1991]: Trade in Intermediaries + Trade in Ideas

$$g^{*4} = \frac{2\delta H - \rho\Lambda}{\Lambda\theta + 1} = 2\delta H_A{}^{*4}$$

$$\overline{x}_i *^4 = \left[\frac{2(\alpha+\beta)\delta H_Y^{1-\alpha} L^{-\beta}}{\alpha} \right]^{\frac{1}{-\alpha-\beta}}$$

$$P_A *^4 = \frac{\alpha H_Y^{\alpha-1} L^{\beta} x^{1-\alpha-\beta}}{2\delta}$$

$$= \left[\frac{\alpha L^{\beta}}{\delta(\alpha+\beta)^{1-\alpha-\beta}} \cdot \frac{\left(H_Y *^4\right)^{\alpha-1}}{2} \right]^{\frac{1}{\alpha+\beta}}$$

$$H_A *^4 = H - \frac{r\Lambda}{2\delta}$$

Figure I

Intermediate Goods Market
Cases 1 vs. 2

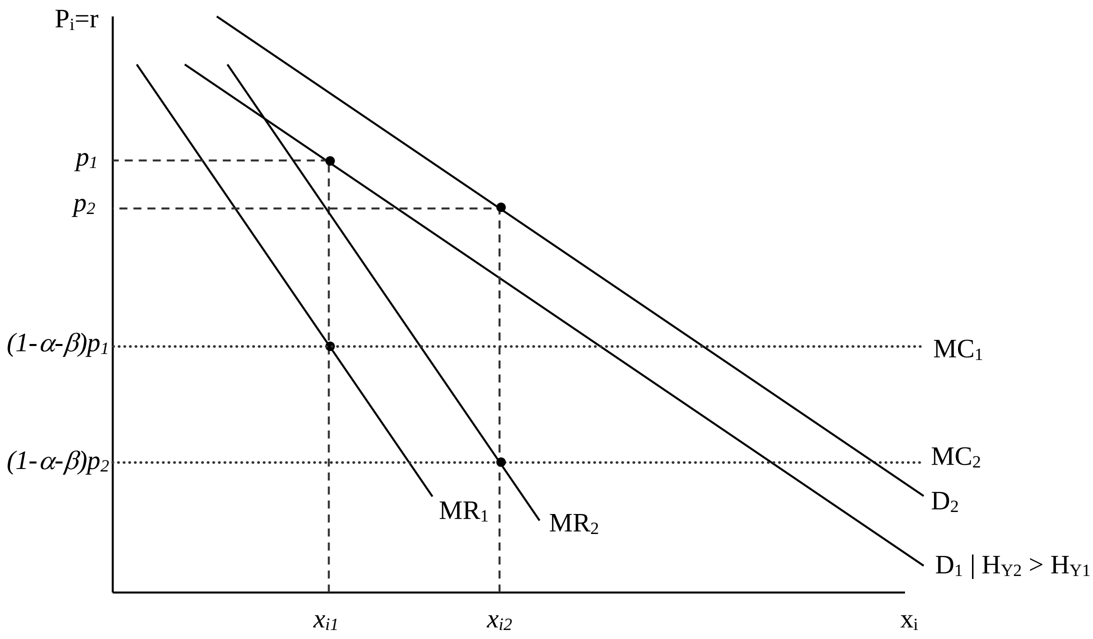


Figure II

Intermediate Goods Market

Cases 1 vs. 2 vs. 3 vs. 4

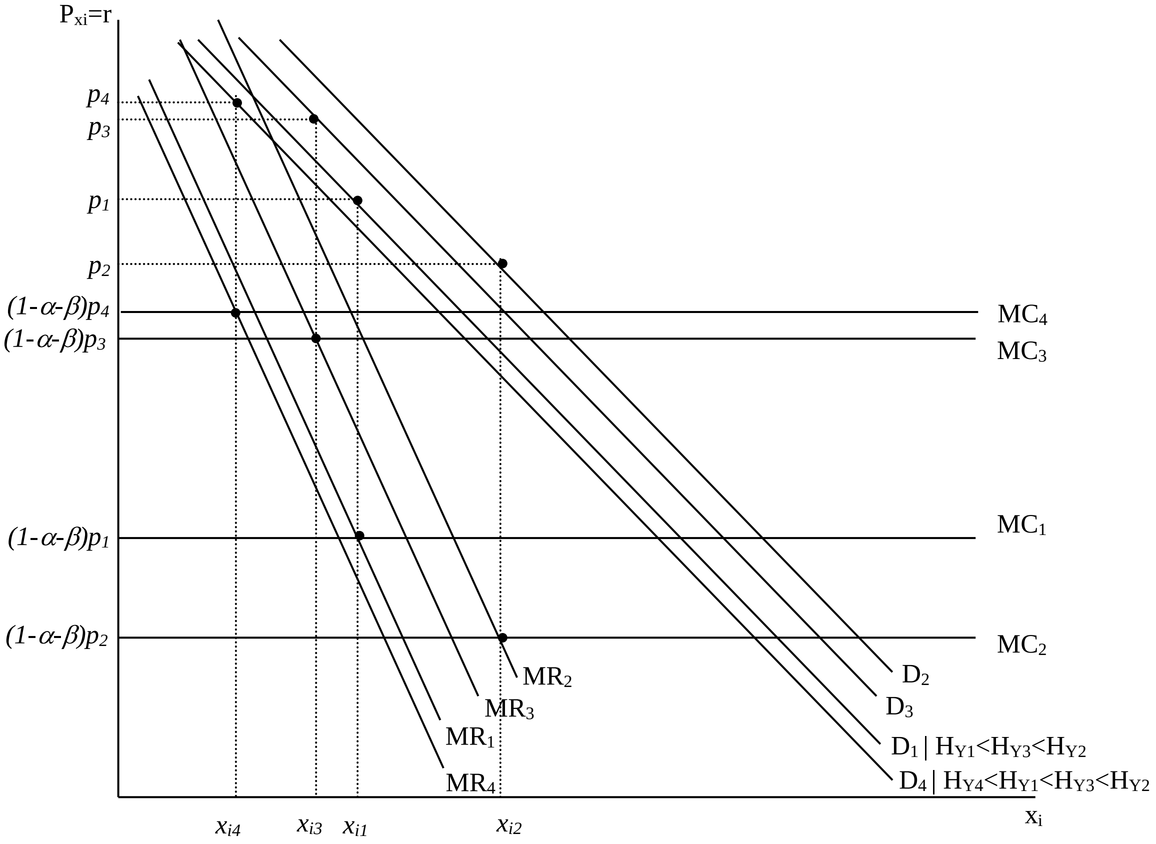
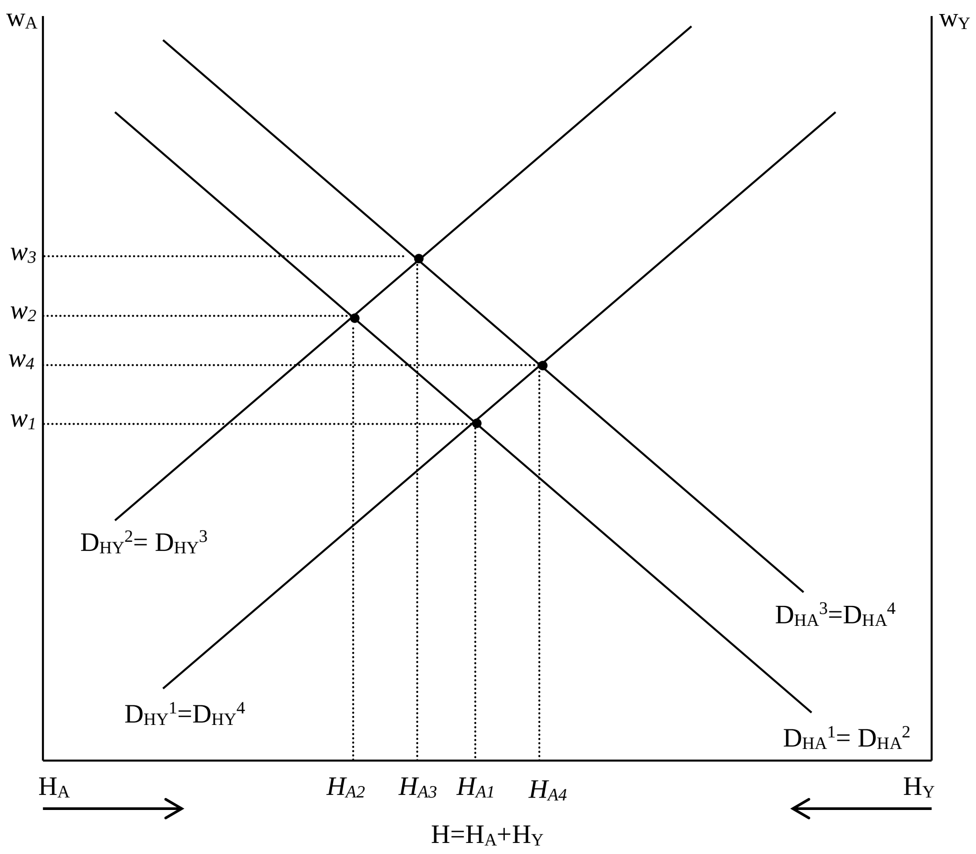


Figure III
Human Capital Market
Cases 1 vs. 2 vs. 3 vs. 4



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Endnotes

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1 For example, Aghion and Howitt [1998] in their advanced text, *Endogenous Growth Theory*, present the incorrect Romer and River-Batiz [1991] results as part of the chapter, "Growth in Open Economies." [Aghion and Howitt, 1998, p. 374]

2 See Markusen, et. al. [1995], *International Trade: Theory and Evidence*, McGraw Hill, ch. 11, or any other good intermediate trade textbook for an exposition on imperfect competition and trade.

3 This is the point where we diverge analytically from Rivera-Batiz and Romer [1991]. They assume that trade in goods implies that $w_H = 2P_A\delta A$ (see Rivera-Batiz and Romer, 1991, p. 543-4). We simply allow P_A to adjust endogenously to the new market conditions.

4 Although this is not explicitly stated with in Rivera-Batiz and Romer [1991], it is certainly the case if countries can not produce redundant goods after they are allowed to trade.

5 There is no need to consider Bertrand competition here because it would necessarily result marginal cost pricing and zero profits to intermediate producers. This effectively removes any incentive to conduct research such that $H_A = 0$ and $\dot{A} = \dot{K} = 0$. In other words, if technological change is the engine of growth and without monopoly profits to provide the incentive to research, then there is no research and, as a consequence, no growth. Interestingly enough, the Bertrand version also implies that the two countries are not only identical but also characterised by perfect competition and constant return to scale in its tradable goods, which is the standard "no trade" model.

6 This result is derived from the Euler Equation, $g* = \frac{1}{\theta}\left(r* - \rho\right)$.