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Explicit dynamics finite element modelling of
defective rolling element bearings

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Explicit dynamics finite element modelling of defective rolling element bearings

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Abstract

Rolling element bearings are widely used in rotating machinery across various industries and their failure is a dominant factor that contributes to machinery breakdown, consequently causing significant economic losses. Numerous experimental and analytical studies have been conducted in the past to understand the vibration response of non-defective and defective rolling element bearings, which have localised, extended, and distributed defects. Previous models have focused on simulating the defect-related impulses, which are generally observed in practice in measured vibration signals, and they implement envelope analysis to predict the significant defect-related frequency components.

The work presented in this thesis is focused on developing an understanding of the underlying physical mechanism by which defect-related impulses are generated in defective rolling element bearings. A novel explicit dynamics finite element (FE) model of a rolling element bearing having a localised outer raceway defect, line spall, was developed and solved using a commercially available FE software package, LS-DYNA. In addition to simulating the vibration response of the bearing, the dynamic contact interaction between the rolling elements and raceways of the bearing were modelled. An in-depth investigation of the rolling element-to-raceway contact forces was undertaken and variations in the forces, as the rolling elements traverse through the defect, were analysed. The contact force analysis has also led to the development of an understanding of the physics behind the low- and high-frequency characteristic vibration signatures generated by the rolling elements as they enter and exit a defect.

It was found that no impulse-like signals are generated during the gradual de-stressing or unloading of the rolling elements as they enter into a defect, which explains the low-frequency characteristics of the de-stressing event. In contrast, a burst of multiple, short-duration, force impulses is generated as the rolling elements re-stress between the raceways in the vicinity of the end of a defect, which explains the high-frequency impulsive characteristics of the re-stressing event. Based on the results of the FE analysis of the rolling element bearing, a mathematical model was developed to predict the gradual de-stressing of the rolling elements as they enter into a raceway defect.

Experimental testing on a rolling element bearing, commonly used in the railway industry, and having a line spall machined on its outer raceway was undertaken. The numerically modelled vibration response obtained using the FE model of the rolling element bearing was compared with the experimentally measured data, and a favourable agreement between the modelled and measured results was achieved. Numerical rolling element-to-raceway contact forces were compared with corresponding analytical results calculated using a quasi-static load distribution analytical model presented in this thesis.

A parametric study to investigate the effects of varying radial load and rotational speed on the vibration response of the bearing and rolling element-to-raceway contact forces was undertaken. It was found that the magnitude of the defect-related vibration impulses and contact forces generated during the re-stressing of the rolling elements increases with increasing load and speed.

The modelled contact forces were correlated with bearing vibration signals, and it was found that the amplitude of the contact forces and acceleration produced during the re-stressing of the rolling elements is much greater than when the rolling elements strike the defective surface. In other words, although a rolling element can impact the surface of a defect and generate a low amplitude acceleration signal, a much higher acceleration signal is generated when the rolling elements are re-stressed between the raceways as they exit from the defect. These higher acceleration signals, generated

during the re-stressing phase, are the ones that are generally observed in practice, and subsequently used for bearing diagnosis.

The work presented in this thesis has provided definitive physical and quantitative explanations for the impulsive acceleration signals measured when a bearing element passes through a defect.

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Statement of Originality

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Nomenclature

Roman Symbols

| | |
|------------|--|
| a_y | acceleration of a node within the FE model of the rolling element bearing in the global cartesian y -direction |
| b | half-contact width at the interface of two contacting isotropic elastic solid bodies |
| b', b'' | extremities of contact width $2b$ at the rolling element-to-raceway contact interfaces within a rolling element bearing |
| b_x, b_y | half-contact width at the rolling element-to-outer raceway contact interfaces within a rolling element bearing in the global cartesian x - and y -directions, respectively |
| B | bending stiffness of a plate / the outer ring of the FE model of the rolling element bearing |
| c | local material sound speed |
| c_b | velocity of bending waves |
| D_c | outer diameter of the cage within the FE model of the rolling element bearing |
| D_i | diameter of the inner raceway of a rolling element bearing |
| D_o | diameter of the outer raceway of a rolling element bearing |
| D_p | bearing pitch diameter |
| D_r | diameter of the rolling elements within a rolling element bearing |
| E' | equivalent modulus of elasticity of two contacting isotropic elastic solid bodies |
| E_1, E_2 | modulus of elasticity of isotropic elastic solid bodies '1' and '2' |
| F | Hertzian contact force at the interface of two isotropic elastic solid bodies |
| f_{bpi} | inner raceway defect frequency or ball pass frequency inner raceway |

| | |
|--------------------------|---|
| f_{bpo} | outer raceway defect frequency or ball pass frequency outer raceway |
| f_c | cage (rotational) frequency |
| $F_{dj(\text{grad})}$ | gradual variation in the contact forces at the rolling element-to-raceway contact interfaces within a defective rolling element bearing |
| F_{dj} | contact force at a j th rolling element-to-raceway contact interface within a defective rolling element bearing |
| F_{dx} | horizontal rolling element-to-raceway contact force for a defective rolling element bearing in the global cartesian x -direction |
| F_{dy} | vertical rolling element-to-raceway contact force for a defective rolling element bearing in the global cartesian y -direction |
| f_{noise}^i | rolling element-to-inner raceway rolling contact noise frequency |
| f_{noise}^{i-o} | beating noise frequency |
| F_j | contact force at a j th rolling element-to-raceway contact interface within a non-defective rolling element bearing |
| F_{max} | maximum force at a rolling element-to-raceway contact interface within a rolling element bearing along the load line (y -axis) |
| f_{noise}^o | rolling element-to-outer raceway rolling contact noise frequency |
| f_{rc} | ring frequency of a cylindrical shell |
| f_s | shaft rotational (run speed) frequency |
| F_x | horizontal rolling element-to-raceway contact force for a non-defective rolling element bearing in the global cartesian x -direction |
| F_y | vertical rolling element-to-raceway contact force for a non-defective rolling element bearing in the global cartesian y -direction |
| H_a | height of the adapter within the FE model of the rolling element bearing |
| h_c | thickness of the cage within the FE model of the rolling element bearing |
| H_d | depth (height) of the outer raceway defect within a rolling element bearing |
| h_i | thickness of the inner ring within the FE model of the rolling element bearing |
| h_o | thickness of the outer ring within the FE model of the rolling element bearing |
| I | impulsive force |

Nomenclature

| | |
|------------|---|
| i | imaginary unit ($= \sqrt{-1}$) |
| j | rolling element |
| K | contact stiffness at the interface of two isotropic elastic solid bodies |
| k_{zm} | modal wavenumbers |
| k_{cs} | contact or spring stiffness at the interface of two contacting segments in an FE model |
| K_{dj} | stiffness at a j th rolling element-to-raceway contact interface within a defective rolling element bearing |
| l | length of two contacting isotropic elastic solid bodies |
| L_{10} | life of a rolling element bearing |
| L_d | length of a localised raceway defect |
| L_e | length of an extended defect |
| l_{fe} | smallest characteristic dimension of an element within an FE model |
| l_r | length of the rolling elements within a bearing |
| m | axial mode numbers |
| m_1, m_2 | masses of two segments in contact within an FE model |
| n | circumferential mode numbers |
| N_r | number of rolling elements within a bearing |
| n_s | rotational speed of a rolling element bearing |
| N_w | window length |
| o' | initial point of contact between two non-conformal isotropic elastic solid bodies |
| P_{\max} | maximum pressure at the interface of two contacting isotropic elastic solid bodies |
| Q | quality factor of a second-order notch filter |
| r_c | mean radius of a cylindrical shell |
| R'_d | curvature difference of two contacting isotropic elastic solid bodies |
| R' | curvature sum of two contacting isotropic elastic solid bodies |

| | |
|--------------|--|
| R_x | equivalent radius of curvature of two contacting isotropic elastic solid bodies in the global cartesian x -direction |
| R_z | equivalent radius of curvature of two contacting isotropic elastic solid bodies in the global cartesian z -direction |
| S_d | profile of the outer raceway defect within a rolling element bearing |
| T | time period of defect-related impulses |
| t | time vector |
| u_y | displacement of a node within the FE model of the rolling element bearing in the global cartesian y -direction |
| V | stressed volume of the bearing material |
| v_y | velocity of a node within the FE model of the rolling element bearing in the global cartesian y -direction |
| W | radial (vertical) load in the global cartesian y -direction |
| w_a | width of the adapter within the FE model of the rolling element bearing |
| $x(t)$ | time-varying signal |
| $\hat{x}(t)$ | analytic signal |
| Z | number of cycles of repeated (stress) loading within a rolling element bearing |
| z_0 | depth at which maximum stress at the rolling element-to-raceway contact interfaces occurs |

Greek Symbols

| | |
|----------------------|---|
| α | contact angle within a rolling element bearing |
| β_j | a factor for introducing gradual changes at the entry and exit edges of a defect within a rolling element bearing |
| δ_1, δ_2 | deformation of isotropic solid elastic bodies ‘1’ and ‘2’ |
| δ | total deformation at the contact interface of two isotropic elastic solid bodies |
| δ_{dj} | total contact deformation at a j th rolling element-to-raceway contact interface within a defective rolling element bearing |
| δ_i | displacement of the inner ring of a rolling element bearing |
| δ_j | displacement at a j th rolling element-to-raceway contact interface within a rolling element bearing |

Nomenclature

| | |
|----------------------|---|
| δ_{\max} | maximum displacement at the rolling element-to-raceway contact interface within a rolling element bearing along the load line (y -axis) |
| δ_o | displacement of the outer ring of a rolling element bearing |
| δ_x, δ_y | deformation at the rolling element-to-outer raceway contact interfaces within a rolling element bearing in the global cartesian x - and y -directions, respectively |
| ϵ | load distribution factor for a rolling element bearing |
| γ_j | a factor that zeros the load distribution outside the load zone within a rolling element bearing |
| κ | Weibull slope for the experimental life of a rolling element bearing |
| λ_b | bending wavelength |
| μ | coefficient of friction between mating bearing components in the FE model of the bearing |
| ω_{mn} | natural frequencies of the outer ring of the FE model of the rolling element bearing |
| ν_1, ν_2 | Poisson's ratio of isotropic elastic solid bodies '1' and '2' |
| $\Delta\Omega$ | band width of a second-order notch filter |
| ω_c | angular velocity of the cage or rolling elements within a bearing |
| Ω_o | notch frequency of a second-order notch filter |
| ω_s | angular velocity of the shaft on which a rolling element bearing is installed |
| ψ_0 | initial angular position of the cage within a rolling element bearing |
| ψ_c | angular position of the cage within a rolling element bearing |
| $\Delta\psi_{cw}$ | angular extent of contact width $2b$ at the rolling element-to-raceway contact interfaces within a rolling element bearing |
| $\Delta\psi_d$ | angular extent of the outer raceway defect within a rolling element bearing |
| ψ_d | centre of the outer raceway defect within a rolling element bearing |
| ψ_j | angular position of a j th rolling element |
| ψ'_l | half-angular extent of the bearing load zone centred at ψ_{lc} |
| ψ_{lc} | centre of the bearing load zone |

| | |
|------------------------------|--|
| ρ | material density |
| $\Delta t_{\text{critical}}$ | critical time step for the explicit time integration scheme used in LS-DYNA |
| Δt_{event} | time difference between the consecutive de-stressing or re-stressing events |
| Δt_{stable} | stable time step used in LS-DYNA |
| θ_r | angular spacing between the rolling elements within a bearing |
| Υ | probability of survival of a rolling element bearing |
| ε_0 | maximum orthogonal shear stress in the rolling element-to-raceway contact interfaces |
| ς | diametral clearance within a rolling element bearing |
| ζ | damping ratio |

Miscellaneous Symbols

| | |
|---------------|------------------------------|
| \mathcal{D} | Dirac delta function |
| \mathcal{F} | Fourier transform |
| \mathcal{H} | Hilbert transform |
| \mathcal{K} | spectral kurtosis |
| \mathcal{S} | short-time Fourier transform |

Superscripts

| | |
|---------|--|
| i | inner raceway |
| $i - o$ | inner-to-outer raceway |
| n | exponent — $n = 3/2$ for point, circular and elliptical contacts, and $n = 10/9$ for line and rectangular contacts |
| o | outer raceway |

Subscripts

| | |
|------|--|
| 1, 2 | isotropic elastic solid bodies ‘1’ and ‘2’ |
| b | bending waves |
| bpi | ball pass inner raceway |
| bpo | ball pass outer raceway |
| c | cage for retaining the rolling elements within a bearing |

Nomenclature

| | |
|------------|--------------------------------------|
| <i>cw</i> | contact width |
| <i>d</i> | defective rolling element bearing |
| <i>e</i> | extended defect |
| <i>fe</i> | finite element |
| <i>i</i> | inner raceway |
| <i>j</i> | rolling element |
| <i>lc</i> | centre of the load zone |
| <i>max</i> | maximum |
| <i>o</i> | outer raceway |
| <i>p</i> | bearing pitch |
| <i>rc</i> | cylindrical shell |
| <i>s</i> | shaft |
| <i>x</i> | global cartesian <i>x</i> -direction |
| <i>y</i> | global cartesian <i>y</i> -direction |
| <i>z</i> | global cartesian <i>z</i> -direction |

Abbreviations

| | |
|-------|--|
| 2-D | two-dimensional |
| 3-D | three-dimensional |
| AAR | Association of American Railroads |
| ABMA | American Bearing Manufacturers Association, Inc. |
| ADINA | Automatic Dynamic Incremental Nonlinear Analysis |
| ADORE | Advanced Dynamics of Rolling Elements |
| ANSI | American National Standards Institute, Inc. |
| BEAST | Bearing Simulation Tool |
| BEAT | BEARing Toolbox |
| BPFI | ball pass frequency inner raceway |
| BPFO | ball pass frequency outer raceway |

| | |
|-----------------------|--|
| COBRA | Computer Optimized Ball and Roller Bearing Analysis software |
| CW | clockwise |
| CWRU | Case Western Reserve University |
| DOF | degree-of-freedom |
| EHL | elasto-hydrodynamic lubrication |
| EPW | elements-per-wavelength |
| FE | finite element |
| FFT | Fast Fourier Transform |
| IBDAS | Integrated Bearing Dynamic Analysis System |
| ISO | International Organization for Standardization |
| PSD | Power Spectral Density |
| RailBAM [®] | Railway Bearing Acoustic Monitor |
| RMS | root mean square |
| RPM | revolutions per minute |
| SFM | scale factor on default master penalty stiffness |
| SFS | scale factor on default slave penalty stiffness |
| SK | spectral kurtosis |
| SLSFAC | scale factor for sliding interface penalties |
| STFT | short-time Fourier transform |
| TADS [®] | Trackside Acoustic Detection System |
| Track IQ [™] | Trackside Intelligence Pty. Ltd. |
| TTCI [®] | Transportation Technology Center, Inc. |