THE STRUCTURE OF SEQUENTIAL EFFECTS

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Contents

Abstract

Research into sequential effects has a long and rich history spanning almost one hundred years. In their most general definition sequential effects can simply be considered a dependence of behaviour on the past sequence of events, and are of the most pervasive phenomena in psychology. Some form of sequential effects has been observed in multiple perceptual and cognitive tasks, and across different modalities. In addition, sequential effects have also been observed in electrophysiological studies, with a great deal of similarity observed between EEG and behavioural results, making this a relevant topic for both psychology and neuroscience. This gives sequential effects a great deal of potential as a doorway for elucidating the relationship between human behaviour and neuronal activity, between the mind and the brain.

Yet perhaps in part because of the great diversity of domains in which sequential effects are observed, this is an often fragmented field of research, with a multitude of experimental paradigms used, often leading to some confusion as to how different results are related to each other. One of the main objectives of this work is therefore to begin to unify the field into one coherent whole, and to do so at both a computational and process levels. To begin with, Chapter 2 addresses the computational nature of sequential effects in terms of different types of statistics humans use in different circumstances. In Chapter 3 it is shown that the most results described before in the literature can be explained by only three components, including a wealth of individual differences which had been largely ignored so far.

On a more theoretical level it could be argued that there is a degree of redundancy between the various mathematical models of sequential effects proposed over the years. Models are usually fit to isolated datasets, when it is well known that even minor experimental manipulations can lead to different results, making it unclear how conclusions extend to other settings. Moreover, by virtue of their common mathematical structure, most models of sequential effects suffer from similar difficulties in reproducing key empirical observations. This, together with other considerations, motivates an entirely different approach to modelling sequential effects proposed in Chapter 4. The framework suggested is based on the physics of oscillatory motion, being continuous-time in nature and able to incorporate space, reflecting the fact that both time and space have been found empirically to play a role in sequential effects.

More generally there are two central proposals which unify this dissertation. Firstly that sequential effects are the consequence of two main independent components possibly related to the separate processing of stimuli and responses. Secondly that sequential effects reflect some form of filtering implemented through interaction with an oscillatory system.

Declaration by author

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List of Figures

List of Tables

Introduction

This dissertation is made up of three research chapters, a literature review and a general discussion. The unifying theme is the structure of sequential effects, both at a computational - Chapter 2 - and process - Chapters 3 and 4 - levels. Chapters 2 and 3 are manuscripts prepared for publication: Chapter 2 is an expanded version of a conference proceedings article already published, and a closely related version of Chapter 3 has been submitted to the Journal of Experimental Psychology General. Because of this Chapters 2 and 3 are self-sufficient entities and some degree of redundancy may exist in the introduction to the two chapters in what concerns basic concepts about sequential effects. What follows is a short summary of each chapter and its main conclusions.

Chapter 1: Literature review - A broad scope review of the overall field of research, with particular emphasis on sequential effects in reaction time, the main focus point of this work. Some attention will also be given to sequential effects in EEG despite no electrophysiological data having been collected, the reason being that results throughout are discussed in light of such data acquired by other researchers. Finally, this section also includes a review of most quantitative models of sequential effects proposed so far given the focus on mathematical modelling throughout.

Chapter 2: Humans use different statistics depending on the task - A computational level analysis of the type of statistics humans use in different tasks depending on the number of alternative stimuli. A statistical model is proposed which is able to differentiate between the use of different types of statistics in sequential effects. In addition, an experimental paradigm is introduced which allows for the expansion of the traditional two-alternative forced-choice (2AFC) design to include any number of alternatives with minimal confounding effects. The main conclusion of this chapter is that humans use first order transition probabilities in a 2AFC but switch to using zero-th order statistics - i.e. the relative frequency of the stimuli - in a 3AFC.

Chapter 3: The structure of sequential effects - A principal component analysis of individual reaction time data from over one-hundred and fifty participants performing different versions of a 2AFC. Three latent variables related to sequential effects are identified, two main and one minor. A relationship between the two main components and two separate processing stages of sequential effects identified before is proposed. It is argued that the third minor component is related to processing delays. The same two main components are found for different values of the response-stimulus interval, thereby unifying what were previously thought to be qualitatively different results. Finally, the way in which changes in the relative contribution of the three components explains different patterns of sequential effects, both collectively and individually, is analysed.

Chapter 4: An oscillator-based framework for sequential effects - A new framework for modelling sequential effects at the process level is proposed, based on the physics of oscillatory motion. Although no definitive model is proposed, it is argued that overall the framework shows a great deal of potential for explaining several aspects of sequential effects, some of which are hard to formalise in the context of any previous model, such as response-stimulus compatibility and individual differences. Early successes of the modelling framework include its capacity to naturally produce patterns of results which have been historically hard to capture. In addition, even a single oscillator is found to reproduce key aspects of the latent structure of sequential effects with only minimal assumptions about the nature of individual differences. Some limitations of the general approach are discussed, and a more nuanced view of sequential effects as some form of spatio-temporal filtering is first suggested.

Chapter 5: Discussion - An attempt is made at putting the field of sequential effects into perspective, in particular on a theoretical level, by drawing a distinction between two different views of sequential effects: a classical or statistical view and a two independent component view. This distinction is also useful in putting into context the main results of this dissertation, which overall add some support for the two-component view. The view of sequential effects as reflecting some form of spatio-temporal filtering is expanded upon. Future directions are also discussed, with particular emphasis on how the model proposed in Chapter 4 could be extended. Finally, a short section outlines an overall principle for how the mind works as a pattern formation/detection device, and how sequential effects may be a reflection of this principle at work.

1

Literature review

Most tasks in psychology consist of a long sequence of discrete trials differing in the stimuli which are presented at each time point. Early in the history of psychological research it became evident that the response on a particular trial could depend on the previous sequence of trials, particularly if the sequence exhibits a regular pattern, in which case a strong expectation about the nature of the next trial might develop. These expectations influence responses, often to a greater extent than the properties of the stimuli themselves, thereby potentially confounding results. So from the outset the ordering of the trials was made random, and it was thought that by this procedure the influence of the sequence would, if not altogether disappear, hopefully average out.

It was not long before some researchers turned to the effect of the sequence as a point of interest in itself, irrespective of any concerns harboured about possible confounding effects. One question in particular is whether a random sequence of trials retains any capacity to influence behaviour. Even a random sequence can appear to be regular at times; if such fleeting patterns emerge will they influence expectations or be dismissed as part of a random process? It turns out that humans will persistently show a dependence of behaviour on the last few trials even after a long random sequence. It is this dependence of behaviour on the last few trials of a random sequence which will be referred to throughout as ‘sequential effects’, by contrast with the influence a perfectly regular sequence may have.

Sequential effects are observed in different aspects of behaviour such as judgements (Fernberger, 1920), reaction time (Bertelson, 1961) and predictive guesses (Jarvik, 1951). Moreover, they happen in different modalities (K. C. Squires, Wickens, Squires, & Donchin, 1976), and even across modalities (Ward, 1979), highlighting the very general nature of the phenomenon. Perhaps most striking of all is the capacity sequential effects have of influencing what is actually perceived, as manifest in changes induced in the psychophysical point of subject equality (Maloney, Dal Martello, Sahm, & Spillmann, 2005). Finally, sequential effects are also observed at a neurophysiological level: properties such as the amplitude of event related potentials (ERP) measured with with electroencephalography (EEG) display a dependence on the sequence of events very similar to that which is observed behaviourally (K. C. Squires et al., 1976; Sommer, Matt, & Leuthold, 1990).

Historically, sequential effects have been studied in a diverse range of tasks falling under three main categories:

- Psychophysical tasks: in this case the dependent variable can be any type of judgement made about properties of the stimuli presented.

- Prediction tasks: in this case the proportion of times a particular event is predicted forms the dependent measure
- Speeded decision making tasks: where subjects must react quickly to the next stimulus, in which case sequential effects are observed in reaction time.

A crucial ingredient in designing experiments where sequential effects occur is *uncertainty*, either about the properties of the stimulus itself or what the next stimulus will be. Consider a psychophysical task: if two stimuli are very distinct any judgement of whether they are different will essentially be, save for some glitch, always the same; this situation can described as *stimulus determined* (Senders & Sowards, 1952), in that no amount of structure in the sequence will influence the outcome. The opposite situation is one where the stimuli are objectively equal, in which case uncertainty is maximal by definition; under these circumstances the influence of the sequence of events is greatest, a situation that can be described as *sequence determined*. Most tasks lie at some point between fully stimulus or sequence determined and in these intermediate situations behaviour can depend on both the properties of the stimuli as well as the sequence. In general, the more uncertainty there is, the greater the capacity for the sequence to influence behaviour. This may be considered part of a more general human tendency to use concrete information if it is available, while switching to an attempt at finding a pattern if little or no information available, and finally even finding a pattern where there is not one to be found, such as in the case of the so-called gambler's fallacy (Nickerson, 2002).

In the beginning this literature review will follow a largely chronological order. As it progresses there will be more emphasis on specific themes, while still respecting a roughly chronological order. This is true except in one respect: the empirical and theoretical literatures will be discussed separately, first the former and in the end the latter. To begin with the early empirical literature, from 1920 to 1961, will be reviewed, during which time sequential effects were studied largely in the context of psychophysical judgement tasks or tasks involving predictions about the next event. The focus will then turn to sequential effects in reaction time - the main focus of this work - and the area of choice for research on sequential effects *per se*. Next the literature on sequential effects in electroencephalography (EEG) - observed for the first time in 1970 - will be reviewed. Finally, quantitative models of sequential effects will be given special attention given the focus on mathematical modelling throughout this work. This literature review is broad in scope and so, while all of its content is of some relevance, some topics of particular importance towards understanding this dissertation will be highlighted at the end of this section.

Following the history of research into the subject itself, a somewhat less rigorous approach will be taken when discussing different experiments as to the type of sequential effects found. As we progress, and again mimicking the field, more rigour in contrasting experiments will become necessary, as even seemingly small differences in design have been found to have a strong impact on sequential effects.

1.1 Early history - 1920 to 1960

In order to ensure experimental results were due to the properties of the stimuli of interest, early psychophysical experiments strived to remove what were then termed the ‘time error’ and the ‘space error’, i.e. the influence of timing differences between trials or of differences in the movements necessary to perform each trial (e.g. Fernberger, 1920). The time error was removed by marking the interval between trials with a metronome; the space error was removed by making sure exactly the same motion was performed on each trial, for instance by having stimuli move automatically to within reach rather than have subjects extend their arm to different locations. This tradition is still followed in modern times through the use of computers to time events and complex contraptions that restrict the number of degrees of freedom in subjects’ movements. However, and despite all precautions, subjects do not act in a manner independent of the order of events, even if this is made random.

Some form of sequential effect was possibly observed in early psychophysical experiments in the nineteenth century,[1] but the first study dedicated specifically to the subject was conducted well into the twentieth century (Fernberger, 1920) in a task which involved comparing two different weights in each trial. Fernberger observed that when, on a given trial, the reference weight was judged to be 'lighter', in the next trial the reference weight would be more likely to be judged 'heavier', and vice-versa with 'lighter' judgements more likely to follow 'heavier' ones. In line with the role of uncertainty in sequential effects, this *alternation effect* was greater the closer the two weights being compared were. This was the first description of what would become known as a general human tendency to alternate judgements across many different types of tasks.

Early on, researchers noticed an important ambiguity about what is driving sequential effects: is it the physical magnitude of the stimuli or the judgements made about those same stimuli? For instance, in the example above, does one tend to respond 'heavier' after a judgement of 'lighter' because of a shift in the point of subjective equality, which makes the current one seem lighter by contrast, or because of a tendency to avoid making the same judgement? In an attempt to resolve the ambiguity Turner (1931) studied a serial weight lifting task similar to Fernberger's and contrasted two types of situation: one in which judgements were emitted on two consecutive trials and another in which a judgement trial would follow a trial in which a judgement was omitted. Turner presents evidence which, on the face of it, would have suggested that the effect of judgement and stimulus magnitude could go in opposite directions, i.e. the sequence of judgements induces an alternation bias and the stimulus magnitude a repetition bias[2]. Despite this, Turner argued that the tendency to alternate judgements was due to changes in the point of subjective equality (Turner, 1931), and therefore due to purely perceptual effects. Later, a different experiment suggested that the alternation effect occurs even when the stimuli are equal - an important control overlooked by

[1] A great number of works from this era is in German, a language that the author regrettably does not master.

Turner - demonstrating that sequential effects are at least in part determined by the sequence of judgements (Arons & Irwin, 1932; Preston, 1936). Interestingly Arons and Irwin (1932), though secondarily to the main point of their article, reported individual differences in sequential effects for the first time: some subjects tended to alternate judgements but others actually tended to repeat them.

During late 1937 and early 1938 a large scale experiment in 'telepathy' was performed by the Zenith radio station in Chicago (Goodfellow, 1938). Every Sunday evening, a radio program was broadcast in which the audience would have to guess which of two symbols a group of 'telepathic senders' - ten university students locked inside a room - was focussing on, with five symbols to be guessed in total. There was no feedback about the real symbols being focussed on, merely a signal from the host indicating that the next trial was on. Participants wrote down their five guesses, and then mailed these back to the radio station. Over 20000 participants took part in several versions of the experiment which made this - unless one believes in telepathy - a large scale experiment in random sequence generation. The results of these experiments are relevant to sequential effects research in that the only feedback available to the participants was the sequence that they themselves had generated up to a point. This arguably makes the Zenith radio experiments a sequential behavioural task, but without any objective stimuli. More generally, random sequence generation is closely related to sequential effects if one considers a sequence to be generated in a step-by-step fashion, as opposed to the it being generated previously and then merely written down. As discussed below there is some debate on this point, with followers of the Gestalt school preferring the whole sequence at once interpretation, and followers of the behaviourist school defending a sequential generating process.

The results of the Zenith radio experiments, while providing no evidence for telepathy, did show that the choices made by the radio listeners were above or below the 50% chance level on

[2] Throughout the terms 'bias' or simply 'preference' will be used to refer to mean differences between repeating and alternating trials, irrespective of the sequence before them.

almost every trial of every sequence. This meant that, while insensitive to the 'senders' or the 'true' sequence, the mechanism used by the audience when generating sequences was clearly not random. Goodfellow (1938) analysed the data from the Zenith radio experiments in terms of the frequencies of all the 16 possible types of five-long binary sequences generated by the audience. This was the first time sequences of stimuli longer than three were taken into consideration when analysing sequential effects, and also the first time sequences were grouped two-by-two according to the pattern they represented - e.g. 12111 and 21222. However, Goodfellow, a follower of the Gestalt school of psychology, performed this analysis in the spirit that the sequence consists of one whole percept rather than a string of stimuli, and suggested the explanation for the results was related to the avoidance of symmetry - as per his own and unusual definition[3] - in the sequence.

In 1942, B.F. Skinner re-analysed the results of the Zenith radio experiment, but this time assuming that some form of sequential mechanism was at play (Skinner, 1942). Skinner argued that the data was more parsimoniously explained by a general tendency to alternate judgements rather than the symmetry of the sequence as defined by Goodfellow. Skinner was also the first to rewrite the sequences of stimuli in terms of repetitions and alternations, and recalculated the raw frequencies of each possible sequence as the proportion of times subjects chose to repeat or alternate *after the same preceding sequence*. For instance, if the absolute frequency of ARRA[4] is 0.1, and that of ARRR is 0.3, the recalculated frequencies would be 0.25 and 0.75 respectively. A subset of the Zenith experiments data as calculated by Skinner is shown in Figure 1.1.

Yacorzynski (1941) analysed the production of random binary sequences in a manner similar to that of the Zenith experiments except that participants were psychotic patients. The author reported a control group to produce results similar to those of the Zenith experiments and different to those

[3] Goodfellow counted the number of symmetries in all subsequences of length 2, 3, 4 and 5, while at the same time adding extra weight if symmetries were found in the first three stimuli in order to 'give a little extra weight to the first three items since they have found them particularly important in psycho-physics'. No reference for the importance of the first three stimuli is given, and in fact it is now well known that it is the last few stimuli which carry a greater weight.

[4] Here 'R' stands for a repetition and 'A' for an alternation of the stimuli, with sequences read from left to right.

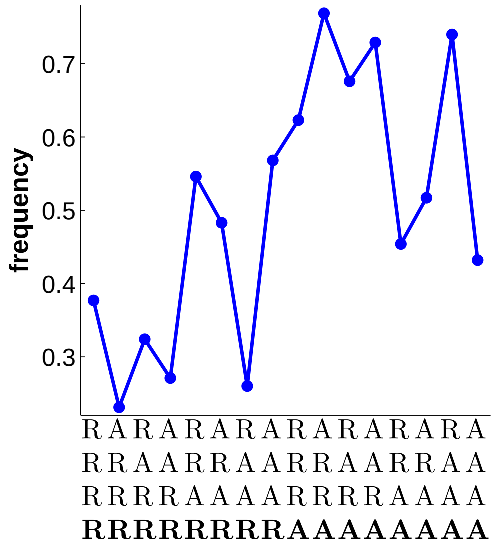

FIGURE 1.1: Data from the Zenith radio experiments as reported by Skinner (1942). Sequences should be read from top to bottom, an ‘R’ standing for a repetition and an ‘A’ for an alternation. Note that the left side of the plot - the repetition curve - contains all sequences ending with a repetition, and the right side all those ending in an alternation. The higher order sequences - the sequences up the last event - are ordered the same way in the left and right sides of the plot. The last event is shown in bold because this is the point at which a dependent measure is usually being taken - reaction time in most cases throughout this work. In this particular case the data consists merely of the relative frequency of sequences generated by the participants in the radio experiment and so the use of bold for the last event is meaningless.

of manic-depressive and schizophrenic patients. Interestingly, he also found the results of the groups with different types of pathology to be significantly similar to each other. Also a follower of the Gestalt school of psychology, Yacorzinsky interprets his results in terms of the abandonment of the principle of symmetry[5] in psychotic patients, a concept borrowed directly from Goodfellow (1938). The study suffers from low sample numbers, making any conclusions doubtful, but it stands as the first study of the potential relationship between sequential effects and different types of psychological disorder. Skinner (1942), in a separate analysis, concludes that Yacorzinsky’s data from psychotic patients is well explained by a tendency to alternate irrespective of stimulus history, i.e. without being influenced by previous stimuli.

Initially sequential effects were studied in tasks where there was no feedback regarding the

[5] Skinner - the father of behaviourism - is quite dismissive of what he quotes as ‘disintegration of perceptual principles’ prompting a reply from Yacorzinsky (Yacorzynski, 1943) - which never used that exact sentence - and counter-reply by Skinner (Skinner, 1943).

outcome of previous trials. However, theories of reinforcement (Hull, 1943) suggested that feedback might influence the relative proportion of repetitions and alternations of judgements. For instance, a successful trial in a prediction task might induce a repetition of the previous guess, with the influence of negative feedback being less clear.[6] Bendig (1951) investigated the effect of reinforcement on sequential effects and found the number of alternations to decrease proportionally with the number of 'reinforcements', i.e. successes. Surprisingly though, Bendig does not analyse data on a trial-by-trial basis, and merely counts the total number of successes and of alternations during a task, while at the same time analysing correlations between the two. It is worth noting that there seems to be some confusion in the literature at this point regarding what a 'reinforcement' is: while Bendig takes a successful guess to be a reinforcement, other authors consider it to be simply the presentation of a given stimulus (e.g. Jarvik, 1951).

Most experiments up to 1950 had found a tendency to avoid repetitions in humans (Fernberger, 1920; Thorndike, 1927; Turner, 1931; Arons & Irwin, 1932; Preston, 1936; Goodfellow, 1938; Irwin & Preston, 1937; Solomon, 1949) as well as mice (Heathers, 1940). This seemed to fit well with the popular theory at the time that a refractory period existed after a particular action which inhibited its performance for a some time, a concept termed 'reactive inhibition' (Hull, 1943). This theory had its roots in the physiological properties of neurons where a refractory period exists after each impulse during which the production of a second impulse is inhibited. Whether or not these physiological effects extended to higher cognitive faculties was a matter of debate (Dodge, 1926; Thorndike, 1927).

One clear prediction of the reactive inhibition hypothesis was that the tendency to avoid repetitions should depend on the interval between the trials. In particular, a shorter interval between trials should lead to a greater tendency to alternate responses. Solomon (1949) analysed the influence of the interval between trials in a coin toss guessing task. Solomon used two different intervals - 15

[6] Note that, if negative feedback was perfectly symmetrical to positive feedback, the two effects would cancel out on average. So in order to postulate an effect of positive reinforcement on sequential effects, it is necessary to assume some degree of asymmetry between both types of reinforcement.

sec and 8 min - between stimuli and found no evidence of an effect of the inter-stimulus interval on the tendency to alternate predictions, with both groups displaying an alternation effect. In any case it soon became obvious that the assumption that humans tended to alternate was not always true, with some researchers finding the very opposite, i.e. a tendency to repeat guesses (Senders & Sowards, 1952; Day, 1956).

Whether or not the reactive inhibition theory was true, the question remained of whether sequential effects were due to temporary effects and therefore dependent on the interval between trials. Starting with Senders and Sowards (1952), several authors turned to autocorrelation analysis in order to analyse sequential effects, a method which allows conclusions to be drawn about the presence or absence of serial correlations in the data but says little or nothing about what type of dependence on the sequence may be present (Senders & Sowards, 1952; Abelson, 1953; Day, 1956). Day (1956) in particular analysed the effect of the inter-stimulus interval in a psychophysical task using auditory stimuli at 50% discrimination threshold and found the interval between stimuli to have a significant impact on autocorrelations. When the interval was short - 1.6 to 2.1 sec - there were significant serial correlations in the data; when it was long - 4.2 to 10.6 sec - there were no such serial correlations. An almost equal result was obtained by Abelson (1953).

The 1950's mark a turning point in the history of sequential effects, with the beginning of a differentiation between low level physiological and higher level cognitive effects emerging. As early as 1949 Solomon states sequential effects in a coin tossing task are best explained by 'subjects' conception of chance' rather than 'basic response mechanisms' (Solomon, 1949). This is echoed by Jarvik (1951) in a work often cited as the first in the literature on the perception of randomness. Jarvik describes what he coins a 'negative recency effect': the longer a run of the same stimulus is, the greater the tendency is for subjects to guess the next rial will alternate (see Figure 1.2). Jarvik compares this phenomenon to the 'gambler's fallacy' experienced by gamblers who irrationally expect an alternation to be more likely after a repetition.

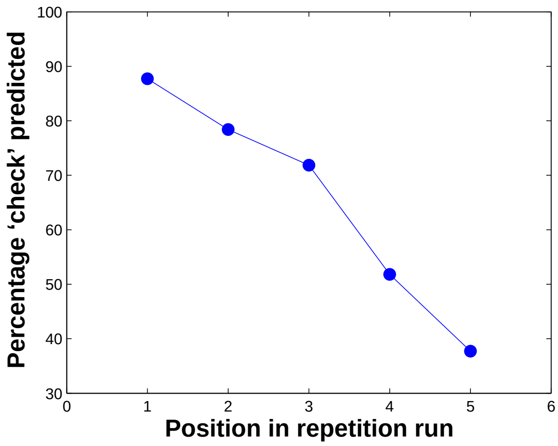

FIGURE 1.2: Data from Jarvik (1951) showing predictions made as a function of position in runs of 5 consecutive equal stimuli. There were two possible symbols - 'check' or 'plus' - but the proportion of 'check' was always greater, with different percentages used - 60, 67 and 75% - and the runs shown are only of 'check' symbols. The task consisted of guessing what the next symbol in a random sequence of the two possible symbols would be. Each symbol was read out to subjects by the experimenter at approximately 4 sec intervals, each trial commencing with a prediction made by the subject when cued by the experimenter. Note the overall shift of the curve towards high percentages of 'check' due to long-term probability learning effects since sequences were richer in 'check' symbols.

Cognitive effects such as those suggested by Jarvik (1951) were hard to reconcile with a mechanistic view of passively decaying traces of activity such as the reactive inhibition theory. Furthermore, it soon became clear that top-down modulation of sequential effects was possible by changing the verbal instructions given to subjects (Goodnow, 1955), even in reaction time tasks (Kirby, 1976). It was also found that prediction and reaction time tasks could yield very different results under the same conditions (see below). Overall, this meant that the fields of sequential effects and randomness perception would remain largely separate, despite some early promising hints that the way humans perceive randomness was related to a sequential process (Skinner, 1942). Sequential effects *per se* would continue to be studied mostly in the context of reaction time tasks, with randomness perception making use of prediction tasks or tasks in which subjects have to produce random sequences.

Many of the early studies in which sequential effects were first observed were primarily concerned with varying the probabilities of the different stimuli (Jarvik, 1951; Hyman, 1953; Bertelson, 1961) leading to long-term probability learning effects. For instance, in prediction tasks subjects adjusted their guessing strategies to match the probabilities of the intervening stimuli quite closely (e.g. Hake & Hyman, 1953). On the other hand, average reaction times for entire sequences were shown to depend on the relative probabilities of the intervening stimuli, i.e. reaction times were faster for a sequence rich in one particular stimulus, as opposed to one with balanced probabilities (Hyman, 1953; Bertelson, 1961). Several authors would eventually argue that the effects of long-term probability and short-term sequential effects are not separable (Laming, 1968; Falmagne, 1965; Kornblum, 1969; Audley, 1973). In particular, Audley (1973) demonstrated mathematically that long-term probability learning effects emerge naturally as a consequence of short-term probability effects in a simple model - a geometric average of the previous sequence of events - which has since been found to underpin all quantitative models of sequential effects (see below).

Sequential effects in one form or another would continue to be observed in a wide range of different experimental paradigms such as: psychophysical tasks within (Verplanck, Collier, & Cotton, 1952; Day, 1956; Treisman & Williams, 1984; Maloney et al., 2005) and across modality (Ward, 1979), affective judgements (Willingham, 1959), a Simon task (Notebaert, Soetens, & Melis, 2001), categorization of diverse stimulus properties such as loudness (Garner, 1953; Jesteadt, Luce, & Green, 1977; Parducci, 1964; Staddon, King, & Lockhead, 1980), just to cite a few examples. A review of this extensive literature would be too lengthy and is not the aim here. Moreover, many of these studies do not go much beyond reporting some form of sequential dependence, with results seldom analysed in detail or put into wider context.[7] Reaction time studies would eventually emerge as the paradigm of choice when studying sequential effects *per se*, and more advanced theories would be developed based largely on these studies. Whether conclusions are extensible to other experimental settings is often unclear, and so throughout this work the expression ‘sequential effects’ should strictly speaking be taken to refer to sequential effects in reaction time. Notwithstanding this, strong commonalities have now been observed in the pattern of sequential effects across different experimental settings - including electroencephalographic (EEG) studies - which points to the universality of at least some of the conclusions from reaction time studies, which are reviewed next.

1.2 Sequential effects in reaction time

The study of human reaction times in general has a long history extending back well into the 19th century (H. Johnson, 1923). Despite this, the first mention of sequential effects in such studies was made with respect to error rates, not reaction times: Hansen (1922) observed that errors in sequential tasks were higher for the ‘stimulus that was absent the longest’. Beyond this early superficial observation, the first mention of sequential effects in reaction time was only made in 1953 and only in 1961 was the first dedicated study of the subject conducted (Hyman, 1953; Bertelson, 1961).

1.2.1 The dominance of information theory

During the 1950’s, in the context of the dominance of information theory (Shannon, 1948), the trend was towards attempting to explain human reaction time in sequential tasks as a function of the information content of the sequence of stimuli (Hick, 1952; Hyman, 1953; Grossman, 1953). This relied on a simplified view of a human subject as a ‘channel’ between a source - the stimuli - and an output - the responses. Reaction time was assumed to be proportional to the time required to extract information from a sequence, and therefore proportional to its information content (Hick, 1952). The quantitative and precise nature of the model and its predictions was attractive and it had

[7] There are some notable exceptions to this though, such as the work of Maloney et al. (2005)

the side-effect of turning attention away from the sequence itself. This is perhaps best illustrated by Hyman's (1953) statement that 'the successive stimuli do no alter S's knowledge of the statistical properties of the stimulus series as a whole' and further that one must exclude 'those situations wherein S gains new knowledge concerning the statistical structure of the stimulus series as the series progresses in time'. Therefore, the trade-off for the gains made in mathematical precision was that entire sequences should be viewed as a whole, a view reminiscent of that held by followers of the Gestalt school of thought two decades before.

At first the information content of a sequence seemed to provide a good fit to overall reaction time (Hick, 1952; Hyman, 1953). Over time however it became clear that many other factors affected reaction times, such as stimulus-response compatibility (Bertelson, 1963), response-stimulus interval (Bertelson & Renkin, 1966; D. J. Hale, 1967), or even if one or two fingers were used to respond (Hannes, 1968), all of which did not affect the information content of a sequence. Moreover, several studies started to focus on the role of the sequence of events (e.g. Bertelson, 1961) revealing differences in reaction time to repetitions or alternations of stimuli which pointed to a dependence of reaction times on the sequence itself. Finally, Kornblum (1967, 1968) demonstrated that the correlation between the amount of information and reaction time was an artefact: the way information content was varied in previous experiments meant that the probability of alternations of stimuli varied concomitantly. Kornblum constructed sequences of equal information content and different proportion of repetitions to alternations and showed it was the latter which was responsible for changes in overall reaction time.[8]

Further criticism of the way in which information theory had been used in analysing human reaction times is provided by Laming (1968). This is a somewhat surprising fact given that Laming's dissertation on the subject is titled 'Information theory of choice reaction times'. The key to this

[8] Kornblum uses a response-stimulus interval (RSI) of 140 ms, a value which was known to induce a strong preference for repetitions. Had he used a longer RSI his results would have likely been inconclusive. Kornblum was surely aware of this fact, as the effect of the RSI was well known by 1968, but fails to mention it. Nevertheless, his conclusions stand inasmuch as demonstrating that information content is not determining reaction times.

conundrum lies with the distinction made by Laming between ‘communication theory’ and ‘information theory’, the former being Shannon’s version as per his 1948 seminal article (Shannon, 1948), and the latter Laming’s own definition.[9] Upon closer inspection the model proposed by Laming is a sequential sampling random walk model which the author relates to a measure of information different from that which was used by Shannon (Kullback, 1959), and thus the use of the term ‘information theory’.

Starting in the 1960’s, researchers would finally turn away from information theory and begin analysing reaction times as a function of the sequence of events itself, rather than its information content, which gave rise to the field of research into sequential effects in reaction time.

1.2.2 A repetition effect in reaction time

As mentioned previously, the first observation of any kind of sequential effect in reaction time was made by Hyman in 1953. Hyman found a repetition effect - i.e. faster reaction time to repetitions of the same stimulus - in tasks with three or more alternative stimuli (Hyman, 1953). In contrast, in a task with only two possible stimuli, Hyman found the opposite: a slight alternation effect. Unfortunately, no quantitative evidence for the effect was given and it would be another eight years before some form of systematic investigation of sequential effects in reaction time was conducted, again in part due to the dominance of information theory during those years.

In 1961, Bertelson published what may be considered the inaugural article of the field of sequential effects in human reaction time (Bertelson, 1961). Bertelson used for the first time what would become the standard experimental paradigm in the field: the speeded two-alternative forced choice task, which he refers to as ‘self-paced 2-choice serial responding task’, and for which an

[9] The details of the distinction Laming attempts to make and its validity are beyond the scope of this review. Detailed critiques of the use of information theory in psychology can can be found in Laming (2001) and Luce (2003).

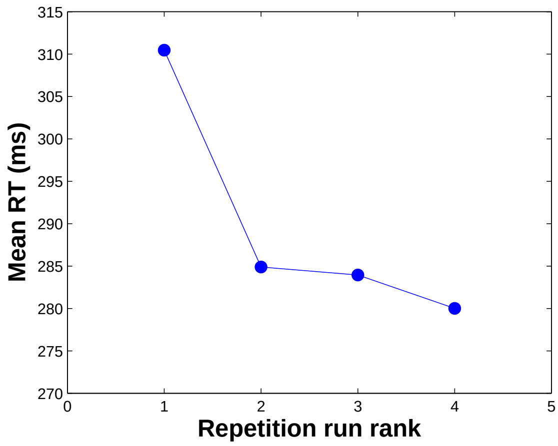

FIGURE 1.3: Repetition effect discovered by Bertelson (1961). Data was adapted from the original article and shows the decrease in reaction time with increasing length of a run of the same stimulus. Stimuli consisted of two light bulbs separated by 4.5 cm and a fixed response-stimulus interval of 50 m was used. However, since Bertelson does not take into account the time spent pressing the key, the true RSI is likely to be over 100 ms (Vervaeck & Boer, 1980). Bertelson analyses sequences with different proportions of the two stimuli, but the data shown is only for the 50:50 condition.

electrical apparatus was purposefully built. The task involved a fixed period between the moment a response is made and the onset of the next stimulus - the response-stimulus interval (RSI) - which effectively made the interval between stimuli a variable dependent on the reaction time.[10] The task is 'self-paced' in the sense that the subject can make it go faster by responding quicker. It is noteworthy that Bertelson measures the start of the response-stimulus interval from the moment the response key is lifted, not depressed, arguably making the true RSI longer than the values stated. Bertelson nonetheless stands out for his rigour in stating that this is the case clearly, with several authors failing to clarify this point.

One of Bertelson's objectives was to investigate whether sequential redundancies would improve performance in repetitive tasks. For this purpose the transition probabilities were manipulated while keeping the frequencies of both stimuli constant, allowing for the construction sequences rich in repetitions or alternations of events but balanced with respect to the frequency of the stimuli.[11] Bertelson describes for the first time what he coined a 'repetition effect': not only are

[10] Let RT denote the reaction time, RSI the response-stimulus interval and ISI the inter-stimulus interval. Then for this type of task ISI = RT + RSI.

reaction times overall faster to repetitions relative to alternations, but they decrease with increasing length of a run of the same stimulus (see Figure 1.3).

In order to explain the repetition effect, Bertelson postulates a 'facilitation' mechanism: a response to a particular stimulus would make responding to the same stimulus faster, possibly due to a residual trace of activity. The obvious implication is that by increasing the response-stimulus interval - and consequently the average inter-stimulus interval - the repetition effect would disappear. In order to investigate this possibility, Bertelson varied the RSI systematically and found a strong repetition effect with a 50 ms RSI which was greatly reduced when a 500 ms RSI was used instead, thereby lending some support to the idea of a decaying facilitation mechanism. Bertelson was the first to analyse the effects of the interval between stimuli in the context of a reaction time task, a type of analysis which was to be repeated afterwards several times in the context of theories of decaying traces of activation, often with conflicting results (see below).

After Bertelson, several other authors reported sequential effects in a variety of reaction time tasks (Williams, 1966; D. J. Hale, 1967; Schvaneveldt & Chase, 1969; Remington, 1969; Kirby, 1972). The results of these experiments were often compared with respect to whether a repetition or alternation effect was found, without due attention to experimental design differences (Kornblum, 1973). This led to some confusion in the field regarding which type of effect was found - repetition or alternation - and for what values of the interval between trials.[12] Differences often overlooked included a forewarning signal a fixed time before stimulus onset (Williams, 1966; Schvaneveldt & Chase, 1969), irregular intervals between trials (Welford, 1959; Williams, 1966), differences in stimulus-response compatibility and even errors in experimental design such as not taking into account the time spent pressing the response button (Bertelson, 1961; Kirby, 1972).

[11] Hyman (1953) also manipulates transition probabilities in a sequence but in a way which resulted in a concomitant variation of the base rates, making results hard if not impossible to interpret.

[12] This situation led Kornblum (1973), in his landmark review of the state of the field, to be quite dismissive of the issue of the dependence of sequential effects on the interval between trials. Kornblum dedicates two paragraphs to the subject on what is otherwise a very detailed review.

Despite the confusion generated by the multitude of different experimental designs a pattern emerged where a repetition effect tended to be found in experiments with a very short interval between trials - 50 to 100 ms - and an alternation effect when the interval used was longer - more than 1 sec. In some cases there was no significant alternation effect when a long interval was used but a repetition effect was consistently found with a short interval, with one notable exception (Schvaneveldt & Chase, 1969). However, the method section of Schvaneveldt and Chase (1969) is scant in detail[13] and it is not clear if a constant RSI or a fixed interval between stimuli (ISI) was used since the authors refer to it as 'inter-trial interval' and yet it is hardly possible for a human to respond in serial to stimuli separated by 100 ms. In the ensuing discussion an attempt will be made to restrict the analysis to experiments where a fixed response-stimulus interval was used.

1.2.3 Reconciling positive and negative recency effects

Despite having been identified in different settings, the negative recency effect - i.e. gambler's fallacy - observed in prediction tasks and the repetition effect in reaction time needed reconciling. When predicting humans tended to alternate whereas when reacting they tended to be faster if the next event was a repetition. At first sight this would imply humans react faster to events they did not predict, when intuitively humans should react quicker to events which are predicted. Setting aside other experimental differences, the prediction tasks in which the negative recency effect was found and the reaction time tasks in which the repetition effect was identified differed in one crucial aspect: the interval between trials. Most prediction tasks had intervals of multiple seconds - e.g. 4 sec for Jarvik (1951) - while reaction time tasks had used mostly intervals of less than one second (e.g. Bertelson, 1961). There was a possibility then that the repetition effect was operating at short inter-stimulus intervals and the negative recency effect was acting at longer intervals.

[13] The authors state that 'The ITI [inter-trial interval] variable represents the transition from the serial to the discrete paradigm', yet fail to define ITI or the difference between a 'discrete' and 'serial' paradigm. From other sources, in a discrete paradigm the interval between trials tends to be long and little care is taken in keeping it constant; by contrast, in a serial paradigm either the interval between trials or the response-stimulus interval is kept fixed. However, this information does little to resolve the ambiguity.

In order to investigate whether a negative recency effect could be obtained in reaction time it was necessary to increase the inter-stimulus interval to values close to the range used in prediction tasks, something which could be achieved by increasing the response-stimulus interval. Bertelson (1961) had already observed that the negative recency effect observed with an RSI of 50 ms was all but gone with an RSI of 500 ms. Bertelson and Renkin (1966) investigated the effect of varying the RSI in more detail using four values: 50, 250, 500 and 1000 ms. However, the authors used an experimental design different from that used by Bertelson in 1961, with overlapping geometric figures as stimuli. It is now known that this type of stimulus, with its reduced spatial mapping between stimuli and response buttons, tends to extend the repetition effect into longer RSI values (see below), and this may have been the reason why Bertelson and Renkin (1966) continued to observe a repetition effect even with an RSI as high as 1000 ms. Another analysis of the dependence of sequential effects on the RSI was performed by D. J. Hale (1967), this time using the numbers '1' and '2' as stimuli. While arguably these stimuli also have a reduced spatial compatibility with responses, the range of RSI values used in this case was extended up to 2000 ms and this allowed Hale to observe for the first time a transition from a repetition to an alternation effect as the RSI is increased.

At first sight the transition from a repetition to an alternation effect in reaction time as the RSI is lengthened seemed to support the theory that a facilitating trace was present only when the RSI was short and a more cognitive effect was operating when the RSI was long. However, it was not clear how the alternation effect observed in reaction time was related to that which was observed in prediction tasks, as both effects had never been observed in the same experiment. In order to clarify this relationship Hale designed an experiment in which subjects had to predict - as well as react to - the next stimulus, something which was only possible with an RSI of 2 sec in order to allow enough time for a prediction to be made (D. J. Hale, 1967). The analysis of predictions as a function of run length revealed that, as the number of repetitions of the same stimulus increased, subjects increasingly predicted an alternation (Figure 1.4, left panel). However, reaction times as

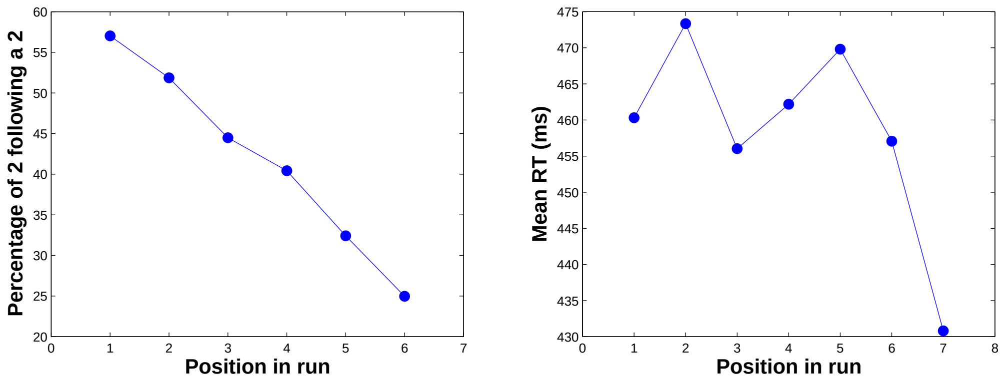

FIGURE 1.4: Data adapted from D. J. Hale (1967) highlighting the seemingly paradoxical results of an experiment where subjects had to predict, as well as respond to, consecutive stimuli. Stimuli for the task were the numbers ‘1’ and ‘2’ and a fixed RSI of 2 sec was used in the case of the data shown. Left panel - percentage of times the number ‘2’ as a function of the length of a run ‘2’. Right panel - Mean reaction times as a function of the length of the same runs as shown for the prediction data. The results from the predictions follow the negative recency effect discovered by Jarvik (1951). However, it seems logical that reaction times should be longer for the predicted stimuli and shorter for the non-predicted stimuli. If this was the case, reaction times should on average increase with run length, when they in fact stay approximately constant or even decrease a little.

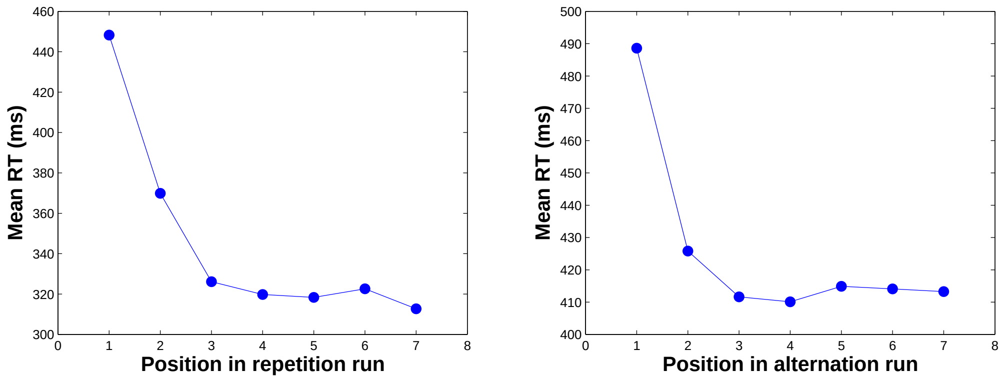

FIGURE 1.5: Data from D. Hale (1969) showing a recency effect for both repeating and alternating runs. Left panel - Reaction times as a function of position in repeating runs. Right panel - the same but for alternating runs. Stimuli were the numbers ‘1’ and ‘2’ and the response-stimulus interval was 100 ms. Based on this data, Hale suggests that the mechanism responsible for processing repetitions must be different from that responsible for processing alternations.

a function of run length did not show a corresponding increase, as expected if we assume humans respond faster to predicted events, and in fact decreased to some extent (Figure 1.4, right panel). The implication of this finding was that subjects were responding quicker or at least at the same speed to stimuli which they are predicting less and less, a seemingly paradoxical result, as pointed out by Hale himself.

One of the main pillars of the facilitation theory was the idea that reaction times decreased with each repetition of the same stimulus because residual activation would somehow quicken the recruitment of the same neural pathways. However, as first observed by Bertelson (1961), reaction times also decreased with the length of *alternating* runs, a fact hard to reconcile at first sight with a simple facilitation trace. D. Hale (1969) provides the clearest evidence for a decrease in reaction times as a function of the length of both repetition and alternation runs (Figure 1.5). Hale observed the two effects in the same task with a 100 ms RSI, a value which tends to induce faster overall reaction times to repetitions. So it seemed that even when humans strongly prefer repetitions they are still sensitive to alternations. Based on this, as well as significantly different error rates to repetition and alternation runs, Hale argues that 'repeated stimuli are processed differently from alternated stimuli'.

Other authors after Hale would argue for the need to postulate separate mechanisms for the detection of alternations and repetitions, based on very different types of analysis but centred on what is essentially the same idea (Laming, 1968; Maloney et al., 2005). The core of the argument is simple but compelling and can be summarised as: if there is a positive recency effect making reaction times faster to repetitions; a negative recency effect making reaction times faster to alternations; and both are operating at the same time, then these have to be separate unless we postulate a mechanism able to generate an expectancies to two different events at the same time. Bertelson (1963) also makes the case for separate mechanisms for the processing of responses to the same stimulus and to different stimuli, albeit from a different perspective. Bertelson studied the effect of stimulus-response (S-R) compatibility and found it to influence reaction times to alternations only, not repetitions. Based on this, he postulated that a faster mechanism bypassing stimulus processing would be used in the case of repetitions, whereas a slower mechanism involving the mapping of stimuli to responses would be used in alternations.

1.2.4 Looking at all possible histories of events

Up to 1969 the debate about sequential effects focussed heavily on the impact of the last event by comparing average reaction times to repeating and alternating trials. If events beyond the last two were considered, this was usually in the context of the analysis of runs of the same stimulus (e.g. Bertelson, 1961; D. J. Hale, 1967). Occasionally runs of alternations or even more complex types of run, such as the length of the interval between two occurrences of the same stimulus, were also considered (D. Hale, 1969). These analyses were guided more or less explicitly by theoretical considerations, with any other type of sequence dismissed - sometimes explicitly such as in D. Hale (1969) - as uninformative.

The inconsistencies in the type of data analysis used in sequential effects research generated much confusion and ambiguity in the literature. Finally, Remington (1969) suggested a systematic method[14] for analysing and plotting sequential effects data. Instead of focussing only on differences in reaction time to repetitions and alternations, Remington looked at all possible sequences of five stimuli including the one being responded to. Amongst other things this allowed for a clearer realisation that strong effects of the sequence beyond the last event were possible, even when no difference between repeating and alternating trials was present.[15] Another advantage of Remington's method is that it provided a model and/or theory free data analysis, allowing researchers to come back and possibly re-interpret results at a later stage. Given the paucity of data displayed by researchers before 1969, it will never be know exactly how these experiments fit with more recent sequential effects research.

To be fair to history, it is worth mentioning that Schvaneveldt and Chase (1969) first looked at

[14] Note that a complete and systematic analysis of sequence data had nevertheless been performed as early as 1938 by Goodfellow in the context of a random sequence generation experiment (the Zenith radio telepathy experiments).

[15] The hallmark pattern of sequential effects - discussed below - obtained with a long RSI, the cost-benefit or inverted 'v' pattern discussed throughout this dissertation, would fall into this category. Considering only the effect of the last event in this case would be an almost literal case of scientific myopia.

all possible histories of four stimuli[16] when investigating sequential effects. However, Remington provided a clear rationale for his choice of five-long sequences based on significance analysis, and also proposed a revolutionary new method for plotting data. Unfortunately, Remington's tree plots, while presenting all the data, are very hard to read and it would be another eleven years before Vervaeck and Boer (1980) introduced the type of plot which is still in use today (see Figure 1.1 for an example) and which will be used throughout this work. The development of better data analysis techniques allowed for the emergence of a more complex theory of the relation between low-level facilitation and higher-level cognitive effects, which is discussed next.

1.2.5 Subjective expectancy and automatic facilitation

A far back as 1961, Bertelson suggested that two kinds of processes were possibly at play in sequential effects: the first would be a passive decaying 'facilitating' trace responsible for what he termed the 'repetition effect'; the second type of process, depending on 'subjective probability', was effectively the expectation from the part of the subject about what the next stimulus would be (Bertelson, 1961). Despite this early suggestion, the idea of two separate mechanisms does not take hold until the field abandoned its habit of looking solely at differences between repeating and alternating trials or specific types of run and started analysing data in more detail (Schvaneveldt & Chase, 1969; Remington, 1969).

At first the effect of looking at all possible histories of stimuli was to shift the field towards subjective expectancy - often termed subjective probability - and away from passively decaying trace theories. Part of the reason for this was the often conflicting results regarding the effects of varying the interval between trials, upon which the theory of a passively decaying trace hinged (see above). Another reason was that the new data analysis methods highlighted with greater clarity the dependence of sequential effects on events as far back as five stimuli, which in turn put the effect

[16] Their study was published only a few months before Remington's.

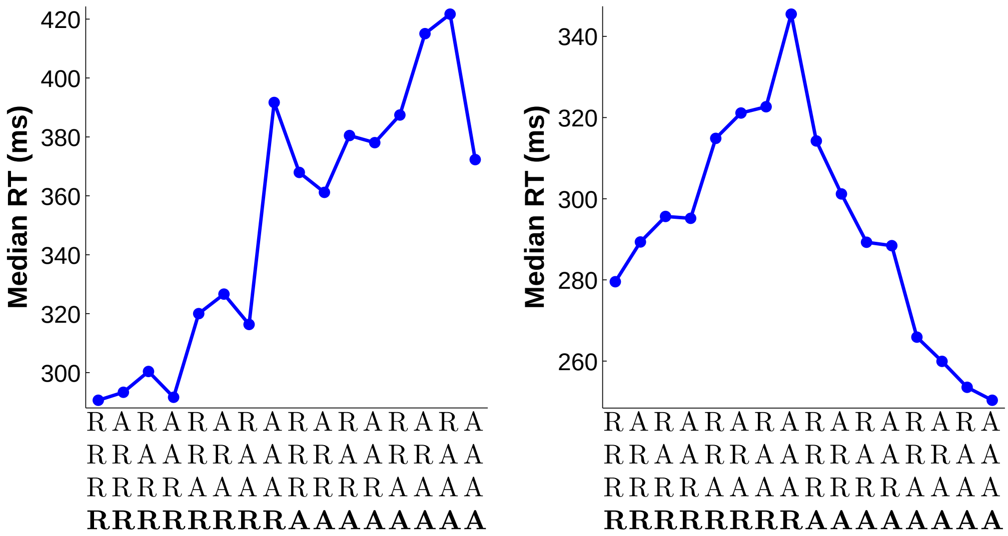

FIGURE 1.6: Benefit-only and cost-benefit patterns of sequential effects. Data is from Soetens et al. (1985). Left panel - Pattern of results obtained with a short - 50 ms - RSI and often referred to as 'benefit-only'. Right panel - Pattern of results obtained with a long - 1000 ms - RSI and commonly referred to as 'cost-benefit'. The benefit-only and cost-benefit patterns of sequential effects have traditionally been associated with automatic facilitation and subjective expectancy respectively (see main text).

of repeating and alternating trials into perspective as one facet of a more complex phenomenon. Together these developments led some researchers to reject the notion of a facilitating trace in favour of an exclusively expectancy based account (Schvaneveldt & Chase, 1969; Remington, 1969; Keele, 1969; Kirby, 1972).

Thus far the mark of a cognitive expectancy effect had been taken to be simply the alternation effect observed with long intervals between trials, an effect considered analogous to the 'gambler's fallacy' (Jarvik, 1951). The more detailed sequence analyses conducted after Remington (1969) allowed for the development of a more nuanced theory of the effects of expectancy. Kirby (1972) first observed that, just as reaction times decreased with increasing run length, so did they increase for *interruptions* of runs, and that this was true both of repeating and alternating runs. In other words, reaction times seemed to increase - relative to the average - when a stimulus was expected and decrease when a stimulus was not expected. This meant that, if one assumes expectations to be generated by the preceding sequence of events, a particular sequence - say XXXX[17] - has a *benefit* if the next event turns out to be a particular stimulus - X in this case - and a *cost* if it turns out to be the other stimulus - Y in this case. This led to the traditional pattern of sequential effects observed with a long RSI to be named as *cost-benefit*. When plotted the traditional way, a cost-benefit curve looks like an inverted 'v', the hallmark of subjective expectancy (Figure 1.6, right panel).

Eventually it became clear that, when a very short response-stimulus interval was used, a cost-benefit pattern was no longer observed, and instead a substantially different pattern of sequential effects was obtained (Kirby, 1976; Vervaeck & Boer, 1980; Soetens et al., 1985). Again, as in the case of expectancy, what had been previously described simply as a 'repetition effect' now revealed itself to be a far more complex phenomenon. The pattern of sequential effects obtained when the RSI is short (Figure 1.6, left panel) displays approximately equal slopes[18] of the alternation and repetition curves - i.e. left and right side of the plots shown in Figure it is followed by a repetition or an alternation, leading to the term *benefit-only* being coined to describe the pattern in Figure 1.6 (left panel).[19]

The benefit-only pattern of sequential effects observed when the RSI was short led to the re-birth of the concept of facilitation, now termed 'automatic' facilitation, albeit in a different form (Kirby, 1976; Vervaeck & Boer, 1980; Soetens et al., 1985). Originally the facilitation mechanism had been proposed to reduce reaction times to the same stimulus (Bertelson, 1961). However, some authors now argued that the facilitation mechanism happened irrespective of the next stimulus (Laming, 1968; Vervaeck & Boer, 1980). In other words, some sequences would induce faster or slower reaction times to the next event no matter what it is. This seemed to fit with the benefit-only pattern of sequential effects, although it ignored a strong difference in overall reaction time to repetitions and alternations[20] which was not predicted by the theory (see Figure 1.6, left

17 Here X and Y are meant to denote the two possible stimuli.

18 This is only true if the sequence AAAA is excluded. Note how this point constitutes a clear violation of a linear trend. Soetens et al. (1985) in fact exclude this data point from their analysis and attribute it to a residual manifestation of subjective expectancy.

19 The absence of a baseline effectively meant that it was unclear whether the pattern was benefit-only or cost-only, or even something in between.

20 Such differences in overall reaction time are visible in plots as changes in 'height' between the left and right sides of the plot since these include respectively all sequences ending with a repetition and all those ending in an alternation.

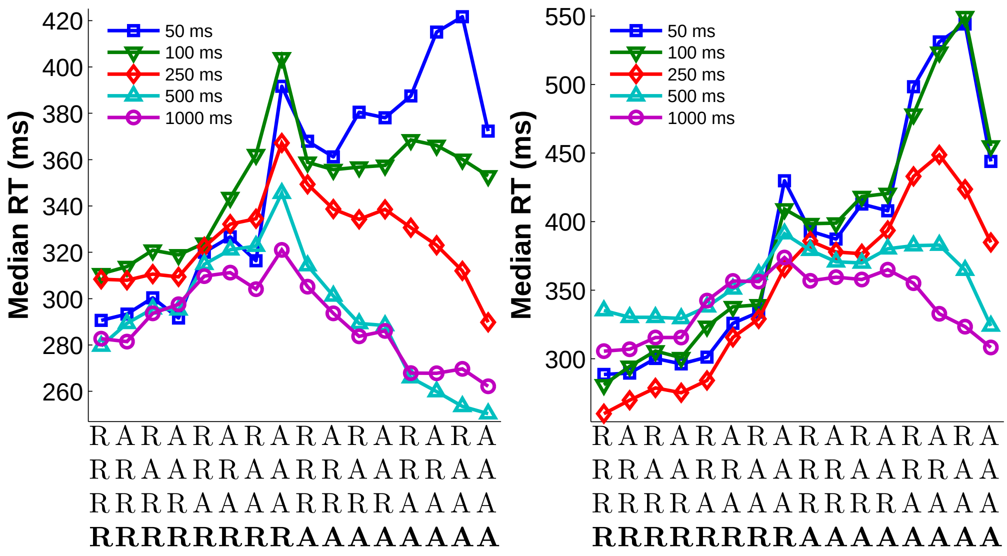

FIGURE 1.7: The dependence of sequential effects on the response-stimulus interval. Left panel - Results for different values of RSI of an experiment with compatible stimulus-response (S-R) mapping. Right panel - Results from an experiment with incompatible S-R mapping. In both panels the RSI values used are: 50, 100, 250, 500 and 100 ms. Both experiments were equal in every respect except for the S-R mapping: stimuli consisted of two horizontally separate dots and responses were made with the same finger of both hands. In the compatible mapping the button on the same side as the stimulus was used to respond, and in the incompatible mapping the opposite side button was used instead. Note the gradual transition from a benefit-only pattern of results for a short RSI to a cost-benefit pattern with a long RSI. This balance is shifted in the incompatible case, with a benefit-only pattern observed for longer values of the RSI.

panel). Automatic facilitation in its new form also raised obvious questions regarding what possible mechanism could affect reaction times irrespective of the next stimulus. Vervaeck and Boer (1980) suggest a model based on hypothetical neural pathways and their interactions,[21] which was then given a mathematical treatment by Soetens, Deboeck, and Hueting (1984), the merits of which will not be discussed here.

Soetens et al. (1985) performed what is perhaps the most comprehensive analysis of sequential effects conducted up to that point and since. The authors define quantitative ways of measuring automatic facilitation and subjective expectancy and study the effects of varying the RSI in two different tasks differing solely with respect to stimulus-response compatibility: in one task subjects responded with by pressing a button positioned on the same side as the corresponding stimulus;

[21] Vervaeck and Boer (1980) also created the plot of sequential effects data which has been used ever since. This ingenious way of plotting data greatly improved visualisation of sequential effects and allowed for a reliance on visual pattern recognition to compare results.

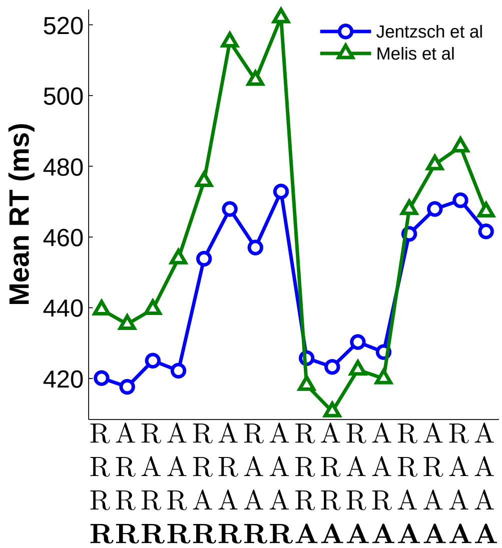

FIGURE 1.8: Reaction time results falling outside the typical benefit-only or cost-benefit patterns of sequential effects. Data from the second experiment of Jentzsch and Sommer (2002) is shown together with the results of an experiment conduced on an elderly group of subjects included in Melis et al. (2002). Notice that in both cases there is an almost two-tiered dependence on the second-to-last event and whether it was a repetition or an alternation.

in the second task subjects had to respond with a button on the *opposite side*. The results of both experiments are shown in Figure 1.7. The authors discuss three 'rules' for the operation of sequential effects: first, automatic facilitation dominates at short response-stimulus intervals and subjective expectancy at long values, the point of transition between the two occurring between 70 and 160 ms; second, making the response-stimulus mapping less compatible shifts the point of transition between automatic facilitation and subjective expectancy towards higher values of the RSI; third, practice tends to reduce sequential effects, and this effect is more pronounced for automatic facilitation. This view of sequential effects has remained influential until recent times, with several authors using it to frame their results (e.g. Jentzsch & Sommer, 2002; Gao, Wong-Lin, Holmes, Simen, & Cohen, 2009).

1.2.6 A different pattern of sequential effects

Most empirical results found in the literature seem to fall somewhere along the continuum between the benefit-only and cost-benefit patterns described by Soetens et al. (1985). However, a qualitatively different pattern of sequential effects is sometimes obtained when the RSI used is short, i.e. 50 ms or less (Melis et al., 2002; Jentzsch & Sommer, 2002). In these cases reaction time seems to be largely a function of the second-to-last event and whether this was a repetition or an alternation (see Figure 1.8). Whatever the mechanism responsible for these results it seems that *age* is a crucial factor: Melis et al. (2002) contrasted the results of two groups, one of young and one of elderly participants, when performing the same experiment with a 50 ms RSI. The young group displayed a typical benefit-only pattern observed in most short-RSI experiments; in contrast, the elderly group displayed a pattern consistent with a dependence of reaction times on the second-to-last event. The authors attribute these differences to age-related losses in processing speed. Jentzsch and Leuthold (2005) further argued that the dependence of reaction time on the second-to-last event is purely response-related and the product of a conflict between neural signals associated with the different responses which would occur when the stimuli alternated in the previous time step. The end result would be that an alternation in the second-to-last trial would inhibit responses on the last trial. More generally there has been some debate in the literature regarding the locus of sequential effects, and to which extent these effects are associated with the stimuli or the corresponding responses.

1.2.7 The locus of sequential effects

As discussed in the beginning of this section, some early attempts were made to clarify whether sequential effects were due to the properties of the stimuli or the judgements made about those same stimuli. A closely related question is whether sequential effects are driven by the sequence of stimuli, the sequence of responses, or both. Bertelson (1965) was the first to tackle this problem by designing an experiment in which multiple stimuli were mapped to the same response button. This resulted in three types of trial: *different*, in which consecutive stimuli as well as corresponding responses are different; *equivalent*, where stimuli are different but the same response is made; and *identical*, where both stimuli and responses were the same. Contrasting results from identical and equivalent trials allowed for some conclusions to be drawn regarding whether stimuli had an effect independently of responses. Bertelson finds a significant difference between identical and equivalent trials for some subjects but not for others. While these results provided some support for an effect of stimuli *per se*, they are otherwise interesting in that they represent some of the first evidence for individual differences in sequential effects.

Other attempts were made to identify whether sequential effects, and in particular a repetition effect, were associated with perceptual, motor or more central stages of processing (Smith, 1968; Pashler & Baylis, 1991; Soetens, 1998) but these were plagued by difficulties with issues such as the spatial mapping between stimuli and responses as well as the dependence of sequential effects on the the RSI, both factors well known to affect whether a repetition effects is observed or not. Overall it seemed that both stimulus and response related effects were involved in sequential effects, in which case the difficulties with determining their locus from reaction time data were somewhat unsurprising: once multiple signals are joined together in order to produce behaviour it becomes a conceptually difficult problem to disentangle them, and this would be aggravated if sequential effects also depended on more central processing stages. However, it was soon discovered that sequential effects could also be observed in electroencephalography (EEG), raising the prospect that questions about the locus of sequential effects could be answered directly by measuring different neural signals.

1.3 Sequential effects in EEG

The following is a somewhat more cursory examination of the literature on sequential effects in EEG when compared to the review of sequential effects in behaviour above. Nevertheless, this quick summary of the literature has its place here as, despite electrophysiological data not having been collected for this dissertation, results are extensively discussed in light of such data obtained by other researchers, in particular Jentzsch and Sommer (2002), in the context of support for the hypothesis of two separate processing stages involve in sequential effects.

Tueting, Sutton, and Zubin (1970) studied the shape of the event-related potential (ERP)[22] in a task involving a random sequence of high- and low-pitch tones, and where subjects had to predict the outcome of each trial. The effect of overall stimulus probability, as well as the proportion of repeating to alternating trials, on the shape of the ERP were examined, with specific focus on the amplitude of the P300 component.[23] In addition to a dependence of the amplitude of P300 on the relative frequency of the stimuli, a significant difference was found between repeating and alternating trials, the first evidence of some form of sequential effects in EEG.

K. C. Squires et al. (1976) performed the first detailed analysis of sequential effects in the ERP, focussing on its overall shape rather than just the amplitude of the P300 component. It was known beforehand that the shape of the ERP depended on how common an event was: in tasks with highly asymmetric stimulus probabilities a ‘rare’ event elicited an ERP of a particular shape different from that which was observed when the event was ‘frequent’. One challenge was to see if it was possible to go the opposite way: to predict whether an event was rare or frequent from an analysis of the shape of the corresponding ERP. For this purpose the authors had previously developed a ‘discriminant function’ which, after parameter fitting based on a training set, attributed a linear

[22] The ERP is a series of positive and negative going shifts in cortical electrical potential occurring after a stimulus is displayed. A review of of the subject can be found in Luck (2005).

[23] P300 is a positive going shift in potential occurring approximately 300 ms after stimulus onset (see Polich (2007) for a review).

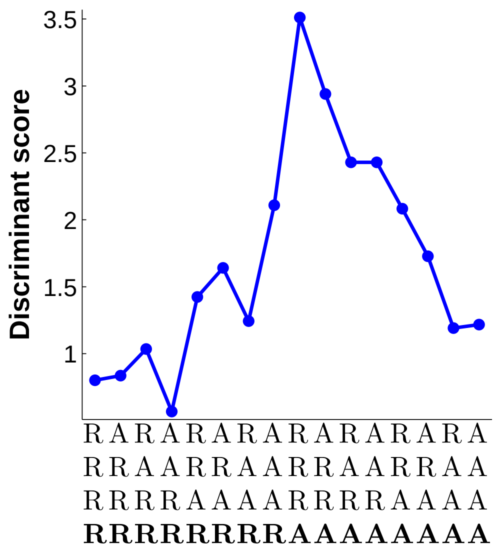

FIGURE 1.9: Sequential effects in the ERP as per K. C. Squires et al. (1976). The ordinate axis shows the mean score obtained from a discriminant function developed to distinguish two standard shapes of the ERP, one related to infrequent and another to frequent events. The task consisted of a sequence of two types of tone, one high and one low pitch, both occurring with the same frequency. The objective of the task from the subject's perspective was to count and report the number of high pitch tones.

score to the ERP of new trials which reflected how close it was to the typical rare or frequent ERP shapes (K. C. Squires & Donchin, 1976). By making use of this discriminant function it was possible to predict accurately whether a stimulus was rare or frequent in 81% of cases. The authors also noted that, in the remaining 19% of cases where the prediction failed this did now appear to be due to noise but rather to the fact that in some cases a rare event induced a frequent ERP and vice-versa. Moreover, upon closer inspection, this seemed to depend on the previous sequence of trials.

K. C. Squires et al. (1976) then studied the shape of the ERP in a task involving a sequence of high- and low-pitch tones with equal frequencies. The authors analysed discriminant scores as a function of the sequence of trials in the style of the literature on sequential effects (Remington, 1969). The results, shown in Figure 1.9,[24] display clear sequential effects similar to cost-benefit patterns observed in reaction time. In order to improve the predictions of the discriminant function, it was necessary to account for the way in which it depended on sequential effects. For this purpose the authors proposed a compact model of sequential effects which nonetheless displays many of the features from other more detailed models discussed below. The model is effectively a linear combination of three sources of information: (1) a geometric average of the last five stimuli; (2) an 'alternation factor' accounting for alternations, essentially a linear function of the number of previous alternations; and finally (3) the long-term probability that a stimulus will occur. The best fitting model explained 78% of the variance in discriminant score.

The study of sequential effects in EEG continued with several studies analysing different aspects of the way in which the ERP or one of its components depended on the sequence of events. Average ERPs as a function of the preceding five stimuli were often shown in tree diagrams analogous to the plots of Remington (1969). K. Squires, Petuchowski, Wickens, and Donchin (1977) extended their discriminant score analysis to a visual task and showed the results to be similar to those obtained with the auditory task described above. Other studies focussed on the detailed effect of prior probability (Duncan-Johnson & Donchin, 1977); the effect of transitions between blocks of trials with different prior probabilities (R. Johnson & Donchin, 1980); the specific components - P3a or P3b - of the ERP related to sequential effects (Munson, Ruchkin, Ritter, Sutton, & Squires, 1984); the effects of age (Ford, Duncan-Johnson, Pfefferbaum, & Kopell, 1982) in P300 latency as a function of the sequence as well as in schizophrenic patients (Duncan-Johnson, Roth, & Kopell, 1984); and whether or not P300 expectancies were conscious or automatic (Sommer et al., 1990; Matt, Leuthold, & Sommer, 1992); to name just a few studies.

The similarity between sequential effects in ERP amplitude and those found in reaction time gave rise to the idea that both were the reflection of a common expectancy mechanism (but see Matt et al. (1992)). However, most if not all EEG studies were conducted with long intervals between stimuli, whereas sequential effects in reaction time were known to change considerably when short intervals were used. Part of the reason why short intervals were avoided were baseline problems:

[24] Note that results are plotted in the current style in order to facilitate comparison with other results, but originally the authors used the tree plots of Remington (1969).

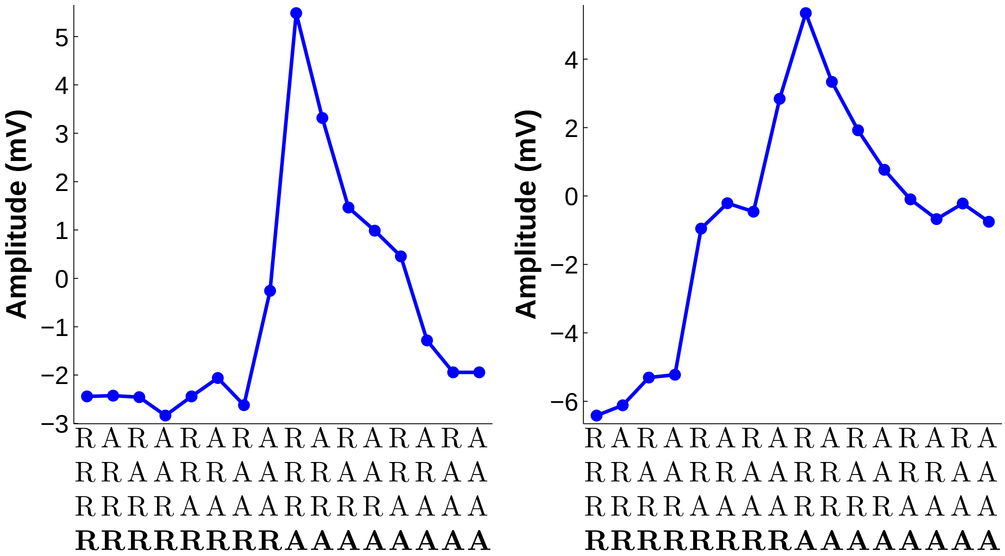

FIGURE 1.10: Sequential effects in the ERP (P300) when short and long response stimulus intervals are used. Left panel - P300 amplitude as a function of the previous sequence of stimuli obtained with a short - 40 ms - RSI. Right panel - Results obtained with a long - 500 ms - RSI. The experiments were the same in every other respect; stimuli consisted of two LED lights displaced horizontally. Note that a special procedure was used in order to correct for overlap between adjacent ERPs in the 40 ms case (see main text).

when the interval between successive trials is shortened the ERPs associated with consecutive events begin to overlap making it difficult to set an initial baseline against which to take amplitude measurements. Sommer, Leuthold, and Soetens (1999) made use of a procedure to correct for the overlap between neighbouring ERPs and this allowed the authors to compare sequential effects in P300 amplitude with a short - 40 ms - and long - 500 ms - RSI. The results, shown in Figure 1.10, revealed a similar pattern in both cases. So despite clear differences in sequential effects observed in reaction time when the RSI was long and when it was short, P300 amplitude seemed to show a similar pattern in both cases.[25]

Before the experiments of Sommer et al. (1999) some debate existed about whether expectancy effects were also present when the RSI is short of if they were altogether absent, perhaps due to lack of time to build up. If expectations were present when the RSI is short, then presumably

[25] While the authors consider the results shown in Figure 1.10 to be similar, it might be argued that there are some non-trivial differences between the two, possibly due to distortion of the 40 ms results induced by the procedure used to correct for overlap.

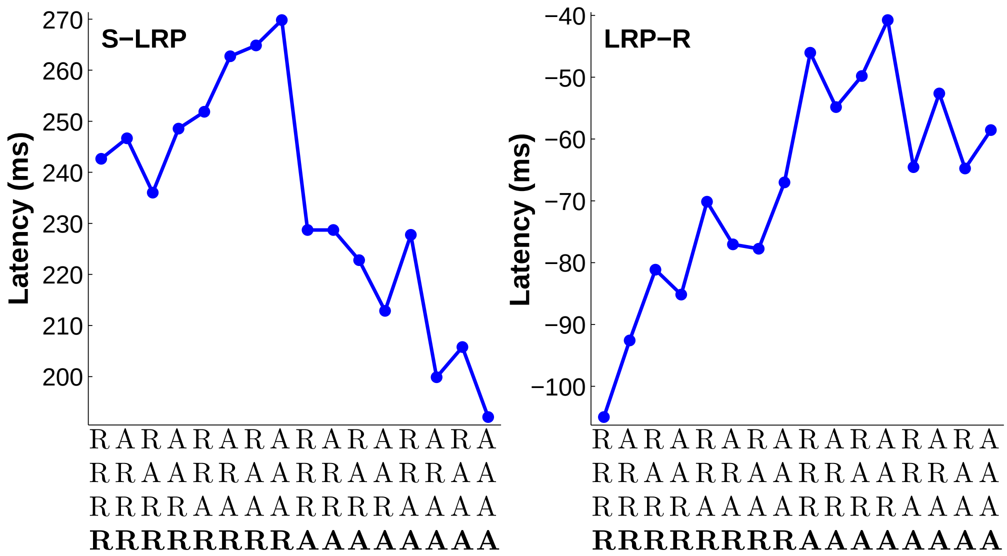

FIGURE 1.11: Evidence for separate processing stages involved in sequential effects according to Jentzsch and Sommer (2002). Left panel - Time between stimulus onset and the rise of the lateralised readiness potential as a function of the previous sequence of stimuli (S-LRP). Right panel - Time between the rise of LRP and the moment a response is made (LRP-R). The behavioural task was a 2AFC with two vertically displaced dots as stimuli and a 700 ms RSI. S-LRP and LRP-R are though to reflect the separate processing of stimuli and associated responses respectively.

these were masked in reaction time by the interference of lower-level effects, resulting in a benefit-only pattern of sequential effects. Soetens et al. (1984, 1985) argued for this hypothesis based on the fact that reaction times to the sequence AAAA were relatively short even when a 50 ms response-stimulus interval was used (Figure 1.6, left panel), revealing some degree of sensitivity to alternations. That a cost-benefit pattern - considered to be the hallmark of expectancy - was observed in P300 when the RSI was short, despite failing to manifest itself in reaction times, seemed to vindicate the masking hypothesis. Sommer et al. (1999) suggest that signals related to expectancy integrate with ‘possibly response-related’ pathways at a late stage; if the RSI is too short this integration might not have time to occur, in which case reaction times would be fully determined by response-related effects - i.e. automatic facilitation.

Sequential effects in EEG are not restricted to the ERP and P300: they have also been observed in the lateralised readiness potential (LRP) (Leuthold & Sommer, 1993; Jentzsch & Sommer, 2002). The LRP consists of a negative shift in potential occurring just before a response is made in the pre-motor cortex area contra-lateral to the hand which will be used to respond. The first observation of sequential effects in the amplitude of the LRP was made by Leuthold and Sommer (1993) but the most detailed study of the subject was performed by Jentzsch and Sommer (2002). The authors investigated not only the amplitude of the LRP but also the time between stimulus onset and the moment the LRP reached a threshold amplitude - S-LRP - and the time between the LRP and the moment a response is made - LRP-R. The rationale for this decomposition is that the time before the onset of LRP is thought to index pre-motor processing time, whereas the time between the onset of LRP and the moment a response is made is thought to measure motor processing time. S-LRP and LRP-R as measured by Jentzsch and Sommer (2002) are shown in Figure 1.11.

The fact that one can measure pre-motor and motor processing times separately seems to imply a serial processing view of sequential effects and of processing in the brain in general, a simplistic view not shared by the author of this dissertation. An investigation into the validity of S-LRP and LRP-R as valid measures of different processing stages is nevertheless beyond the scope of this work. Besides, irrespective of these considerations, there is now substantial evidence pointing to the fact that S-LRP and LRP-R are meaningful constructs (Maloney et al., 2005; M. H. Wilder, Jones, Ahmed, Curran, & Mozer, 2013; M. Jones, Curran, Mozer, & Wilder, 2013), and whether or not these can be conceptualized as motor and pre-motor ‘processing’ times becomes a secondary consideration if one is careful not to over-interpret results. Therefore, and inasmuch as S-LRP and LRP-R can be taken to be a manifestation of separate processes involved in sequential effects, the results of Jentzsch and Sommer (2002) provide the first evidence for a decomposition of sequential effects into different components. Furthermore, it would seem that these two components are associated separately with the processing of stimuli and of responses[26], providing an answer to questions regarding the locus of sequential effects: it would appear that both stimuli and responses contribute towards sequential effects.

[26] This point will be discussed in more detail in Chapter 3.

Interestingly S-LRP and LRP-R are somewhat symmetrical, with S-LRP displaying faster processing times to alternations and LRP-R to repetitions (see Figure 1.11). It is only natural to consider what the role of these two processing stages might be in producing different patterns of sequential effects such as those observed when the RSI is varied (see Figure 1.7). In addition, some mentions have been made in the literature of the fact that, under the same experimental circumstances, individual participants may differ with respect to whether they display faster reaction times to repetitions or alternations. One possibility then is that different contributions from stimulus and/or response processing may be responsible for the differences observed both across individuals and when the RSI is varied. Attempting to answer some of these questions is a main focus point of this dissertation, in particular Chapter 3, where individual differences in sequential effects are used as a tool in order to infer what independent contributions towards sequential effects may exist.

1.4 Individual Differences in Sequential Effects

To the author's best knowledge, there has not been a single study dedicated specifically to the topic of individual differences in sequential effects. Some passing mentions do exist, often to point out that different subjects display differences with respect to whether they display a repetition or alternation effect (Arons & Irwin, 1932; Bertelson, 1965; Kirby, 1976; Kornblum, 1968). In one case a model was fit to different participants separately but the significance of the individual differences observed is not analysed in detail (Falmagne, Cohen, & Dwivedi, 1975). The avoidance of the subject is all the more striking since, foreshadowing some of the results presented here, individual differences are not only considerable but clearly meaningful in that they *mimic the way sequential effects depend on the RSI.*

As mentioned above, it is a difficult problem to infer the contributions of separate stages of sequential effects from just a few results obtained under different experimental circumstances. Individual differences, if they are found to be meaningful and not just due to noise, could be useful in in this respect, in that it may be possible to infer latent components from the patterns of covariance across multiple subjects, the principle underlying techniques such as principal component analysis (PCA). In chapter 3 of this work PCA will be conducted on a dataset of over one hundred and fifty individual participants with the aim of identifying meaningful latent variables and to relate these to what is already know from the empirical literature about separate processing stages involved in sequential effects.

1.5 Quantitative models of sequential effects

As far back as the 1960's attempts were made to formalise ideas about the mechanisms underlying sequential effects by developing mathematical models. At the core of these models are usually one or more variables representing the state of expectation of the subject with respect to the the next stimulus, upon which reaction times are assumed to depend: the more someone expects a particular event the shorter the reaction time will be. This state of expectation is also referred to by some authors as 'preparation' or 'subjective probability', reflecting in the latter case an interpretation of expectancy as an estimate of the objective probability of occurrence of the next stimulus. This subjective probability estimate is generally incorrect in the sense that it does not correspond to any objective measure of the probability of the next stimulus based on the frequency of events.

The focus on expectations means that most models have sought to reproduce the cost-benefit pattern of sequential effects observed in experiments conducted with a relatively long RSI. There have been few attempts at explaining sequential effects observed when the RSI is short (but see Soetens et al., 1984; Jentzsch & Sommer, 2002) - the so-called benefit-only pattern often interpreted as the product of lower-level facilitation mechanism. With respect to the more general dependence of sequential effects on the RSI, a single attempt was made at tackling this complex problem (Gao et al., 2009). This means that even the most successful models of sequential effects discussed below are incapable of reproducing a benefit-only pattern of sequential effects and are, strictly speaking, models of subjective expectancy. This *status quo* is maintained by the prevalent notion that sequential effects observed when the RSI is short are due to a fundamentally different mechanism (Kirby, 1976; Soetens et al., 1985; M. Wilder, Jones, & Mozer, 2009).

Methodologically, models of sequential effects have traditionally been developed outside more general reaction time modelling frameworks such as sequential sampling models (Stone, 1960) and are therefore often unable to produce reaction time distributions (e.g. M. Jones et al., 2013). The main reason for this is that sequential effects reflect inter-trial differences whereas reaction time modelling is more often focussed on intra-trial variation and the shape of reaction time distributions (Ratcliff & Smith, 2004). The two approaches are not necessarily incompatible though: one can in principle substitute an estimate of the subjective probability for the objective probability in decision making models, in which case sequential effects would presumably manifest themselves in a dependence of the mean of reaction time distributions as a function of the sequence, an approach taken by some authors (Laming, 1968). Generally speaking, even when models are able to produce reaction time distributions, these are seldom fit to their empirical counterparts (but see Falmagne, 1965).

Another aspect of empirical results which is usually set aside or otherwise given little consideration is error rates, although there are some exceptions to this rule (e.g. Cho et al., 2002). This is arguably for relatively benign reasons as error rates have been shown time and time again to follow a similar trend to reaction times, while at the same time yielding a much noisier measure due to the very low error rates in many experiments - sometimes as low as 1 or 2% - which means the number of reaction time data points relative to errors can be in a proportion close to 100:1. Early studies often mentioned error rates in order to dispel the possibility that sequential effects were the product of some form of speed-accuracy trade-off, in which case error rates should display some form of negative relationship with reaction times. A few models actually predict error rates (e.g. Cho et al., 2002) which tend to follow closely reaction times. In some cases there are hints of differences between trends in reaction times and error rates (e.g Soetens et al., 1985; Jentzsch & Sommer, 2002) which are left unaddressed. Whether it is possible to decouple reaction time and error rates in sequential effects, and under which circumstances this might happen, is a question left open.

A careful review of the history of sequential effects modelling reveals some degree of redundancy. Most models consist of combinations of two types of geometric mean over the past sequence of events: one mean taken over the sequence of stimuli themselves and the other over the sequence of repetitions and alternations (discussed in detail below). Such a combination was proposed for the first time more than forty years ago by Laming (1969) in a model which arguably encapsulates the fundamental properties of all sequential effects models to come.[27]

1.5.1 The geometric average or exponential filter

In its many incarnations the geometric moving average, geometrically weighted mean, exponentially weighted moving average or simply exponential filter is at the core of every model of sequential effects. In the context of discrete time models this would more correctly be referred to as a ‘geometric’ mean, since it corresponds to a discrete progression[28] but it will be referred to throughout as ‘exponential filter’, in part because a continuous time model will eventually be proposed here. In discrete time, and in its recursive form, the exponential filter can be written as

$$x(n+1) = (1-\alpha)S_n + \alpha x(n) \tag{1.1}$$

[27] Most recent works fail to cite the work in question despite sharing almost the same mathematical structure. Part of the reason for this may have been Laming’s 1968 dissertation, misleadingly titled ‘Information theory of choice reaction times’, which may have turned attention away from his work.

[28] The geometric progression can be interpolated by an exponential function so this is a somewhat pedantic point.

where α is a constant varying between 0 and 1 effectively determining the rate of exponential decay (implicit in this case) and S_n represents the stimulus at trial n coded as either a 0 or 1 corresponding to the two possible stimuli. In non-recursive form, the exponential filter can be written as

$$x(n) = \sum_{i=0}^{N} \theta^i S_{n-i}$$

where θ is a number between 0 and 1 and N is the length of the sequence so far. In this form, one would need to know all the elements from the beginning of the sequence up to trial n in order to calculate $x(n)$, a rather wasteful procedure considering that, once i is large enough, θ^i becomes negligibly small. In the particular case of sequential effects it seems that stimuli beyond the last five do not contribute towards reaction times (Remington, 1969) implying that θ is small enough to make θ^5 negligibly small.

While the mathematical form of the exponential filter remains constant it is sometimes applied to the sequence of stimuli and other times to the sequence of repetitions and alternations, also coded as 0's and 1's. In fact, the most successful models of sequential effects proposed so far have made use of both at the same time (Laming, 1969; M. Jones et al., 2013). In some models the exponential filter is not mentioned explicitly, but one can deduce that exponential decay is occurring due to some form of constant rate leaking (Cho et al., 2002; Gao et al., 2009). In other models recursion relations of the form in (1.1) apply to distributions and this tends to produce a similar effect on the respective moments (Falmagne, 1965; Yu & Cohen, 2008; M. Wilder et al., 2009).

Next a selection of mathematical models of sequential effects is briefly discussed. Part of the reason for this review is an attempt to highlight the commonalities between the different models proposed over the years. In fact, most models below produce results which are either equal or at least well approximated by some combination of two types of exponential filter: one applied to the sequence of stimuli and the other to the sequence of repetitions and alternations.[29] This equivalence is sometimes explicit (Laming, 1969; Yu & Cohen, 2008; M. Wilder et al., 2009; M. Jones et al., 2013) and sometimes left implied (Falmagne, 1965; Cho et al., 2002; Gao et al., 2009). Because of this common mathematical structure the set of models below can be interpreted as representing a particular view of sequential effects, one that will be later challenged in this work.

1.5.2 Capturing a repetition effect - Falmagne (1965)

Falmagne (1965) constitutes the first attempt at constructing a formal model specifically addressing sequential effects. Falmagne draws upon the concept of preparation for the next stimulus, first suggested by Bertelson (1961), and represents this with a discrete variable $K_{i,n}$: on each trial n the subject is either prepared for stimulus i, in which case $K_{i,n} = 1$, or it is not, in which case $K_{i,n} = 0$. A second discrete variable $E_{i,n}$ represents whether stimulus i was presented at trial n. The model can accommodate any number of stimuli and so both E and K can be vectors of 0's and 1's of any length. If the complete state of the system at trial n is known, then the reaction time distribution is defined and is

$$J(t|E_{i,n} = 1, K_{i,n} = 1, W_{n-1}) = K(t)$$
$$J(t|E_{i,n} = 1, K_{i,n} = 0, W_{n-1}) = \bar{K}(t)$$

where W_{n-1} is a vector representing the state of all variables up to trial $n-1$, $K(t)$ is a distribution of fast reaction times and $\bar{K}(t)$ is a distribution of slow reaction times. The model itself is a Markov chain where the state of preparedness is a hidden variable, and where we further assume

29 An analytical proof of this fact not given here for those cases in which the equivalence to some form of exponential filter is not explicit (e.g. Falmagne, 1965; Cho et al., 2002)

the previous history of stimuli presentation up to trial $n-1$ to be unknown. So in order calculate the probability $P_{n,i}$ that the subject is prepared for stimulus i on trial n ones must integrate over all possible histories W_{n-1}

$$P_{n,i} = P(K_{i,n} = 1) = \sum_{W_{n-1}} P(K_{i,n} = 1|W_{n-1})P(W_{n-1})$$

If a stimulus i is presented at time n so that $E_{n,i} = 1$ the resulting reaction time distribution at trial n is

$$J(t|E_{n,i} = 1, W_{n-1}) = P_{n,i}K(t) + (1 - P_{n,i})\bar{K}(t)$$

which is effectively a linear combination of the fast and slow reaction time distributions. Note that even though $K(t)$ and $\bar{K}(t)$ are both symmetrical, their combination will be asymmetrical, and this is meant to account for the asymmetry in empirical reaction time distributions.

The probability that $K_{i,n} = 1$ depends on the value of $K_{i,n-1}$ and $E_{i,n-1}$: if $K_{i,n-1} = 0$ and $E_{i,n-1} = 0$, i.e. the subject was unprepared and the stimulus was not observed, then $P(K_{i,n} = 1) = 0$; if $K_{i,n-1} = 1$ and $E_{i,n-1} = 1$, i.e. the subject was prepared and the stimulus was observed, then $P(K_{i,n} = 1) = 1$. If $K_{i,n-1} = 1$ and $E_{i,n-1} = 0$ then $P(K_{i,n} = 1) = 1 - c'$; finally if $K_{i,n-1} = 0$ and $E_{i,n-1} = 1$ and $E_{i,n} = 1$ then $P(K_{i,n} = 1) = c$. These transition probabilities result in the following recursion relations for the $P_{i,n}$

If $E_{i,n-1} = 1$

$$P_{i,n} = (1 - c)P_{i,n-1} + c$$

and if $E_{i,n-1} = 0$

$$P_{i,n} = (1 - c')P_{i,n-1}$$

As with the $P_{i,n}$, similar recursions can be obtained for the reaction time distributions, so if $E_{i,n} = 1$

$$J_{i,n} = (1 - c)J_{i,n-1} + cK(t)$$

and if $E_{i,n} = 0$

$$J_{i,n} = (1 - c')J_{i,n-1} + c'\bar{K}(t)$$

Falmagne is particularly interested in how the mean reaction time varies as a function of the length of runs of the same stimulus in order to reproduce the repetition effect discovered by Bertelson (1961). According to the model the reaction time distribution after k repetitions of the same stimulus is given by

$$J_{i,n+k} = (1 - c)^k(J_{i,n} - K(t)) + K(t)$$

Falmagne attempts to fit different order moments of this distribution to their empirical equivalents obtained from a reaction time task with six possible alternatives. The model is successful at capturing a repetition effect, i.e. the decrease in mean reaction time with increase in repetition run length. However, it is far less successful in capturing higher order moments of the reaction time distributions. Laming (1968) criticizes Falmagne's model on the basis of its incapacity to capture the decrease in reaction time with increasing length of an alternation run, or put simply its incapacity to detect alternations.

1.5.3 Two geometric means - Laming (1969)

In Laming's own words the 'essence of the model, mathematically speaking, is that subjective probability relating to the signal to be presented on a given trial can be represented as the sum of two geometric moving averages over preceding events in the experiment, one moving average taken over preceding signals, the other over the preceding sequence of repetitions and alternations' (Laming, 1969). The geometric average over preceding events is given by

$$x_{f,n} = \sum_{i=0}^{\infty} \theta_f^i S_{n-i}$$

where f is a label to identify this as the 'frequency' average, θ is a parameter between 0 and 1, n is the n-th trial and S_n is the stimulus presented at trial n coded as a 0 or a 1. The second moving average is given by

$$x_{a,n} = \sum_{i=0}^{\infty} \theta_a^i [S_{n-i} + S_{n-i-1}]$$

where a stands for 'alternation' average and $[S_{n-i} + S_{n-i-1}]$ is the sum modulo 2 of the values inside square brackets and is equal to 0 if stimuli are the same and 1 if they are different.

The two geometric means are used as parameters for the posterior distribution of y, the subjective probability taken to be an estimate of the objective probability p, in a manner that will be made clear shortly. Laming calculates, based on the information content of the sequence (Kullback, 1959), that the posterior distribution over y after n stimuli is given by

$$\pi(y) = \frac{(N+1)!}{(N-x)!x!}(1-y)^{N-x}y^x$$

which is a Beta distribution with parameters $(N-x+1, x+1)$. N and x must be bounded in order to limit accumulation of information. x will be a combination of the exponential averages shown above and is therefore limited given that θ^i tends to 0 as $i \to \infty$. N is limited to a finite quantity which depends on the θ parameters and reflects a limited storage capacity. The model also incorporates prior information meant to account for the significant effect of previous blocks of trials described by Bertelson (1961). When all types of information are considered, N is given by

$$N = N_f + N_{f,0} + N_a + N_{a,0}$$

where $N_f = \frac{1}{(1-\theta_f)}$ and $N_a = \frac{1}{(1-\theta_a)}$, with $N_{f,0}$ and $N_{a,0}$ representing prior information. x for a particular trial is given by

$$x_n = x_{f,n} + x_{f,0} + (1-S_n)(x_{a,n} + x_{a,0}) + S_n(N_a - x_{a,n} + N_{a,0} - x_{a,0})$$

Model predictions for reaction times and error rates are then derived by substituting y - the subjective probability - for p - the objective probability - in the equations of a random walk decision model developed by Laming (1968) and calculating expectations relative to $\pi(y)$. Presumably though, this subjective probability could be substituted for the objective probability in any decision making model.

The fit of the model to empirical data is only analysed in terms of reaction times and error rates as a function of runs of the same stimulus, as well as regression coefficients. This is unfortunate as the model allows for detailed predictions based on the previous sequences of events, and this was the year - 1969 - in which sequential effects data began to be analysed in more detail. Based on current knowledge, and on the similarities with the model of M. Wilder et al. (2009), Laming's model is expected to fit well the cost-benefit pattern of sequential effects.

Arguably, Laming's model encapsulates the essence of all models to come. In particular, it incorporates key features of the models by Cho et al. (2002), Yu and Cohen (2008), M. Wilder et al. (2009) and M. Jones et al. (2013). This is all the more remarkable since the model was conceived at a time when empirical information about sequential effects was still scarce; for instance the cost-benefit pattern of sequential effects associated with expectancy had not been described yet. In fact, the decision to incorporate two geometric means in the model seems largely based on the regression analysis performed by Laming (1968) and the observation therein that regression equations 'would have represented the influence of the sequence more accurately if they had contained two sets of coefficients, one to represent the subjective estimate of signal frequency, the other to represent the subjective likelihood of an alternation of the signal'.

1.5.4 An ordered memory scanning process - Falmagne (1975)

The model proposed by Falmagne et al. (1975) is an extension of the model proposed by Theios and Smith (1972), itself an extension of the model by Falmagne (1965) described above. The difference lies in the fact that the model assumes some form of matching between each stimulus and an ordered template stored in memory. If there are two stimuli - 1 and 2 - this template can be in two states: (1,2) and (2,1). The model assumes an ordered memory scanning process, with the next stimulus - say 1 - first attempting a match to the first item stored in memory and then to the second. The order of the template can change with a probability which depends on what its order was in the previous trial and on what the current stimulus is. In practice the difference relative to the model suggested by Falmagne (1965) is essentially the way the new model predicts errors. We will not go into much detail into this model, but the work in which it is presented stands as the only example in the literature where a model is fit to individual participant data rather than data pooled from multiple participants.

1.5.5 The biased leaky accumulator - Cho et al (2002)

The model proposed by Cho et al. (2002) is based on the leaky competitive accumulator decision-making model of Usher and McLelland (Usher, 2001). The model is described by the following set of stochastic differential equations

$$\begin{aligned}\frac{dx_1}{dt} &= -kx_1 - \beta f(x_2) + \rho_1 + \xi_1 + b_1 \\ \frac{dx_2}{dt} &= -kx_2 - \beta f(x_1) + \rho_2 + \xi_2 + b_2\end{aligned} \tag{1.2}$$

where x_1 and x_2 are two decision making units each corresponding to a particular stimulus; k determines the constant rate - i.e. exponential - decay of each unit; β is a mutual inhibition term; f is an 'activation function' given by $f(x_i) = \frac{1}{1+e^{-G(x-d)}}$ where G is the gain and d the offset; ρ_i represents whether a particular stimulus i is present or not; ξ_i are Gaussian random noise terms. The b_i are the key terms making this a model of sequential effects and represent biases induced by the sequence of stimuli.

Each trial is separated into two stages: a first one during which the stimuli are absent - i.e. $\rho_1 = \rho_2 = 0$ - meant to represent the RSI; and a second stage, modelling the response, in which the ρ_i - representing the stimuli - are assigned randomly one of two values with $\rho_1 = 1 - \rho_2$. Reaction time is calculated as the time taken for one of the units to reach a fixed threshold in the manner usual in decision making models (Ratcliff & Smith, 2004). Model predictions are therefore means of 'reaction time' distributions, one for each of the sixteen possible sequences of stimuli.

The biasing terms b_i are kept fixed throughout each trial and are updated at the end of the response period. The b_i are a function of the history of stimuli, and consist of repetition (R) and alternation (A) detectors. These detectors can be of different types with regard to two factors: firstly, whether detection of repetitions or alternations is independent for each unit (I) or shared across both (S); secondly, whether the detector uses the last stimulus - one-back - or the two last stimuli - two-back when detecting repetitions or alternations. There are eight possible combinations of detectors which the authors tested systematically for fit to the results of a single experiment. The combination exhibiting the best fit was found to be IR1-SA2, i.e. a combination of an independent one-back repetition detector with a shared two-back alternation detector. IR1 determines that the b_i corresponding to a particular decision unit should be incremented depending on whether the corresponding stimulus was observed in the last trial; SA2 determines that the decision unit corresponding to stimulus opposite of that observed last should be incremented every time an alternation is detected.

This model is capable of reproducing successfully the cost-benefit pattern of sequential effects commonly observed with a long RSI, as well as being able to capture variations in preference for repetitions or alternations. However, like most models suggested here it is unlikely to be able to explain the benefit-only pattern of sequential effects observed when the RSI is short. In addition, the model relies on the explicit hard-coding of most of its relevant properties, such as the capacity to detect alternations, instead of letting these properties emerge naturally, arguably limiting its informativeness to some extent.

1.5.6 Dynamic belief model - Yu and Cohen (2008)

The dynamic belief model - DBM - proposed by Yu and Cohen (2008) uses repetitions and alternations of stimuli as the raw input. In the binary sequence fed into the model a value of 1 represents a repetition and a 0 an alternation of individual stimuli. Denoting the two possible stimuli as X and Y, this means that every time an XX or a YY is observed in the sequence, this will be replaced by a 1; conversely, XY and YX will be replaced by a 0.[30]

The model itself works by keeping a running distribution - $P(\gamma_t|\mathbf{x_t})$ - over the range of possible values of a binomial parameter γ which determines the probability of observing a repetition or an alternation in the next trial. At each time point t, the posterior distribution from the previous time step - $P(\gamma_{t-1}|\mathbf{x_{t-1}})$ - is linearly combined with a beta 'reset' prior $P_0(\gamma) = Beta(a, b)$ to form an 'iterative prior' $P(\gamma_t|\mathbf{x_{t-1}})$, according to

$$P(\gamma_t \mid x_{t-1}) = \alpha P(\gamma_{t-1} = \gamma \mid \mathbf{x}_{t-1}) + (1-\alpha)P_0(\gamma) \tag{1.3}$$

the parameter α determines how much information from the previous time step is carried through to the next, in effect regulating the rate of exponential decay implied by the recursion in (1.3). Model predictions are calculated as the expected value of the iterative prior

$$P_t = \int \gamma P(\gamma_t|\mathbf{x_{t-1}})d\gamma$$

Finally, the posterior distribution for the next time step is calculated in the usual way by taking into account the likelihood of the current event, according to

30 Note that models in which the sequence represents the individual stimuli also use 0's and 1's as the effective input into the model, in which case they represent the stimuli themselves

$$P(\gamma_t \mid \mathbf{x_t}) = P(x_t \mid \gamma_t)P(\gamma_t \mid \mathbf{x_{t-1}})$$

A prior preference for alternations or repetitions can be introduced in the model by varying a and b, the parameters of the Beta reset prior. It is worth noting that the authors interpret α as representing the rate of change in the environment as opposed to the rate at which the observer forgets the past. In practice the difference in interpretation of α is of no consequence. What matters is that the model is equivalent to an exponential filter of the sequence of repetitions and alternations, as demonstrated by the authors.

DBM is operating on a sequence of repetitions or alternations of stimuli. This means the model never 'sees' the raw stimuli, and that consequently it does not have access to information about base rates, only the relative proportion of repetitions and alternations. This has consequences in that there is a notable lack of detail when attempting to fit the typical cost-benefit pattern of sequential effects. On the other hand, a strong point of DBM is its capacity to capture a preference for either repetitions or alternations, which few other models can.

1.5.7 Adding base rates to DBM - Wilder et al (2009)

DBM2 is an extension of DBM meant to capture a sensitivity to the base rates of stimuli as well as the relative proportion of repetitions and alternations. Instead of just one binomial parameter - γ - tracking the relative probability of seeing a repetition or an alternation, the model makes use of a second binomial parameter - ϕ - tracking the relative proportion of the two possible stimuli. So instead of a univariate distribution over one binomial parameter, the model keeps a running bivariate distribution - $P(\gamma, \phi)$ - over two binomial parameters. Finally, a mixture parameter w allows for different proportions of both types of information to be combined when calculating the likelihood of the next stimulus, given by

$$P(x_t = X|\gamma_t, \phi_t, x_{t-1} = X) = w\phi_t + (1-w)\gamma_t$$
$$P(x_t = X|\gamma_t, \phi_t, x_{t-1} = Y) = w\phi_t + (1-w)(1-\gamma_t)$$

where X and Y are the two possible stimuli, γ_t is the probability that the next event will be a repetition, ϕ_t is the probability of the next stimulus and w is a mixture parameter controlling the proportion of the two types of statistics used. The rest of the model is analogous to its parent DBM: DBM2 also keeps a running iterative prior, defined as a mixture of the posterior distribution for the previous step and a reset prior,

$$P(\gamma_{t+1}, \gamma_{t+1}|\mathbf{X_t}) = (1-\alpha)P(\phi_t, \gamma_t|\mathbf{X_t}) + \alpha P_0(\phi_{t+1}, \gamma_{t+1})$$

the posterior for the next step is calculated the usual way as

$$P(\phi_t, \gamma_t|\mathbf{X_t}) \propto P(x_t|\phi_t, \gamma_t, x_{t1})P(\phi_t, \gamma_t|\mathbf{X_t})$$

Model predictions are calculated as the expected value of the iterative prior with respect to the likelihood function, according to

$$P(x_{t+1}) = \int_\gamma \int_\phi P(x_t|\phi_t, \gamma_t, x_{t1})P(\gamma_{t+1}, \gamma_{t+1}|\mathbf{X_t})d\phi d\gamma$$

DBM2 is in effect equivalent to a linear combination of two exponential filters: one at the level of individual stimuli and the other at the level of repetitions and alternations. With w set to 1, the model reduces to an exponential filter of the sequence of stimuli; when it is set to 0 the model is effectively equivalent to DBM, which again is equivalent to an exponential filter of the sequence of repetitions and alternations. The model is successful in capturing additional details in the typical cost-benefit pattern which are thought to reflect a sensitivity to the base rates of stimuli (M. Wilder et al., 2009). Nevertheless, DBM2 suffers from a problem not discussed by the authors: γ and ϕ are not independent. Increasing the proportion of either stimulus above the usual 50% will clearly have consequences in that repetitions are expected to increase relative to alternations. Conversely, varying the proportion of repetitions to alternations has consequences for the base rates: in the limit of a fully alternating sequence the base rates must be equal. This makes it hard to define a distribution for the reset prior $P_0(\phi, \gamma)$ which will introduce a preference of repetitions or alternations in a manner analogous to DBM. The authors sidestep the issue by using a uniform reset prior and difficulties encountered when fitting empirical results with alternation bias are dealt with through the use of a constant added to all alternating trials.

1.5.8 Tackling RSI dependence - Gao et al (2009)

The model proposed by Gao et al. (2009) is an extension of that by Cho et al. (2002) to which three additional biasing mechanisms were added: a bias due to post-response residual activity, a bias from expectations, and a bias from response conflict monitoring. The end result is a very complex model with fourteen parameters of which four were kept free and the rest adopted from Cho et al. (2002), Usher (2001), or other sources. The mechanics of the model is otherwise the same as in Cho et al. (2002), the difference being that additional biasing terms similar to b_i were added to equations 1.2.

Being continuous time in nature, much like Cho et al. (2002), the model allows for a parameterisation of the response-stimulus interval. The authors take advantage of this in order to tackle for the first - and to the best of our knowledge only - time the complex issue of RSI dependence. Even with the added biases, the success in approximating the dependence on sequential effects by fitting the data of Soetens et al. (1985) (shown in Figure 1.7) is questionable. Furthermore, it will be demonstrated here that the effects of the different biasing mechanisms can be explained in a considerably more parsimonious manner. The choice was therefore made not to go into much detail about this model here.

1.5.9 Different types of statistics - Gokaydin et al (2011)

This model was created specifically in order to analyse the types of statistics people are tracking when analysing sequences, and is described in detail in Chapter 2. In essence it is a model allowing for the analysis of the role of different types of statistics in sequential effects. These different types of statistics are calculated by attributing exponentially decaying weights to past events in line with the usual assumptions of sequential effects models.

1.5.10 Joint learning model - Jones et al (2013)

This model builds more explicitly than most on the idea that two separate types of information, the base rate of stimuli and the rate of repetitions (or equivalently alternations), underlie sequential effects. The authors contrast two sub-models, one in which the two types of statistics are learned in ‘parallel’ - i.e. separately - and one in which they are learned in a joint fashion, both models sharing a common structure. At each time point, the model holds an estimate for the true base and repetition rates. For the parallel learning model, and depending on the outcome of each trial, the base rate estimate w_{base} is updated according to

$$\Delta w_{base} = \epsilon_{base}(E_{n-1} - w_{base})$$

where Δw_{base} is the increment change to w_{base}; E_{n-1} is the outcome of the previous trial, encoded in this case as either -1 or 1; and ϵ_{base} is the 'learning rate' and takes a value between 0 and 1. Successive iteration of the update rule results in an exponentially weighted moving average of the sequence of stimuli,

$$w_{base} = \sum_{k=1}^{n-1} \epsilon_{base}(1 - \epsilon_{base})^{k-1} E_{n-k}$$

Model predictions are linearly scaled as usual in order to fit reaction time data according to

$$RT = \beta_0 - \beta_{base} w_{base} E_n$$

where β_0 and β_{base} are the parameters of a linear transformation and the negative sign on β_{base} reflects the fact that the reaction time should be shorter the larger the probability assigned to the next event.

The situation for the repetition rate is analogous to that of the base rate. The update rule for w_{rep} is

$$\Delta w_{rep} = \epsilon_{rep}(E_{n-1}E_n - w_{rep})$$

where the notational convenience of encoding events as ± 1 can be seen in that it allows for repetitions and alternations to be mapped to the values $\{-1, 1\}$ simply by writing $E_n E_{n-1}$. Again this update rule results in an exponentially weighted moving average, but this time of the sequence of repetitions and alternations,

$$w_{rep} = \sum_{k=1}^{n-2} \epsilon_{rep}(1-\epsilon_{rep})^{k-1} E_{n-k}E_{n-k-1}$$

Simply combining the two types types of information - w_{base} and w_{rep} results in a prediction for reaction times given by

$$RT = \beta_0 - \beta_{base}w_{base}E_n - \beta_{rep}w_{rep}E_nE_{n-1} \tag{1.4}$$

This is the parallel learning model, which is in effect equivalent to DMB2 (see above), and much like its predecessor suffers from an incapacity to produce an alternation bias which is compensated for through the use of a constant parameter. Once this parameter - δ - is included the full model is given by

$$RT = \beta_0 - \beta_{base}w_{base}E_n - \beta_{rep}w_{rep}E_nE_{n-1} + \delta E_nE_{n-1} \tag{1.5}$$

where, depending on the sign of δ, the last term in (1.5) induces a constant preference for repetitions or alternations.

The authors attempt to equate the two types of statistics to two different processing stages identified before with EEG and encapsulated in the patterns known as S-LRP and LRP-R (see Figure 1.11). In this view LRP-R, the motor component of sequential effects, would be responsible for tracking the base rates of stimuli; S-LRP, the pre-motor element of sequential effects, would be in charge of keeping track of the repetition rate. In order to demonstrate the equivalence between processing stages identified with EEG and different types of statistics, the model is fit to either LRP-R or S-LRP with only predictions resulting from w_{base} in the former case and only from w_{rep} in the latter.

When fitting LRP-R the parameter δ can be set at 0 since an exponential filter exhibits a repetition bias naturally. In contrast it is not possible to fit S-LRP with only the w_{rep} arm of the model without using δ to account for the alternation bias displayed by S-LRP. This is analogous, though made clearer, to the problems encountered by M. Wilder et al. (2009) when fitting datasets with an alternation bias, and serves as the main practical motivation for the joint learning model described next, in which the arbitrary parameter δ is removed. In the joint learning model w_{base} and w_{rep} combine in order to form a single prediction $\hat{E}_n$ according to

$$\hat{E}_n = w_{base} + w_{rep}E_{n-1} \tag{1.6}$$

updating each rate is now done according to the error of this prediction. For w_{base} this is

$$\begin{aligned} \Delta w_{base} &= \epsilon_{base}(E_n - \hat{E}_n) \\ &= \epsilon_{base}(E_n - w_{base} - w_{rep}E_{n-1}) \end{aligned} \tag{1.7}$$

and for w_{rep} it is

$$\begin{aligned}\Delta w_{rep} &= \epsilon_{base}(E_n - \hat{E}_n)E_{n-1} \\ &= \epsilon_{base}(E_n - w_{base} - w_{rep}E_{n-1})E_{n-1} \\ &= \epsilon_{base}(E_nE_{n-1} - w_{rep} - w_{base}E_{n-1}) \qquad (1.8)\end{aligned}$$

Mathematically speaking the joint learning model induces an alternation bias in w_{rep} through the last term in (1.8) which introduces a negative bias in the updating of w_{rep} in the following manner: since w_{base} exhibits a repetition bias, i.e. it is shifted towards E_{n-1}, it is on average positive and this induces a negative influence on w_{rep}, biasing it towards alternations. On a conceptual level the authors base the model on theories of joint error correction when multiple cues are present and refer to the terms in (1.7) and (1.8) which depend on the opposite rate - w_{base} or w_{rep} depending on the case - as the 'cue-competition' terms. This relies on an interpretation of the two types of statistics as cues, when strictly speaking only one type of stimulus is usually present in sequential effects experiments. The authors acknowledge this by stating that 'a similar mechanism may be in play with sequential effects, where the 'cues' are aspects of the trial sequence'.

Much like its parallel learning counterpart, the joint learning model suffers from a similar problem in that the cue competition mechanisms is really only necessary in order to explain S-LRP, not LRP-R. Moreover, the correspondence between different types of statistics and S-LRP/LRP-R now rests on the two processing stages depending on each other. Somewhat surprisingly, the authors still take the joint learning model as providing 'strong support for the separate-stages hypothesis'. The model is nevertheless successful at a practical level in that it produces an alternation bias without any extra arbitrary parameters (M. Wilder et al., 2009) or sacrificing the learning of the base rates of stimuli (Yu & Cohen, 2008). Testing the model on a dataset of 158 individual differences discussed in Chapter 3 (not shown) revealed it can provide a good fit to results obtained with long RSI values, including those with a repetition or alternation bias, but is unable to reproduce short

RSI results.

1.6 Summary

While not all of the content above is of specific relevance to the research presented in the next three chapters, the choice was made to give an overview of the field as a whole, extending into arguably more marginal topics such as the relation between sequential effects and the perception of randomness. Part of the reason for the breadth of this literature review is fact that an entirely different perspective on sequential effects is suggested in this dissertation which, if found to be true, will be of consequence to the field as a whole. Nevertheless, some of the topics discussed above are of particular relevance to understanding the following chapters, of which three are highlighted:

- The dependence of sequential effects on the RSI (see Figures 1.6 and 1.7) and the differences between the results observed with a long RSI - the 'cost-benefit' or inverted 'v' pattern - and those observed with a short RSI - the so-called 'benefit-only' pattern. Multiple references will be made to this topic throughout, but it is of particular relevance for understanding Chapter 3.

- The decomposition of sequential effects into two separate processing stages and the patterns thought to be associated with these - S-LRP and LRP-R (see Figure 1.11). These will be referred to extensively in Chapters 3, 4 and the final discussion in the context of the theory of two independent components of sequential effects.

- The two types of exponential filter - one over the sequence of stimuli and the other over the sequence of alternations and repetitions - thought to be of central importance in sequential effects. This is of particular importance to understanding Chapters 2, 4 and the final discussion.

2

Human use different statistics depending on the task

Adapted from:

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Abstract

Research into sequential effects has almost exclusively been conducted in the context of sequential tasks with two possible alternatives. Of the few studies that focussed on more complex tasks with a greater number of alternatives, none has analysed results in detail as a function of all possible stimulus histories in the manner commonly done in binary tasks. Similarly to its empirical counterpart, the theoretical literature shows the same focus on two-alternative tasks, with few models able to accommodate more than two elements. With this in mind, the objective here is to begin to bridge the gap between sequential effects observed in tasks with different numbers of sequence elements by comparing an experiment with two and one with three alternatives. In order to achieve this goal a new experimental design is proposed which minimises any confounding effects associated with increasing the number of alternatives. The main objective of this work is to compare the sequential effects observed in both experiments from a computational point of view. Significant differences are found in the nature of the statistics used as the number of alternatives increases from two to three. These results may be of wider relevance for the understanding of how humans track events in a sequence.

2.1 Introduction

Most experiments in psychology consist of long series of trials differing in the nature of the stimuli presented. The objective is usually to compare the effect of different stimuli by taking the mean of some behavioural measure across all trials where the same stimulus was presented. However, early on in the history of psychological research it became clear that behaviour does not depend just on the properties of the stimulus on a particular trial but also on the *history* of previous trials. One of the first observations of some form of *sequential effect* was in a task where a series of pairs of weights was presented to subjects, which were asked to judge which one was heavier (Fernberger, 1920; Turner, 1931). It was found that humans tended to alternate their judgements, i.e. if on a previous trial the reference weight was deemed ‘heavier’ people would tend to judge ‘lighter’ on the current pair. This effect was greatest when the current pair was almost equal in weight, i.e. near the point of subjective equality, and hardly present when the difference in weight was considerable, illustrating a general principle: some degree of uncertainty is necessary on order for sequential effects to occur (Senders & Sowards, 1952).

Sequential effects have been observed in a wide range of tasks (e.g. Jarvik, 1951; Bertelson, 1961; Maloney et al., 2005) but reaction time studies would eventually emerge as the paradigm of choice when studying these effects *per se* (Bertelson, 1961; D. J. Hale, 1967; Laming, 1968; Schvaneveldt & Chase, 1969; Kirby, 1976; Soetens et al., 1985; Jentzsch & Sommer, 2002; Cho et al., 2002). Experiments usually conform to a two-alternative forced-choice (2AFC) experimental paradigm with variation in details such as the stimuli used, the response scheme or the response-stimulus compatibility, or other aspects. In a typical task, subjects are presented with a random sequence of two possible stimuli, denoted here as X and Y. On any given trial participants are asked to press a button to indicate which of the two stimuli was presented, as quickly as possible. Once a response is made the next stimulus appears after a fixed period of time termed the response-stimulus interval (RSI). The instruction to respond as quickly as possible is crucial since it seems to induce in subjects an attempt at predicting the next stimulus in order to respond faster. This anticipation of the next event is reflected in reaction times, which are quicker for expected stimuli and slower for unexpected ones (Kirby, 1972). In this context we may define sequential effects in terms of the dependence between the RT to the current stimulus and the past history of stimuli in the experiment.

A key question to ask is: what is driving expectations about the next event? There is a consensus in the literature that sequential effects reflect people’s tendency to detect patterns, even if those patterns exist only across a few trials and are in fact the outcome of a purely random process.[1] The reason for this is that sequential effects seem to be particularly strong when the previous sequence of events presents a perfectly regular repeating or alternating pattern. For instance if the last four observations in the experiment were X→X→X→X - denoted XXXX for short[2] - participants tend to respond faster if the next element in the sequence is an X, and slower if it turns out to be a Y. Similarly, if the last few trials have been purely alternating - XYXY - participants will respond faster if the next stimulus is an X and slower if it is a Y. What is perhaps more surprising is that almost any recent sequence of stimuli will tend to produce some response bias. For instance, if the last four trials in the experiment were XYXX, there would still be a tendency to find significant differences in RT depending on whether the next stimulus was an X or Y (e.g. Remington, 1969; Soetens et al., 1985; Cho et al., 2002).

One open question in the literature relates to how sequential effects generalize to a situation with more than two possible alternatives, and how this compares with the two alternative case. As we will see, increasing the number of possible alternatives raises experimental as well as theoretical difficulties which call for both a new experimental design as well as a new modelling approach. With this in mind, the introduction to this article is structured as follows: firstly, some fundamental concepts about sequential effects and the way data is usually presented are reviewed; following this the need for a new experimental paradigm is discussed; finally, the rationale for a new modelling approach will be presented. Ultimately the goal will be to compare a 2AFC and 3AFC with respect to the statistical nature of the sequential effects present in both cases, and to do so with minimal confounding effects.

1 Indeed there are some similarities between the literature on sequential effects and the literature on human randomness perception (see Nickerson (2002) for a review).

2 Sequences are often coded in terms of repetition or alternations of stimuli, in which an ‘R’ will replace every instance of XX an YY, and an ‘A’ will replace both XY and YX. The individual stimulus X/Y notation will be preferred here as alternations are not defined for a task with three alternatives.

2.1.1 Some fundamental concepts

Two different types of sequential effects are traditionally considered: first and higher order. First order effects relate to the effect of the last stimulus and whether it represents a *repetition* or an *alternation* relative to the stimulus before it. Higher order effects refer to the influence of the *higher order sequence*, i.e. the sequence of events before the last stimulus.[3] Traditional sequential effects plots (Vervaeck & Boer, 1980) make visualising both first and higher order effects easy. Note that the left side of the plot (see Figure 2.4 for example) contains all sequences ending with a repetitions and the right side all those ending with an alternation. Moreover, all possible eight higher order sequences are ordered in the same way on both halves of the plot. So first order effects are visible as differences in 'height' between the left and right halves, whereas higher order effects are visible as differences within each half.[4] Recall that sequential effects are thought to be the product of an expectation-based mechanism and that any sequence - regular or not - produces a degree of expectancy regarding the next stimulus. This is reflected in both short reaction times to expected events as well as a relatively long reaction times to unexpected ones (Kirby, 1972). One way to put it is that any sequence has a *benefit* or a *cost* depending on what the next stimulus turns out to be. When plotted the traditional way, this trade-off results in an inverted 'v' shape, often termed a *cost-benefit* pattern, usually considered to be the hallmark of an expectation-based mechanism (Soetens et al., 1985). It is worth mentioning that this pattern of results is only observed when the response-stimulus interval is relatively long, i.e. 500 ms or more. When the RSI is short sequential effects tend to be considerably different. This does not pose a problem for our study since short RSI results are usually not considered to reflect the tracking of the sequence in a statistical sense (Yu & Cohen, 2008; M. Jones et al., 2013), and all our experiments are conducted with a relatively long 800 m RSI.

[3] The usefulness of this distinction is questionable since both types of effects are not independent but rather part of a more general dependence on the sequence of stimuli, but it is often useful in describing results.

[4] In order to avoid confusion with different order statistics discussed below first order effects will be referred to as a preference for repetitions or alternations or alternatively as a bias towards either. Henceforth the word 'order' will only be used to refer to statistics.

Early studies of sequential effects in reaction time often focussed on situations where more than two alternative stimuli are possible (e.g. Falmagne, 1965; Schvaneveldt & Chase, 1969; Kirby, 1975) but results were usually only analysed in the context of what was then termed a 'repetition effect', i.e. the tendency for reaction times to be quicker to repetitions of the same stimulus (Bertelson, 1961). No systematic analysis, i.e. including all possible sequences of stimuli, of a task with three possible stimuli has been conducted so far, making it important to clarify how data will be organised in this case. In a 2AFC data is usually presented as the mean or median reaction time as a function of the last five stimuli, including the one being responded to (Remington, 1969). There are 32 different five-long sequences of two possible elements, but these are usually grouped two-by-two according to the pattern they represent - e.g. the sequence XYXYX consists of the set $\{10101, 01010\}$ - so only 16 sequences are usually shown. For a sequence with three elements there are 243 possible five-long sequences which will also be grouped according to the pattern displayed, except that each set will now contain six elements, e.g. XYZZY = $\{01221, 02112, 10220, 12002, 20110, 21001\}$. The sole exception to this is the sequence XXXXX = $\{00000, 11111, 22222\}$. Note that this way of organizing data, for both a 2AFC and 3AFC, has the extra benefit of compensating for any systematic differences in RT to either the different stimuli or corresponding responses, the reason being that the sequences in each group include all possible permutations of stimuli, which in turn represent permutations of the corresponding responses.

2.1.2 The need for a new experimental paradigm

Traditional 2AFC paradigms do not generalise well to more than two sequence elements. Issues arise at the level of the stimuli used, the corresponding responses, or the spatial compatibility between the two. Some of these issues will be discussed with the final goal of proposing a methodology which, by minimizing possible confounding effects, ensures that the differences observed between a 2AFC and a 3AFC are solely the product of the different number of sequence elements.

Our first objective is that all responses take approximately the same amount of time to perform and no sequence of responses is preferred *a priori*. Traditional response schemes for a 2AFC include the use of the index and middle finger of the dominant hand (e.g. Cho et al., 2002) or responding with the index fingers of both hands (e.g. Soetens et al., 1985). Extending the latter scheme would require a third hand and is therefore not possible. With respect to using additional fingers of the same hand, this could potentially induce a preference for left-to-right or right-to-left sequences of responses, as any person who has tapped their fingers on a table should recognize. While it is unclear whether these effects will have a significant impact on results, it is preferable to make sure they cannot happen in the first place. This will be achieved by having all responses performed with the same finger while at the same time ensuring that the time required to perform each response is the same. The response scheme is as follows: three response buttons are symmetrically positioned around a central resting button and the index finger of the dominant hand will be used to perform all responses. At the start of each trial the response finger should be resting on the middle button, otherwise the trial is invalid; following the appearance of a stimulus the central button is released and a response button pressed; finally, after a response is made, the finger must return to the middle position before the next stimulus appears. This scheme ensures that the distance the finger must travel is always the same for each response, and should minimise any purely response-related effects.

Once we have ensured the response scheme does not induce any systematic biases we must do the same with our stimuli. Traditional 2AFC tasks make use of a wide range of stimuli such as spatially separate lights or dots (Bertelson, 1961; Soetens et al., 1985; Jentzsch & Sommer, 2002), more or less abstract figures occurring in the same location (Bertelson & Renkin, 1966; Cho et al., 2002), numbers (D. J. Hale, 1967), different coloured dots (Jentzsch & Sommer, 2002), or even sounds differing in pitch (K. C. Squires et al., 1976). Out goal is to avoid stimuli which induce a natural preference for any particular ordering, in which case a distinction based on size or even pitch should be avoided, as well as numbers or letters. Perhaps of even greater importance is to avoid issues of spatial compatibility between the stimuli and responses, known to have a strong impact on sequential effects in 2AFCs (Bertelson, 1963; Soetens et al., 1985) as well as tasks with a larger number of stimuli (Alegria & Bertelson, 1970; Kirby, 1975). So a differentiation based on spatial location should also be avoided, further narrowing our choice of stimuli. One possibility would be to use stimuli differing in colour but this has show to affect sequential effects considerably (Jentzsch & Sommer, 2002). So in the end the decision was made to use abstract figures as stimuli, all of which are displayed in the same position in the middle of the screen.

2.1.3 The need for a new model

In this section the need for a new modelling approach will be motivated. We start by discussing what kind of statistical information humans use in order to detect patterns in a sequence. Evidence for what this information might be is is largely indirect so this discussion will be guided by previous modelling efforts. Finally, the representation of memory in the model, and how previous events are forgotten, will also be discussed.

What information do people keep track of?

To the extent that sequential effects can be viewed as a kind of pattern detection, a critical question to ask relates to what information is used in order to define a pattern. There is a remarkable consensus in the literature regarding this point for the particular case of a 2AFC, with most models proposed so far making use of two types of information: the relative frequency of the stimuli on the one hand; and the relative abundance of repetitions and alternations on the other hand (Falmagne, 1965; Laming, 1969; Cho et al., 2002; Yu & Cohen, 2008; M. Wilder et al., 2009; M. Jones et al., 2013). Formally these two ratios can be denoted as $P(X)/P(Y)$ and $P(R)/P(A)$ where $P(R) = P(XX) + P(YY)$ and $P(A) = P(XY) + P(YX)$. Note that there are only two degrees of freedom implicit here because usually in a 2AFC $P(X) = 1 - P(Y)$, which further implies that $P(A) = 1 - P(R)$ in the long run and if the sequence is random. Moreover, the two ratios are *not independent*: increasing the frequency of X or Y clearly constrains the relative abundance of repetitions and alternations, and the reverse is true.[5]

In attempting to generalise the above pieces of information to the case of three sequence elements we are faced with the problem that alternations are no longer well defined. One possibility would be to maintain the definition above where alternations involve only two different stimuli. However, this would leave out sequences including more than two stimuli such as XYZXY, which are not defined either as repeating or alternating. Some authors take an alternating sequence to be one in which the same stimulus does not occur in succession, and according to this definition XYZXY is alternating (e.g Audley, 1973). However, defining alternations in this way overlooks the fact that these may no longer form a clear *pattern*. In addition, it renders the ratio of repetitions to alternations meaningless. We are therefore left in need of an alternative way to define the nature of the information used by humans to keep track of a sequence.

One possible way in which predictions could be made about the next event in the case of an arbitrary number of stimuli is to calculate conditional *transition probabilities*, i.e. the probability of the next stimulus *given* the preceding sequence of stimuli. Depending on the length of the previous sequence considered we speak of transition probabilities of different *order*. For instance, zero-th order transition probabilities are simply the probabilities of the individual stimuli, i.e. $P(X)$, $P(Y)$ and so on. First order transition probabilities take forms such as $P(X|Y)$ or $P(X|X)$ for example. Higher order transition probabilities are possible of course, i.e. $P(X|XY)$, $P(X|XYZ)$ and so on, but only zero-th and first order are considered here for two reasons: firstly because it was found here that second order statistics[6] are not useful in describing sequential effects (not shown); secondly, evidence from linguistics suggests that humans are not able to use statistics beyond first

5 If the frequency of either X or Y is 1 then the sequence must be fully repeating, and the inverse is true for the case of a fully repeating sequence. Finally, a fully alternating sequence implies that $P(X) = P(Y) = 0.5$, except for perhaps a small difference if the length of the sequence is an odd number.

order (Newport & Aslin, 2004; Gebhart, Newport, & Aslin, 2009).

The number of different types of events one must keep track of in order to calculate transition probabilities changes with both the order of the statistics being considered as well as the number of different stimuli in the sequence. To make matters concrete, we will contrast the four possible cases defined by a sequence with either two or three possible elements and transition probabilities of either zero-th or first order. The case of zero-th order statistics is trivial as one must simply keep track of the base rates of the stimuli, i.e. $P(X)$ and $P(Y)$ for a sequence with two elements and $P(X), P(Y)$ and $P(Z)$ for a sequence with three elements. The case of first order statistics is more complex since, in addition to the base rates one must also keep track of the probability of *pairs* of stimuli, due to the way transition probabilities are calculated, e.g. $P(X|Y) = \frac{P(X,Y)}{P(Y)}$. So when using first order statistics on a sequence with two elements there are four probabilities - $P(XX), P(XY), P(YX)$ and $P(YY)$ - that must be tracked in addition to the base rates, for a total of six quantities. When using first order statistics on a sequence with three alternative elements there are three base rates and nine possible pairs - $P(XX), P(XY), P(XZ)$ and so on - for a total of twelve probability values. So it is expected that when using first order statistics the number of values humans must implicitly calculate doubles for a 3AFC in comparison to a 2AFC[7]. This is expected to have consequences with respect to how difficult it is for humans to use higher order statistics in tasks with larger numbers of stimuli.

The next section will explain how memory is handled in the model. It is intuitive that the the recent past is more relevant towards predicting the future when compared to the distant past. Therefore, and in line with all previous models of sequential effects, recent events will be given more weight when estimating the probability of the next stimulus.

[6] We will abbreviate different transition probabilities simply as ‘statistics’ for the sake of brevity.

[7] Strictly speaking these numbers depend on whether humans take into account the loss in degrees of freedom associated with knowing the number of possible stimuli. Taking this into account the number of quantities which need to be tracked would be reduced by two for statistics of any order except zero.

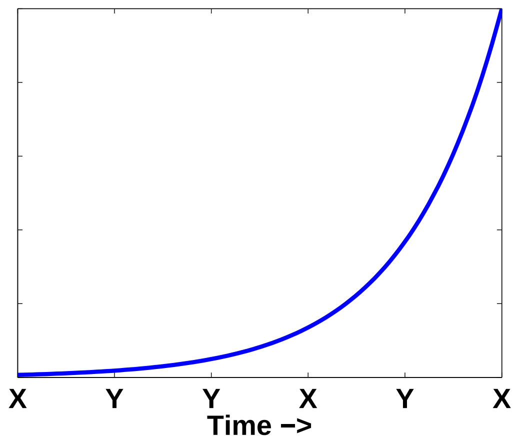

FIGURE 2.1: Illustration of the exponential decay of the influence past events have on expectations about the future. Time flows from left to right and the last stimulus is shown on the far right. In this illustration parameters were chosen so that any stimuli beyond the last five would have little impact on expectations, much like what is expected to be the case in sequential effects.

What do people remember about the past?

Different lines of argument can be invoked in order to justify giving greater weight to recent events when attempting to predict the future. One interpretation is that this reflects the rapid decay of human memory (Wixted & Ebbesen, 1991). Another possibility is that such weighting reflects the fact that the present is more strongly correlated with the recent past, a view preferred by some authors (Yu & Cohen, 2008). Finally, more pragmatic considerations dictate that some form of forgetting is necessary in order to prevent the convergence of all model predictions to fixed values according the long-term frequencies of events. When transposed to the human case such a convergence would imply that sequential effects die-out after a long enough sequence, when in fact humans have been shown to continue to be sensitive to short term variations in the frequency of events, irrespective of long-term statistics. In order to capture this short-term sensitivity the weighting of past events will be assumed to decay exponentially, following most models proposed so far[8] (e.g. Laming, 1969; M. Jones et al., 2013). Figure 2.1 shows an illustration of this principle.

[8] There is some debate as to whether human memory decay takes the form of an exponential or a power-law function (Wixted & Ebbesen, 1991) but this is of little practical consequence for our purposes.

Discounting past events according to an exponential function creates a temporal 'window' of only a few recent elements that can have a significant impact on predictions, since events distant in past are given a negligible weight. The number of such relevant items depends on the λ, the rate parameter in the exponential function $e^{-\lambda t}$: the larger λ is the shorter the 'memory span'. In the specific case of sequential effects it seems that only the last five stimuli have any influence on reaction times (Remington, 1969). A limited memory horizon introduces an issue of *sparsity* when seeking to to calculate transition probabilities. Recall that in order to estimate transition probabilities such as $P(X|Y)$ one must estimate two quantities - $P(X)$ and $P(Y,X)$ - from the respective frequencies of both types of event - X and YX - in the previous sequence. However, a short memory makes it possible that one or both events have not been observed recently, resulting in a predictive probability equal to zero. For instance, in a long repeating sequence such as XXXXX the probability that the next event is a Y will quickly converge to 0 because neither Y nor XY are observed in the recent past. This problem is made more acute as the number of alternative sequence elements increases since the probability of individual events decreases, making it less likely that any particular stimulus or pair of stimuli occurred within the memory horizon. Similarly, using higher order statistics - e.g. $P(X|XYX)$ - will also add to the problem of sparsity since the longer a string of events is the rarer it tends to be. In short, a finite memory span means that past events may have been forgotten. The issue of sparsity is analogous to problems encountered in linguistics when using n-gram models where the number of possible sequences - words or letters in this case - if often very large (Manning & Schütze, 1999). The solution in linguistics is often to introduce Laplace smoothing, i.e. to increase the frequency count of all events by one, ensuring all events will have non-zero probability.

2.2 Model

In this section we will outline a model capable of incorporating both different order statistics and an arbitrary number of sequence elements. Two sub-models will be considered, corresponding to zero-th order and first order statistics. Both models result in different predictions depending on the number of stimuli and these predictions will be compared to empirical results from the novel task described above. Assessing the best fitting model for both a 2AFC and a 3AFC will allow inferences to be made regarding what type of statistics are being used in each case.

2.2.1 Formal details

The model we wish to propose is inspired by n-gram models (Manning & Schütze, 1999) and consists of a combination of the transition probabilities commonly used in such models with exponential discounting of past events. In order to formally construct our model we start by defining transition probabilities in the general case as

$$P(x_t|x_{t-\tau}, ..., x_{t-1}) = \frac{P(x_{t-\tau}, ..., x_t)}{P(x_{t-\tau}, ..., x_{t-1})} \tag{2.1}$$

where t represents discrete time, the x_t represent the stimuli and $\tau > 0$ is the order of the statistics considered. For the zero-th order case predictive probabilities as simply given by $P(x_t)$. The terms on the right side of (2.1) will be estimated from the frequencies of events according to

$$P(x_{t-\tau}, ..., x_t) = \frac{C(x_{t-\tau}, ..., x_t)}{N} \tag{2.2}$$

where $C(.)$ is the number of occurrences of a particular ordered set stretching back to the beginning of time and N is the total sequence length. For the zero-th order model, and letting x_t denote the current sequence element, the predictive probability is simply

$$P(x_t = X) = P(X) \tag{2.3}$$

whereas for the first order model this is

$$P(x_t = X | x_{t-1} = Y) = P(X|Y) = \frac{P(X,Y)}{P(Y)} \tag{2.4}$$

where $P(X,Y)$ is the relative frequency of the ordered pair (X,Y) calculated according to (2.2).

With infinite memory all predictions made by both zero-th and first order models will quickly converge to constant values depending on the long term frequencies of events. Exponential weighting of past events is therefore introduced in order to prevent the model from losing sensitivity to local changes, as well as better approximating human behaviour. This is done by explicitly assigning weights to all events observed so far and having these weights decrease exponentially with distance into the past. So in effect probabilities will not be estimated from their relative frequencies but rather from *weighted averages* calculated according to

$$P(x_{t-\tau}, ..., x_t) = \frac{\sum_{i=\tau+1}^{t} e^{-\lambda(t-i)} \delta[x_{i-\tau}, .., x_i = x_{t-\tau}, .., x_t]}{\sum_{i=\tau+1}^{t} e^{-\lambda(t-i)}} \tag{2.5}$$

where λ is the the rate of decay of the exponential weighting function; the larger λ is, the quicker the model forgets past events, and vice-versa. $\delta[.]$ is the Kronecker delta function, taking a logical expression and returning 1 if the argument is true and 0 if false. Equation (2.5) effectively takes an ordered subsequence of any length and compares it to all sequences of equal length which occurred previously, adding the corresponding weight - given by $e^{-\lambda(t-i)}$ - if a match is found. The normalising constant is simply the sum of all the weights going back to the beginning of the sequence and ensures that $P(x_{t-\tau}, ..., x_t)$ varies between 0 and 1. The probability estimates calculated according to (2.5) are then introduced in Equation (2.1) which, for the zero-th order case, results in

$$P(x_t = X) = P(X) = \frac{\sum_{i=1}^{t} e^{-\lambda(t-i)} \delta[x_i = X]}{\sum_{i=1}^{t} e^{-\lambda(t-i)}} \tag{2.6}$$

and, for the first order model,[9]

$$P(x_t = X | x_{t-1} = Y) = \frac{P(X,Y)}{P(Y)} = \frac{\sum_{i=1}^{t} e^{-\lambda(t-i)} \delta[x_{i-1}, x_i = Y, X]}{\sum_{i=1}^{t} e^{-\lambda(t-i)} \delta[x_i = Y]} \tag{2.7}$$

We are finally left with two models capturing different order statistics and incorporating an exponentially decaying memory of the sequence. In addition, both models can accommodate an arbitrary number of sequence elements. Figure 2.2 shows predictions made by both models for a sequence with two possible elements, such as in a 2AFC. A key point is that the sequences XYXYX and XYXYY are the most defining in distinguishing between the two models: a first order model will attribute a higher probability of seeing an X after XYXY, whereas the zero-th order model will in fact predict a Y in the same situation, thereby predicting a repetition after a string of alternations. The reason for this ‘erroneous’ prediction is that the zero-th order model will always be biased towards the last stimulus so it will always display a preference for repetitions. In fact, a zero-th order model can be interpreted as being sensitive to repetitions only, whereas a first order model detects both repetitions and alternations. Finally, predictions by both models will depend on λ, with two extreme cases useful in guiding our intuitions about this dependence:

[9] Note that in practice some care must be taken to ensure the normalising constants in (2.5) cancel out since a sequence of pairs is always one shorter than a sequence of singlets.

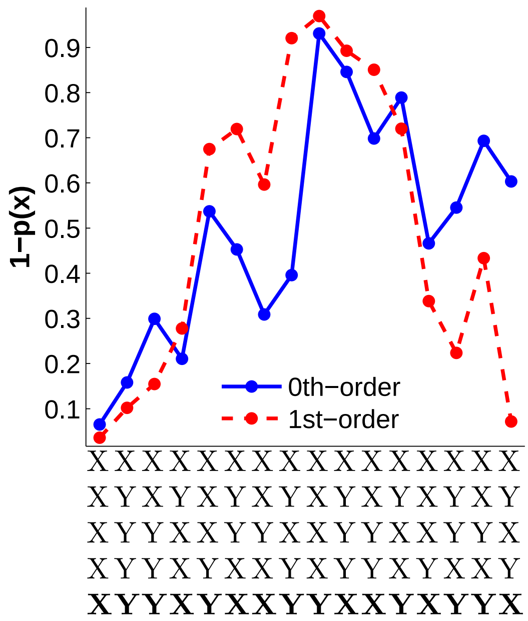

FIGURE 2.2: Model predictions for a 2AFC. Solid blue line - zero-th order model. Dashed red line - first order model. Predictions of both models are shown in terms of $1 - p(x)$ in order to facilitate comparison with empirical data. Results were calculated by running the model on a 3000 long sequence and binning the probabilities at each time step in a way analogous to reaction time data. $\lambda = 0.22$ for both illustrations. Note the stronger repetition bias of the zero-th order model when compared with the more balanced preference for repetitions and alternations displayed by the first order model. Moreover, note the difference between the sequences XYXYX and XYXYY, indicative of the zero-th order model's insensitivity to alternations (see main text).

for very high values of λ, corresponding to a very short memory, model predictions reduce to a trivial dependence on the last event; for values of λ close to zero, corresponding to a near-perfect memory, model predictions will tend asymptotically to the base rates of events, which for a random binary sequence with equal stimulus frequencies are all 0.5.

Figure 2.3 shows model predictions for a sequence with three possible elements. Results are far less intuitive in the three element case, but again a zero-th order model is incapable of detecting anything other than repetitions of the same event, and so predictions are largely a function of the length of the preceding run of elements equal to the last. A first order model shows a more complex pattern depending on the type of transitions present in each five-long sequence, and reveals a sensitivity to other types of patterns such as alternations of two elements. Note that in Figure 2.3

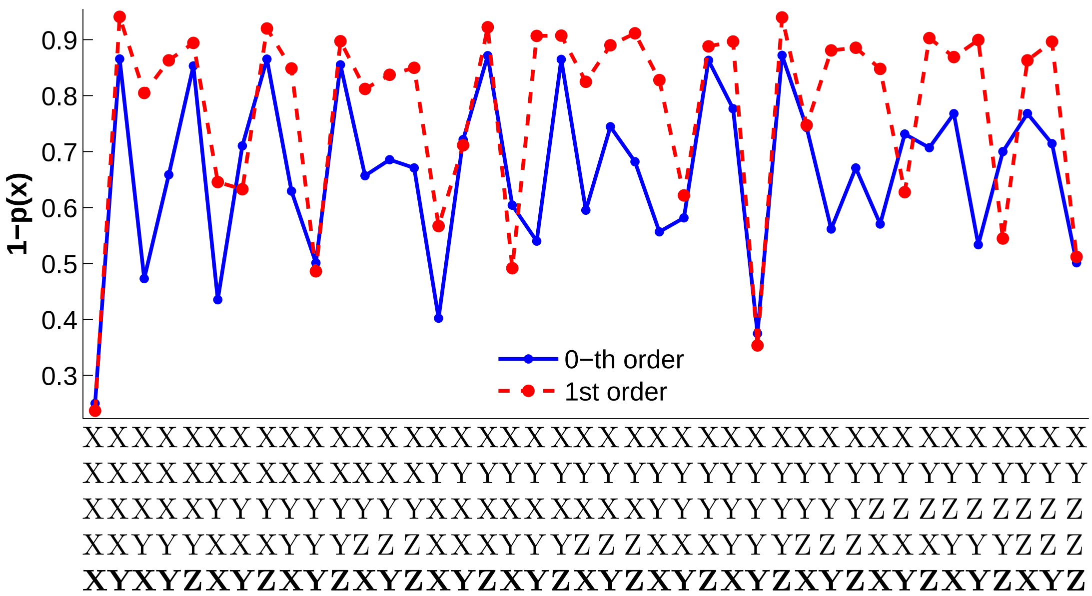

FIGURE 2.3: Model predictions for a 3AFC. Solid blue line - zero-th order model. Dashed red line - first order model. $\lambda = 0.22$ for both illustrations. Model predictions are more complex in this case and harder to interpret. However, there are some crucial differences between both models, which can be again be observed in the relative predictions for sequences such XYXYX and XYYXYY: a zero-th order model attributes almost the same probability to both but a first order model makes a clear distinction between the two, indicating a sensitivity to higher order transitions.

the values of λ were chosen not only to highlight differences between the two models but also to avoid the issues of sparsity described above, particularly in the case of the first order model which, for small enough λ, results in predictions equal to 0 for several of the five-long sequences.

A negative linear relationship between predictive probabilities and reaction times will be assumed throughout, as is usually the case in the literature (e.g. Falmagne, 1965), so $RT \propto 1-P(x_t)$. Model predictions will be linearly transformed according $a - b(1 - P(x_t))$ in order to to account for differences in scale and magnitude between probability values and reaction times. Best fitting parameters will be estimated by minimising the sum of squared deviations between model and data by varying three parameters: a, b and λ.

	0th order model	1st order model
Experiment 1	-3.34	**-0.52**
Experiment 2	-7.46	**-1.56**
Experiment 3	**-3.24**	-7.08

TABLE 2.1: log-likelihood values for the zero-th and first order models (higher values shown in bold) on all three datasets estimated by assuming the data is normally distributed. Experiment 1: 2AFC similar in every respect to Cho et al. (2002); Experiments 2 and 3: 2AFC and 3AFC respectively under the new experimental paradigm.

2.3 Experiments

The results of three experiments are reported here. The first experiment is a control for the overall set-up. The second experiment validates the new experimental protocol as producing results similar to typical 2AFC designs. The third experiment is a 3AFC also conducted under the new experimental design. The overall aim of these three experiments is to compare the results of a 2AFC with those of a 3AFC while ensuring that the differences encountered are solely the product of the different number of sequence elements.

2.3.1 Data analysis and model fits

Experimental results will be presented as median reaction times on all trials in which the subject did not make an error, organised according to the previous sequence. Similarly, model predictions are the median of all predictive probabilities calculated from a random sequence the same length as that which subjects experienced - 1560 stimuli.

Best fitting parameters were obtained in each case by minimising the sum of squared errors between data and model predictions. The quality of fit of each model for each experiment at best fitting parameters was calculated assuming the sixteen median RTs are normally distributed. Under these assumptions, the log-likelihood can be calculated as

$$LL \propto -\frac{SSE}{\sigma^2} \tag{2.8}$$

where LL is the log-likelihood, SSE is the sum of squared errors between data and model and σ^2 is the variance of the sixteen median RTs. Table 2.1 shows the log-likelihood values for each model and each experiment, where the preferred model in each case is shown in bold.

2.3.2 Experiment 1

Experiment 1 was a replication of the experiment performed by Cho et al. (2002). That a similar cost-benefit pattern of results can be obtained validates the overall protocol, ensuring that any minor differences in set-up such as background colour and the use of a response box are not important. Moreover, it adds to our confidence that the results obtained by Cho et al. (2002) are reproducible.

Participants

Five subjects (four female, one male) took part in this experiment. Participants in this and all experiments were volunteers recruited from the University of Adelaide and surrounding community; all gave their informed consent to participating in the experiment; all had normal or corrected-to-normal eyesight.

Stimuli

Stimuli consisted of an upper-case and lower-case 'O', displayed in the same position in the centre of the screen.

Procedure

Subjects sat approximately 60cm away from the computer screen, inside a darkened room. The stimuli were white (approximately 3 cm tall) and the background was gray. Stimuli were displayed using Psychophysics Toolbox 3 and Matlab r2008a on a 15" Macintosh MacBook Pro running MacOSX 10.6. Responses were made using a Cedrus RT-530 response time box, which has one central round button surrounded by four rectangular buttons. The RT box was placed to the right of the computer if the subject was right-handed, and to the left if left-handed.

Responses were made using two fingers, the middle and index of the dominant hand, one placed on the left button and one on the right button of the response box. Subjects were instructed to respond as quickly and accurately as possible to the stimulus by pressing the button corresponding to the stimulus shown (left - 'o', right - 'O'). After pressing the button, the stimulus disappeared and after a fixed response-stimulus interval (RSI) of 800ms, another one appeared. The only feedback was a beep whenever a button was pressed. This paradigm closely replicates Cho et al. (2002), the only differences being that in the present case stimuli were presented on a gray (rather than black) background and that a response box was used, allowing for near-millisecond precision in measuring reaction times.

The experiment consisted of 13 blocks of 120 trials each, with a small break in between each block and a longer break (approximately 10 min) after the seventh block. Each subject was given one block of training before beginning. Data from training blocks was not used in the analysis. Sequences were generated for each block by randomly permuting a sequence with an equal number of both elements. The relative frequencies of both stimuli were equal within each block and so for the whole experiment.

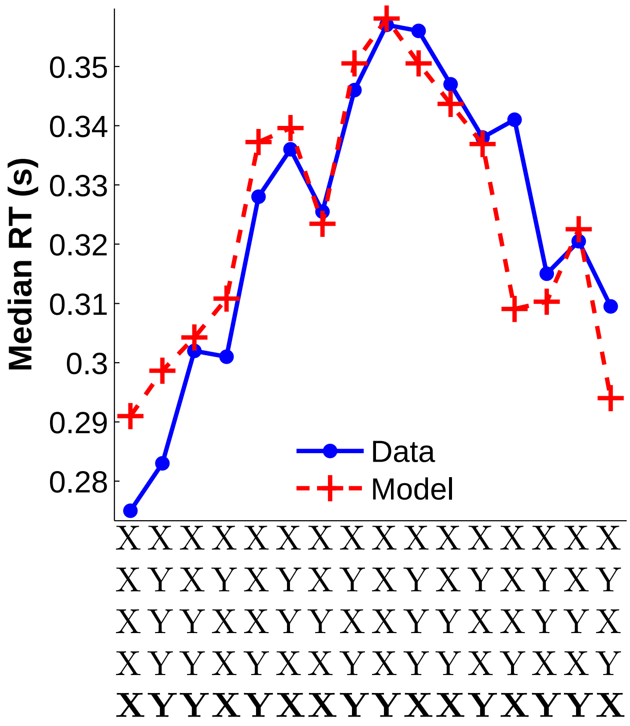

FIGURE 2.4: Results of Experiment 1 together with best fitting model predictions. Solid blue line - median RTs across all participants. Dashed red line - best fitting **first order** model with parameters $a = 0.0816$, $b = 0.2833$ and $\lambda = 0.33$. This experiment followed a protocol close to that used by Cho et al. (2002); stimuli consisted of an upper-case and lower-case 'o' and the RSI was 800 ms.

Results

The results of Experiment 1 are shown in Figure 2.4, together with the best fitting first order model. Results are remarkably similar to that of the original experiment by Cho et al. (2002), and both are typical examples of a cost-benefit pattern. Log-likelihod values, shown in Table 2.1, confirm quantitatively that the preferred model was first order. Notice that alternations are clearly being detected, as subjects show quick reaction times to a perfectly alternating pattern (rightmost sequence); conversely, very slow reaction times are observed when a repetition ends what had been so far a perfect alternating run (eighth sequence from left). As discussed before, there is some variation in the predictions of a first order model depending on λ, but the hallmark of this model is a sensitivity to alternations as well as repetitions of stimuli.

Discussion

Despite a few points of divergence between model predictions and data, the best fitting model is clearly first order as can be seen from the log-likelihood values in Table 2.1. So when performing a normal 2AFC subjects seem to be using predominantly first order transition probabilities in order to predict the next event. In addition, our experimental set-up is validated as producing a typical cost-benefit - i.e. inverted 'v' - pattern of sequential effects. These results are important in that, when we turn to the comparison of a 2AFC with a 3AFC under a new experimental paradigm (see below), there should be no room for doubt that any differences encountered are the consequence of the number of possible stimuli and not minor differences in set-up.

While somewhat marginal to the main objectives of this work, it is worth noting that significant individual differences were found among participants. These differences bear some relevance here because they indicate that, while as a group participants were using first order statistics, this is not necessarily the case for each individual. One subject in particular showed a near-perfect fit to a zero-th order model (see Figure 2.7, right panel) which suggests that individual variation is not all due to noise, but rather reflects meaningful differences possibly related to differences in the statistics used by different participants (see below for a discussion of this point). Further research is necessary to confirm or disprove this hypothesis but it seems likely that individual factors, as well as experimental conditions, could influence the nature of the statistics being used. Some concern is also raised regarding averaging practices prevalent in the literature, as they could be masking important individual differences.

2.3.3 Experiment 2

Experiment 2 is a 2AFC performed under the new experimental paradigm. The aim of this experiment is to confirm that the new experimental design can produce a cost-benefit pattern of sequential effects much like other 2AFC tasks.

Participants

Five participants (four female, one male) took part in this experiment. One of these also participated in Experiment 1 and a different one in Experiment 3.

Stimuli

Stimuli consisted of two geometric shapes, a square and a triangle, displayed in the same position in the center of the screen.

Procedure

The experimental procedure was the same as with Experiment 1 except that responses were made using only one finger - the index of the dominant hand. A central button in the RT box was kept depressed before stimulus onset; shortly after a stimulus appeared subjects moved their finger and responded by pressing one of two side buttons (left - triangle, right - square); after each response, the finger returned to the central position; finally, the next stimulus appeared after a 800 ms interval starting from the moment the response button was pressed. The time between stimulus onset and middle button release and between middle button release and side button press were recorded with reaction time taken to be the sum of both. Feedback consisted of a high pitch beep if everything was all right and one low in pitch as a warning in case the subject forgot to return his/her finger to the middle position in time for the next stimulus.

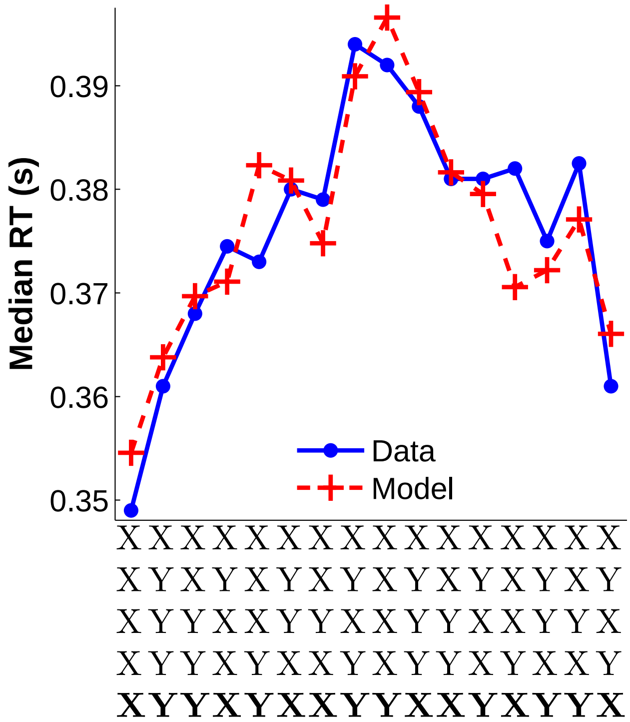

FIGURE 2.5: Results of Experiment 2 together with best fitting model predictions. Solid blue line - median RTs across all participants. Dashed red line - best fitting **first order** model with parameters $a = 0.0793$, $b = 0.3367$, $\lambda = 0.12$. The experiment was conducted according to the new experimental design: responses were made with one finger and stimuli consisted of a square and a circle and the RSI used was 800 ms.

Results

As with Experiment 1, the reaction time pattern obtained with Experiment 2 was best captured by a first order model (see Figure 2.5 and Table 2.1), so participants were also using first order transition probabilities when predicting the next event. Again this is visible in relatively fast reaction times to alternating sequences and slow to interruptions of such a pattern. However, results differed from those of Experiment 1 in that subjects displayed a slightly longer RT to interruptions of an alternating pattern rather than to interruptions of a repeating sequence.

Discussion

The reason for the longer reaction time to the sequence XYXYY (AAAR), when compared with XXXXY (RRRA), is made clear upon inspection of individual results: several subjects displayed an alternation bias, i.e. faster overall reaction times to alternations than to repetitions, implying a greater sensitivity to alternating patterns relative to repeating ones. The flip-side to being more sensitive to alternations is that reaction times will also be greater to violations of an alternating pattern, and this explains the longer RT observed for the sequence XYXYY (AAAR). The reason some subjects display an alternation bias may be the use of just one finger to perform responses as this has been shown before to induce a greater preference for alternations when compared to using two fingers (Hannes, 1968). However, the mean RT pattern across all subjects still displays a slight repetition preference, and this is the predominant bias in individuals. Sporadic mention of individual differences in preference for repetitions or alternations do in fact exist in the literature (e.g. Arons & Irwin, 1932; Bertelson, 1965), though no dedicated study of the subject has been conducted.

There are two main differences between the new experimental paradigm used in Experiment 2 and the classic design of Experiment 1: firstly, under the new paradigm only one finger is used to respond to each stimulus whereas before two fingers were used; secondly, geometric figures were used as stimuli, rather than stimuli differing in size. Although it was designed minimise confounding effects, the new scheme involving just one finger is expected to introduce some additional noise in the data due to the motor error associated with moving the arm as well as the finger. Otherwise no significant differences were observed in mean RT to any of the figures, and the same was true of all responses.

2.3.4 Experiment 3

Experiment 3 was a 3AFC performed with the new experimental design, and was conducted in order to investigate the consequences of increasing the number of stimuli.

Participants

Seven participants (six female, one male) took part in Experiment 3. The higher number of participants relative to the previous two experiments was due to the fact that there there are now many more possible five-long sequences of stimuli and that as a consequence each individual quintet is on average less frequent. Therefore, in order to have a similar number of data points for each sequence relative to a 2AFC, a higher number of trials was necessary, either by increasing the number of participants or the number of trials per participant. Increasing the number of trials could lead to confounding effects from tiredness and so the choice was made to include more participants.

Stimuli

Stimuli consisted of three geometric shapes: a square, a triangle and a circle displayed in the same position in the center of the screen.

Procedure

The experimental procedure was the same as in experiment 2 except that a third response button - top - was now used. The mapping was: left - triangle; right - square; top - circle.

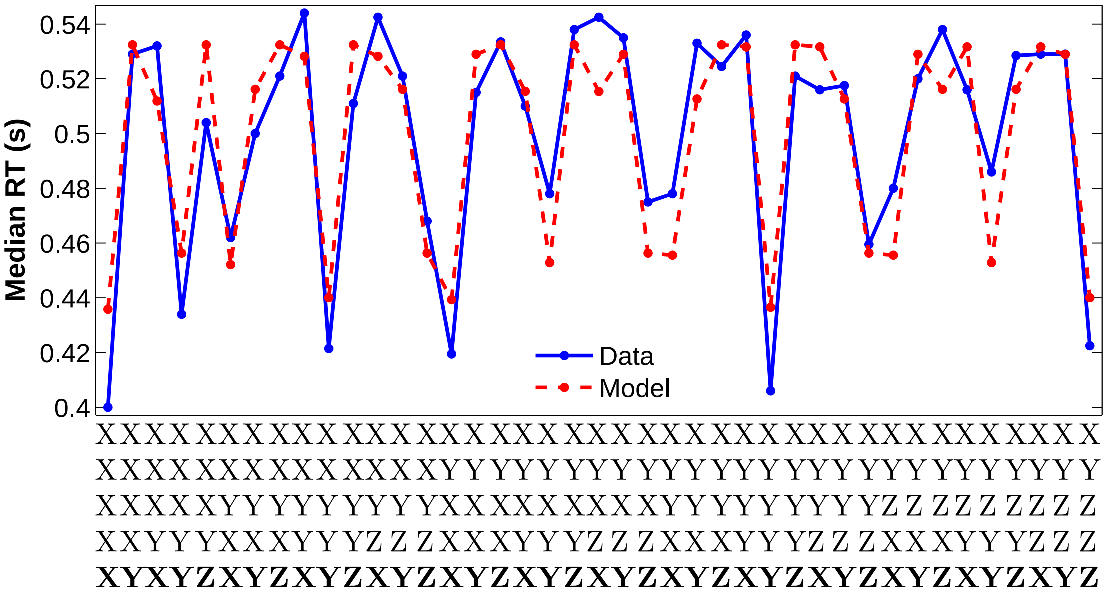

FIGURE 2.6: Results of Experiment 3 together with best fitting model predictions. Solid blue line - median RTs across all participants. Dashed red line - best fitting **zero-th order** model with parameters $a = 0.0969$, $b = 0.4356$, $\lambda = 1.54$. The experiment was conducted according to the new experimental design: responses were made with one finger; stimuli consisted of a square, a triangle and a circle; and the RSI used was 800 ms. Note the increase in the best fitting λ when compared to the two previous experiments, which implies a shorter memory span in a 3AFC and determines that reaction times depend largely on the last two trials and whether these repeated or alternated.

Results

Figure 2.6 shows results for the 3AFC task together with the best fitting zero-th order model. Table 2.1 shows log-likelihood values demonstrating that a zero-th order model is preferred. If subjects were using first order transition probabilities this would be reflected in relatively fast RTs to sequences such as XYXYX (AAAA), a perfectly alternating sequence, when in fact only sequences ending in a repetition display short RTs. In fact the overall pattern consists of an almost two-tiered dependence on the last two stimuli and whether these represented a repetition or an alternation. The reason for this pattern of results is not only that zero-th order statistics were being used, but also that the memory span was very short, as reflected in the best fitting value of λ which was considerably higher in magnitude than those obtained for the previous two experiments ($\lambda = 1.54$ for Experiment 3, compared to $\lambda = 0.33$ and $\lambda = 0.12$ Experiments 1 and 2). As discussed above, λ determines the steepness of the exponential weighting function which in turn determines how long the effective memory span is.

Discussion

It is apparent from our results that subjects changed the nature of the statistics being used when the number of possible stimuli increased from two to three. Specifically, subjects were using first order statistics in a 2AFC and switched to zero-th order statistics in a 3AFC. Results show an almost binary dependence on the last event being a repetition or not and reveal no sensitivity to patterns such as alternations which would have indicated the use of first order transition probabilities. Given that all other experimental factors were held constant between the two tasks, these changes must have happened as a consequence of the added complexity of a 3AFC.

Another important difference between the model fit to a 3AFC when compared to a 2AFC is the higher value of λ obtained, which implies a shorter even horizon, with stimuli beyond the last two having very little impact on reaction times. So it seems that, in addition to changing the nature of the statistics used, the extra difficulty of the task had the effect of shortening participants' effective memory range. A more detailed discussion of the differences between a 2AFC and a 3AFC in terms of statistics used and memory span can be found in the next section.

2.4 General Discussion

In this work the kind of statistics people use when analysing sequences of events, and whether these statistics change with increasing sequence complexity, was investigated. Depending on the nature of the statistics being used, different patterns of sequential effects are expected. Comparing model predictions with experimental results allowed conclusions to be drawn regarding what statistics were being used in two different tasks: a 2AFC and a 3AFC. Care was taken to ensure that the differences encountered between the two tasks were due to the nature of the sequence and not any other experimental differences. Results revealed an interesting change: in contrast with a 2AFC, where subjects were using first order transition probabilities, participants used just the relative frequency of the stimuli in a 3AFC. This is the first time fundamental differences are reported in the nature of the statistics used in sequential effects as a function of experimental circumstances.

In principle, and assuming no drastic change in the environment, the past can be used to some extent in order to predict the future. Once this premise is fulfilled, it is always better to use higher order transition probabilities, as this leads to more accurate predictions about the future (Manning & Schütze, 1999), despite the fact that evidence suggests humans are limited to using first order transition probabilities (Newport & Aslin, 2004; Gebhart et al., 2009). The use of lower order statistics in a 3AFC is therefore sub-optimal, and in need of explanation. It is only natural to assume, given the greater complexity of a 3AFC relative to a 2AFC, that the differences observed between both tasks are due to processing capacity limitations. In order to compute first order statistics the frequencies of pairs of stimuli, as well as their respective base rates, are necessary. In a 2AFC, using first order statistics requires tracking four pairs of events as well as the base stimulus frequencies, for a total of six quantities; in a 3AFC, this number rises to twelve, which is twice as many as a 2AFC.[10] It therefore seems plausible that processing limitations, and in particular a limited working memory capacity, might play a role in determining the type of statistics being used in tasks of differing complexity.

In addition to changing the nature of the statistics used to make predictions, increasing the number of possible stimuli also reduced subjects' memory span as measured by the best fitting exponential decay rate λ. So far the processing demands of different tasks have been discussed in terms of the number of relative frequency values one must implicitly keep track of in order

[10] Again, these numbers could be smaller if the loss in degrees of freedom due to mathematical constraints is taken into account. However, even if this was the case, the quantities to be tracked would be five and ten respectively for a 2AFC and 3AFC, which is still a considerable difference.

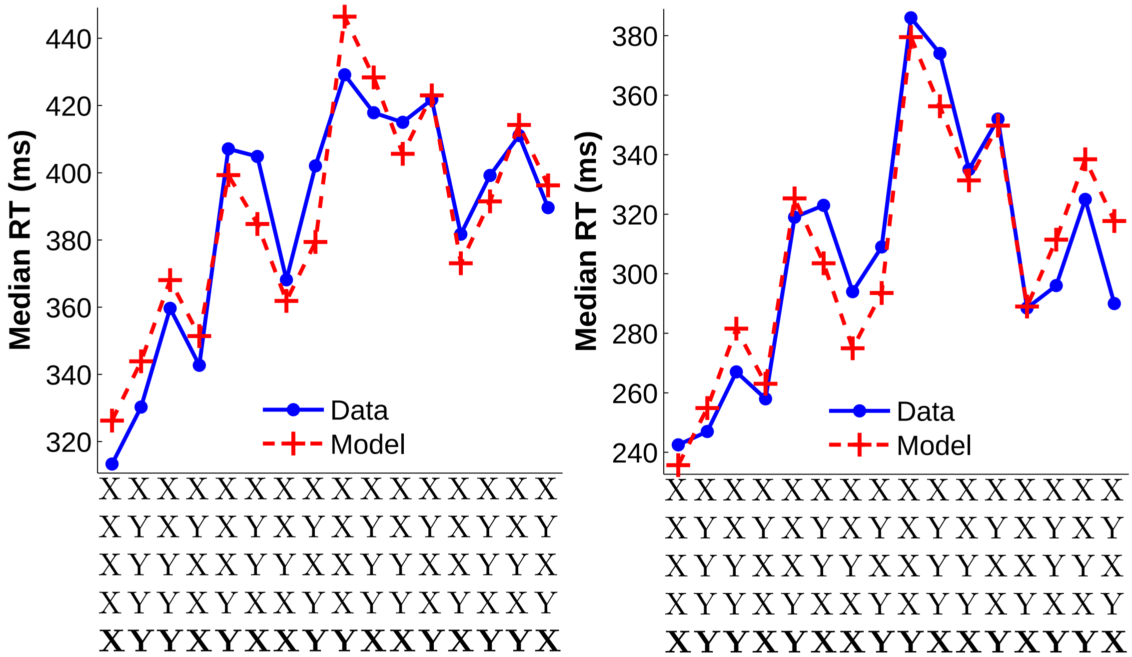

FIGURE 2.7: Evidence for zero-th order statistics in a 2AFC. Left panel - Data from A. D. Jones et al. (2002) together with the best fitting zero-th order model with $\lambda = 0.33$. Right panel - Data from one of the participants in Experiment 1 together with the best fit of a zero-th order model with $\lambda = 0.36$. The experiment performed by A. D. Jones et al. (2002) is described as a 2AFC despite the fact that subjects responded with one button to only one stimulus - an 'X '- and with another button to any other stimulus - a series of upper-case letters - with the frequency of 'X' equal to the sum of those of all other stimuli; the RSI was 1000 ms and the stimuli were displayed for 250 ms.

to calculate different transition probabilities. However, if we assume that what participants keep track of are individual *instances* of events, rather than just their frequency, reducing the memory span would go some way towards alleviating the burden associated with an insufficient memory capacity. If this is the case the increase in λ observed in a 3AFC might also be explained by a limited working memory capacity. Whatever the underlying truth is, it is clear that not only do humans change the statistics they use when faced with added task complexity, but they also shorten their memory range.

The results presented here show that humans use first order statistics in a 2AFC and zero-th order in a 3AFC. It is only natural to ask whether there are any circumstances in which humans use zero-th order statistics in a 2AFC task or, conversely, first order statistics in a 3AFC. These two question will be addressed in turn starting with what evidence there is for zero-th order statistics in a 2AFC. A. D. Jones et al. (2002) performed an experiment with multiple sequence elements which was nevertheless described as a 2AFC given that there were only two possible responses: one button was used to respond to a target stimulus and the other to any one of the remaining stimuli. Interestingly, the pattern of sequential effects obtained was very similar to a zero-th order model (Figure 2.7, left panel). While not being a typical 2AFC in that there are multiple stimuli involved, this experiment is nevertheless still informative in that it adds strength to the argument that the number of stimuli is driving changes in the type of statistics used. More conclusive evidence that the use of first order statistics is possible in a 2AFC comes from the individual results of a participant in Experiment 1 which displays a pattern approximating closely that which is predicted by a zero-th order model (Figure 2.7, right panel). This raises the enticing possibility that different individuals are using different types of statistics when performing the same task. If this is found to be the case, it is interesting to speculate whether these individual differences could be due to variation in processing capacity. Speed of processing has in fact been implicated in changes in sequential effects, albeit only when the RSI is short (Melis et al., 2002). Finally, it is not known whether it is possible for humans to use first order statistics in a task with more than three stimuli, although this is unlikely given the expected increase in processing demands.

The exponentially forgetting function used in our model is clearly a simplification, albeit a common one in the literature (Falmagne, 1965; Laming, 1969; Yu & Cohen, 2008). Furthermore, that humans show such a memory decay should be understood from a correlational point of view, as well as occurring in the face of a *random* environment. Any monotonically decreasing memory function would likely break down if humans were presented with a perfectly regular sequence where the next event is entirely predictable. Under such an input, it would be possible to remember things about the past far beyond the short time span allowed by an exponentially decaying memory. Strong correlations with events far into the past would be made possible, and conversely correlations with recent events could break down. Consider the sequence YYYYYYXXXXXX repeating itself into the past: the next element is clearly a Y, and presumably humans would would be quick to realise this. In this example, the recent past becomes irrelevant towards predicting the future, and any model of sequential effects developed so far would fail at predicting the next element. Therefore, a richer view of the significance of a monotonic memory function is that it is a trace of a more complex mechanism at play. Assuming sequential effects to be the consequence of an attempt at detecting patterns, then is only natural to speculate that whatever pattern detection mechanism the brain is employing, it sees its activity decay in an exponential fashion when it fails to detect a clear pattern. Under this view, sequential effects are the product of presenting the brain, a highly developed pattern detecting machine, with random input.

The simple framework proposed here was never designed as a complete model of sequential effects, but rather as a tool to understand the different types of information humans use when analysing different types of sequence. Sequential effects are a rich and diverse area of study with many different aspects still in need of explanation. Phenomena such as the dependence of sequential effects on the RSI are difficult to explain in the context of any previous model. One aspect in particular which has been left unaddressed so far is individual differences, despite a few passing mentions that these do exist (Arons & Irwin, 1932; Bertelson, 1965; Kirby, 1976). Finally, our simple model falls short of what would ideally be a full statistical model allowing for a more principled model comparison based on inference rather than a comparison of log-likelihood values.

2.5 Conclusion

Sequential effects illustrate how, in dealing with random environments, humans exhibit some striking regularities. The present work adds to this by highlighting the difficulties faced in increasingly complex environments. In particular, it was shown here that humans can change the very nature of the way in which they analyse a sequence in order to cope with the extra complexity. Furthermore, it was shown that humans do not look as far into the past when performing a more complex task. These conclusions are intuitive: the more complex the environment, the harder it is to analyse it, especially in the absence of a clear pattern.

3

The Structure of Sequential Effects

A closely related version is included in:

Gökaydin, D., Ma-Wyatt, A., Navarro, D., Perfors, A. (2015) The Structure of Sequential Effects, manuscript submitted to the *Journal of Experimental Psychology General*

Note: Supplementary information to the article can be found in Appendix A.

Abstract

There is a long history of research into sequential effects, extending more than one hundred years. Yet despite some passing mentions one aspect of sequential effects has been largely overlooked: individual differences. Here principal component analysis is performed on a dataset of 158 individual results from participants performing different experiments with the aim of identifying hidden variables responsible for sequential effects. We find a latent structure consisting of three components related to sequential effects - two main and a minor. A relationship between the two main components and the separate processing of stimuli and of responses is proposed based on previous empirical evidence. It is further speculated that the minor component of sequential effects arises as the consequence of processing delays. Independently of the explanation for the latent variables encountered, this work provides a unified descriptive model for a wide range of different types of sequential effects previously identified in the literature. In addition to explaining individual differences themselves, it is demonstrated how the latent structure uncovered here is useful in understanding the classical problem of the dependence of sequential effects on the interval between successive stimuli.

3.1 Introduction

The survival of any intelligent organism depends on its capacity to predict - and adapt to - changes in its environment. Such predictions are possible because the world is not random but instead it is full of spatial and temporal regularities. Extending these patterns into the unknown allows humans to develop a mental picture of what lies beyond their immediate sensory experience and this induces a state of expectation about what will happen next. This is true in space as well as time, e.g. we expect the laws of physics to apply tomorrow as well as anywhere on earth. Crucially, that we can predict the future to some extent allows us to prepare for it, and react faster to those events we expect. On the other hand if we are surprised by an unusual event we will take longer to adapt and respond to the new situation.

But what if there is no pattern to be found? Short-lived regularities sometimes occur in a random world, but will they still retain the capacity to influence our expectations or will we recognise them as the product of a random process and dismiss them as such? It turns out that humans will persistently shift their expectations based on short-term fluctuations in the environment, as demonstrated in the context of several different behavioural tasks, all of which involve a sequence of trials (Fernberger, 1920; Jarvik, 1951; Bertelson, 1961; K. C. Squires et al., 1976; Maloney et al., 2005). In this context the human tendency to be sensitive to the recent sequence of events manifests itself as a dependence of some measure of performance on the last few trials in the sequence. This phenomenon - often referred to as *sequential effects* - has been studied most extensively in reaction time tasks involving a random sequence of only two possible stimuli (Bertelson, 1961; Falmagne, 1965; D. J. Hale, 1967; Laming, 1968; D. Hale, 1969; Remington, 1969; Schvaneveldt & Chase, 1969; Laming, 1969; Kirby, 1972, 1976; Soetens et al., 1985; Cho et al., 2002; Jentzsch & Sommer, 2002). The effect of the sequence of stimuli has often been found to be stronger than the properties of the stimuli themselves in accounting for variation in reaction times (Kornblum, 1969). A particularly striking example of the power of sequential effects is that they can alter what is actually perceived: in an experiment with an ambiguous percept the recent sequence of trials can determine whether subjects see a pair of stimuli in quick succession as rotating left or right (Maloney et al., 2005). Sequential effects similar to those observed in reaction time experiments have also been observed in event-related potentials (ERPs) measured with EEG (K. C. Squires et al., 1976; Sommer et al., 1990, 1999; Jentzsch & Sommer, 2002) making the overall topic one that lies at the interface between psychology and neuroscience, and of relevance to both fields.

This work is concerned with uncovering the latent structure of sequential effects by studying what has been so far a largely overlooked source of evidence: individual differences. Latent variable analysis of the individual results of over one-hundred and fifty participants performing different experiments will be conducted. Foreshadowing some of the results of this work, two main latent components are identified together with a third minor component. An attempt will be made to relate the two main latent components to previously available evidence for the existence of two separate processing stages involved in sequential effects. Further, it will be speculated that the minor component identified is a consequence of processing delays. Independently of its interpretation, the latent structure uncovered here is of value in establishing a common framework for understanding differences in sequential effects. For instance, it will be shown here that individual differences are closely related to the way sequential effects depend on the interval between the successive stimuli.

With all of the above in mind the introduction to this article is structured as follows: firstly some key concepts related to sequential effects fundamental to understanding this article will be discussed; secondly, different sources of variation in sequential effects will be reviewed; finally previous evidence for two processing stages involved in sequential effects will be given particular attention.

3.1.1 Background

The most common experimental paradigm used to study sequential effects is the sequential two-alternative forced-choice task (2AFC). In a typical experiment subjects experience a long random sequence of two possible stimuli - denoted here as X and Y - one at a time, to which they have to respond with one of two corresponding buttons as quickly and as accurately as possible. When a response is made the stimulus disappears and, after a fixed period of time termed the *response-stimulus interval* (RSI), the next stimulus is shown. The reaction time (RT) is recorded for each trial and these measurements are then grouped according to the last 5-stimuli - including the one responded to - in a 'sliding window' fashion. The mean or median of each set of reaction times to all instances of a particular five-long sequence is then calculated. There are 32 possible five-long binary sequences but these are usually grouped two-by-two depending on the pattern they represent (e.g. XXYYX and YYXXY) for a total of 16 pairs which we will refer to simply as 'sequences'.

Sequential effects are thought to be the product of an attempt at detecting two types of pattern in the sequence: repetitions and alternations (D. Hale, 1969; Maloney et al., 2005). In order to highlight this fact sequences of stimuli are often rewritten in terms of repetitions or alternations (denoted A and R respectively) of individual stimuli. For instance, XYYXY corresponds to ARAA, and the perfectly regular sequences XXXXX and XYXYX will be RRRR and AAAA respectively. This coding scheme makes it easier to illustrate the way in which sequential effects depend on the expectations generated by the previous sequence and the pattern it represents. For instance, consider the incomplete sequences RRR_ and AAA_, i.e. perfectly repeating and alternating patterns so far, and where the blank space represents the next - as yet unseen - event: subjects tend to react faster to an R in the first case and an A in the second because these events continue the local pattern. Conversely, reaction times tend to be slower for violations of the local pattern such as AAAR and RRRA, where the notation used throughout was used, in which the last event on the right is the one being responded to.

Sequential effects do not occur exclusively when the sequence of events displays a perfectly regular pattern. Any sequence - regular or not - will induce a degree of expectation about the next event and this in turn influences reaction times, which will be faster than average for expected stimuli and slower for unexpected ones. One way to put it is that any sequence has a *benefit* if the next stimulus is expected and a *cost* if it is not. The benefit of each sequence is a function of how many events *equal to the next* it includes. For instance, reaction times tend to be faster to ARRR than to AARR because more repetitions happened before the last R in the former case. If the number of events equal to the last is the same in both sequences a recency principle applies, e.g. reaction times will be shorter to AARR when compared to RAAR.[1] In short, one simple way

[1] Note that, as pointed out by Maloney et al. (2005), there is some degree of ambiguity regarding the sequences pairs AARR/RRAR and RRAA/AARA in that it is not clear whether the benefit of one more recent event outweighs that of two more distant ones or vice-versa.

to describe the influence of the sequence is to consider that each event - A or R - *primes* subjects to expect the occurrence of the same event, and that this effect is cumulative. If the cost and benefit effects of each sequence are balanced then overall reaction times to repetitions and alternations will be similar. More often than not however some difference in overall reaction time to repetitions and alternations exists, and this will be referred to throughout as a repetition or alternation 'bias', or simply as a 'preference' for whichever event displays faster reactions times.[2]

The priming view of sequential effects predicts an inverted 'v' shape of data when plotted the traditional way.[3] A similar shape is in fact often observed in empirical reaction time data (e.g. Soetens et al., 1985; Cho et al., 2002), a pattern of results commonly referred to as 'cost-benefit' due to its similarity to the ideal trade-off in the effects of the preceding sequence described above. However, considerable deviations to this ideal scenario exist, with many different patterns of sequential effects observed in reaction time data. There are two main sources of this variation: firstly, sequential effects depend on experimental parameters, and in particular the response-stimulus interval; secondly, even for constant experimental conditions substantial individual differences are observed in the pattern of results. In some cases these differences are relatively small, such as a slight repetition or alternation bias on what is otherwise a clear cost-benefit pattern of results. In other cases the sequential effects observed are so different as to no longer be recognisable as a cost-benefit pattern.

In the next two sections the two main sources of variation in sequential effects will be reviewed in turn, starting with the dependence on the RSI and the classical mechanistic theories used to explain this phenomenon. In order to discuss individual differences it will become necessary to show a sample of the data collected for this work since no previous study of the topic exists.

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2 A repetition or alternation bias can easily be visualised as a difference in 'height' between the left and right halves of traditional plots, since all sequences ending in R are to the left and with A to the right.

3 Notice that the sequences excluding the last event are organised by increasing expected cost if the last event is a repetition, or conversely increasing benefit if it is an alternation. Therefore, monotonically increasing reaction times in the left half where all sequences end with R, and decreasing in the right half where all end with A, are predicted by this simple idealisation of the effects of the sequence.

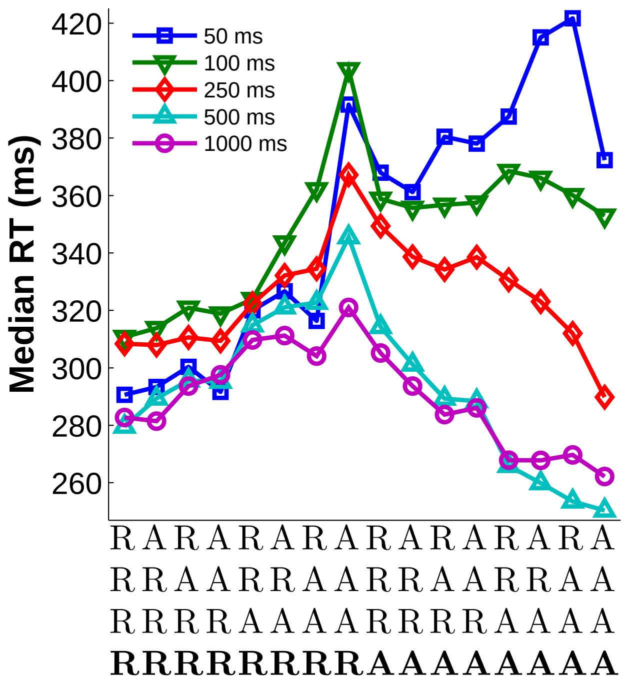

FIGURE 3.1: The dependence of sequential effects on the RSI. Adapted from Soetens et al. (1985) with permission. Each line corresponds to a different set of 10 subjects which performed the same experiment with a different RSI value. Other than the choice of RSI, the experimental protocol was exactly the same, with two horizontally displaced dots as stimuli, similar to Experiment 3 (see Method section).

3.1.2 Experimental differences

A cost-benefit pattern of sequential effects - the inverted 'v' - is often observed in experiments conducted with a long RSI (see Figure 3.1; 1000, 500 and 250 ms RSI) and, as we have seen, this is usually considered to reflect an expectation-based mechanism, often referred to as *subjective expectancy* (Kirby, 1976; Soetens et al., 1985). In contrast, when a short RSI - under 100 ms - is used a substantially different pattern of sequential effects, displaying approximately a positive slope on both the left and right halves of the plot, is observed instead (see Figure 3.1; 50 ms RSI); this was originally considered to reflect a unidirectional effect of the previous sequence, which would induce faster or slower reaction times irrespective of what the next event is, which then led to this pattern of results being coined *benefit-only* (Laming, 1968; Soetens et al., 1985).[4] Given that it no longer seemed compatible with expectations generated by the pattern in the sequence,

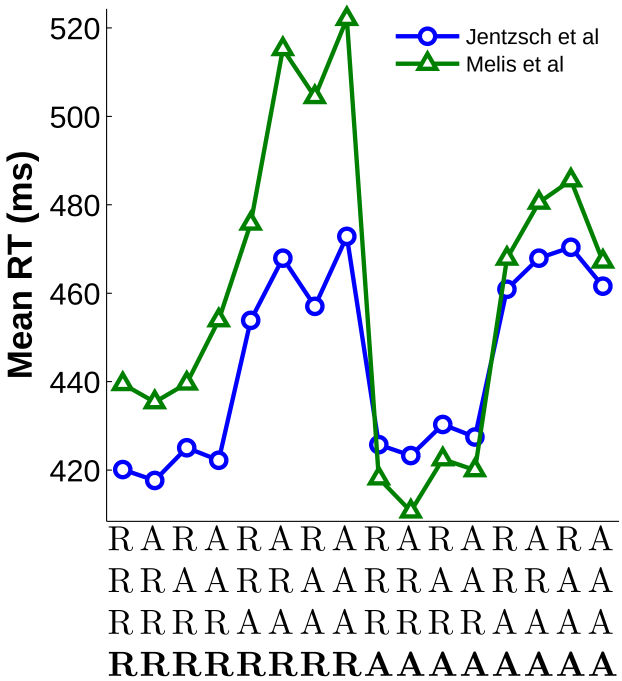

FIGURE 3.2: Unusual patterns of sequential effects. The results of two experiments are shown: the second experiment conducted by Jentzsch and Sommer (2002) and results from an elderly group of subjects included in Melis et al. (2002). Notice the approximately two-tiered pattern of results depending on whether the second-to-last event was a repetition or an alternation, irrespective of the last event. Both experiments were standard 2AFCs with separate dots as stimuli and both were conducted with a 50 ms RSI; the experiment by Jentzsch and Sommer (2002) was considerably longer both in terms of total trials (3960 vs 1560) and number of trials in one block (330 vs 120). Adapted with permission from the authors.

the benefit-only pattern of sequential effects was ascribed to a low-level effect termed *automatic facilitation* (Soetens et al., 1984, 1985). As the RSI is shortened, sequential effects gradually change from a cost-benefit to a benefit-only pattern (see Figure 3.1), and as the classical theory goes this reflects a gradual transition between subjective expectancy and automatic facilitation.

Most empirical results fall somewhere along the continuum between the cost-benefit and benefit-only patterns sequential effects (e.g. Vervaeck & Boer, 1980; Soetens et al., 1985; Cho et al., 2002; Jentzsch & Sommer, 2002; Gokaydin, Ma-Wyatt, Navarro, & Perfors, 2011). However, there are at least two experiments in which a qualitatively different pattern of sequential effects was obtained

[4] We will not go into much detail about automatic facilitation and how it could have a unidirectional effect on reaction times. For the purposes of this study 'benefit-only' should be taken as a label useful in referring to the pattern of sequential effects often observed when the RSI short.

(Jentzsch & Sommer, 2002; Melis et al., 2002). In both cases, shown in Figure 3.2, the pattern of results points to a dependence of reaction times on the *second-to-last* event independently of the last one (Jentzsch & Leuthold, 2005).[5] One hint of what may be behind this unusual pattern of results comes from the work of Melis et al. (2002) in which two groups, one of elderly and one of young subjects, performed the same experiment. When the RSI was long - 1000 ms - both groups displayed a typical cost-benefit pattern; when the RSI was short - 50 ms - the young group displayed a benefit-only pattern of sequential effects as expected but the elderly group produced the results shown in Figure 3.2. Melis et al. (2002) suggest that the underlying variable responsible for the differences observed between age groups is processing speed, of which age is a close correlate.

In addition to the RSI, several other experimental parameters influence sequential effects. These include stimulus-response compatibility (Bertelson, 1965; Soetens et al., 1985), different stimuli (e.g. D. J. Hale, 1967; Soetens et al., 1985; Cho et al., 2002) or different response schemes such as if just one or two fingers are used to respond (Hannes, 1968; Gokaydin et al., 2011), among others. However, the effects of most of these experimental manipulations again seem to fall along the continuum between a cost-benefit and benefit-only patterns (e.g. Soetens et al., 1985). So one simplified way of describing all types of sequential effects is by invoking the spectrum of results shown in Figure 3.1 together with the unusual results shown in Figure 3.2.

All of the results discussed so far are averages taken across a set of participants. However, as we will see next, considerable individual differences in sequential effects are usually present for the same experimental conditions.

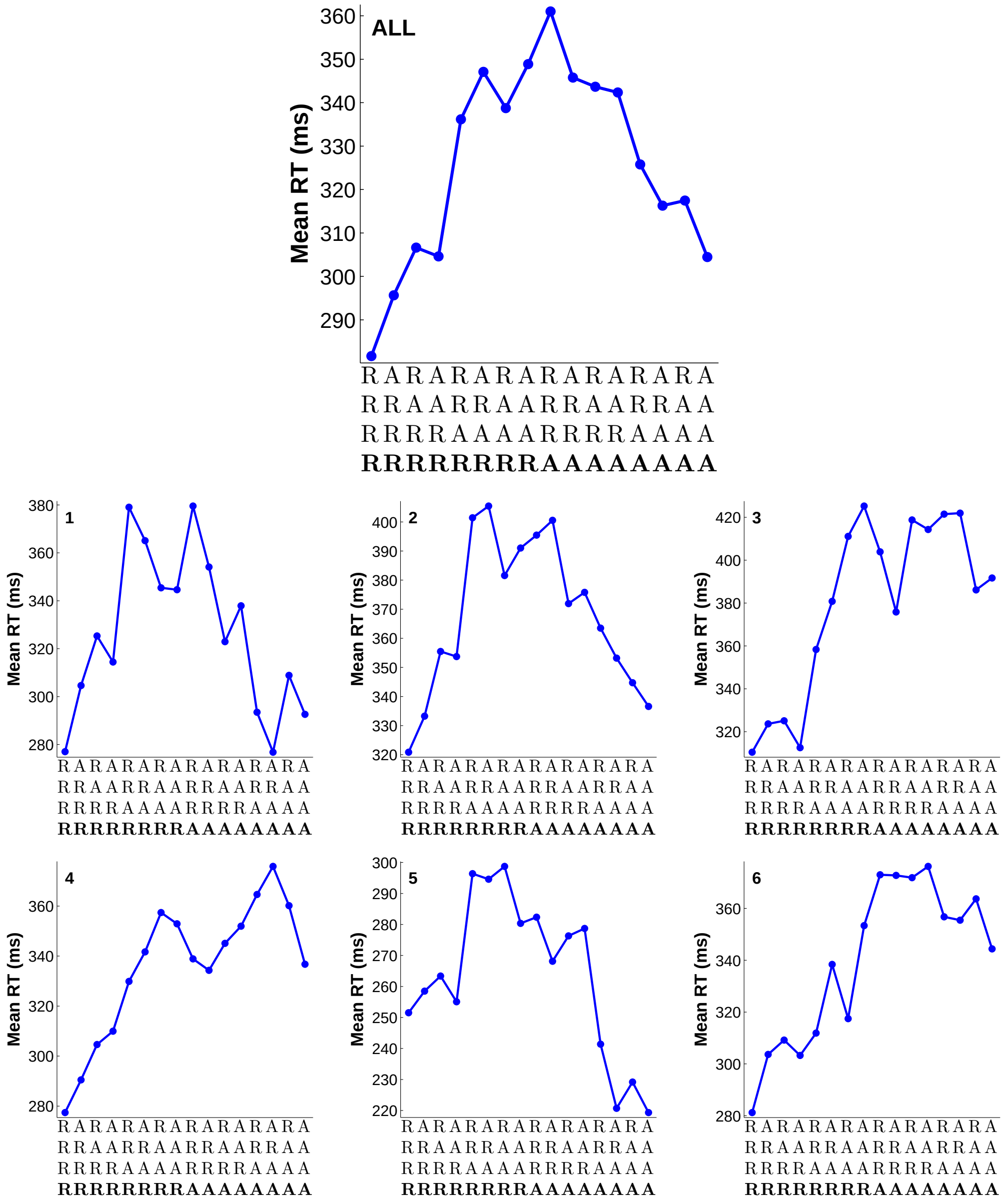

FIGURE 3.3: Individual differences for the same experimental conditions. Top panel (ALL) shows the average RT pattern of all six subjects which performed Experiment 1 with a 500 ms RSI. The bottom panels (1-6) show the results of the same six individuals separately. Note that while the average pattern shows a clear cost-benefit pattern with only a slight repetition bias, the individual subjects that make it up display marked deviations from the average pattern.

3.1.3 Individual Differences

Several mentions of individual differences in sequential effects exist in the literature, but these are usually limited to a passing observation that some individuals display a preference for repetitions and others to alternations (Arons & Irwin, 1932; Bertelson, 1965; Kirby, 1976), with no single dedicated study of individual differences in sequential effects conducted before. This work draws heavily on individual differences but the overarching aim is not to study these *per se* but rather to use individual differences as a tool to elucidate the more general structure of sequential effects.

Individual differences in sequential effects are usually hidden from view due to the common practice of averaging results across a group of participants. Figure 3.3 shows a sample of the data collected for this work illustrating individual variation for the same experimental conditions (Experiment 1, 500 ms RSI). Note how the average pattern or results across all subjects (Figure 3.3, top panel) displays a typical cost-benefit pattern whereas most individual subjects reveal substantial deviations to such a pattern. This is actually the rule rather than the exception, with similar levels of variation in all experiments reported here. Notably, the way in which individual subjects differ from each other for a *fixed* RSI is similar to the way in which collective average results vary with RSI (see Figures 3.1 and 3.11). For instance, the pattern of results of subject 4 in Figure 3.3, which was obtained with an 800 ms RSI, is reminiscent of the benefit-only pattern of sequential effects usually observed when a short RSI is used (see Figure 3.1; 50 ms data). These similarities indicate that not only are individual differences unlikely to be due to noise but also that they may be related to the dependence of sequential effects on the RSI.

If the premise that individual differences are meaningful holds then exploring the patterns of covariance across multiple subjects may be of use in better understanding sequential effects, and this is the main purpose of this work. However, before this analysis is conducted, it is important

[5] Note that under normal circumstances reaction times depend on the second-to-last event too, but *conditional* on the last one. In this case it is as if the last event did not happen.

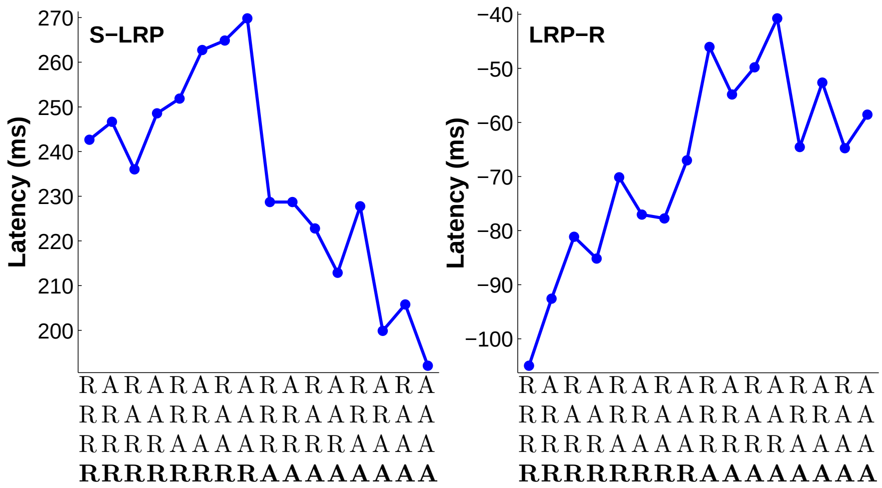

FIGURE 3.4: Reaction time decomposition of sequential effects performed by Jentzsch and Sommer (2002). The time between stimulus onset and the rise of the lateralized readiness potential (LRP) is shown on the left plot (S-LRP); the right plot shows the time between the rise of LRP and the moment a response is made (LRP-R). The LRP peaks just before a response is made and is more negative contra-laterally to the hand which will be use to respond. The time of LRP onset is defined as the moment it reaches a threshold amplitude. Adapted with permission from the author.

to review what is already know about the possible mechanisms underlying sequential effects. As discussed next, there is growing evidence for the existence of two separate processing stages responsible for sequential, and these are likely to play a role in the differences observed in sequential effects, both across individuals as well as when experimental parameters are varied.

3.1.4 Separate processing stages

Some debate has existed in the literature regarding the locus of sequential effects, and in particular whether such effects happen because of the sequence of stimuli, the associated responses, or both (Bertelson, 1965; Soetens, 1998). Overall evidence pointed to the fact that both stimuli and response related effects are involved in sequential effects, but once different signals are joined together it is hard to infer what their individual contributions were from just a few experimental results. A more direct approach was taken by some authors which attempted to observe directly at a neurophysiological level the relative contributions of stimulus and response processing towards sequential effects.

Jentzsch and Sommer (2002) conducted a study of sequential effects in the lateralised readiness potential (LRP) measured with EEG. The LRP is a negative going shift in electrical potential happening just before a response, and located in the pre-motor cortex area contra-lateral to the hand which will be used to respond. It is thought that the time after the occurrence of the LRP is exclusively due to motor processing (Leuthold, Sommer, & Ulrich, 1996). Therefore, by measuring the time between stimulus onset and the rise of LRP - S-LRP - and the time between LRP and the moment a response is made - LRP-R - the authors sought to measure pre-motor and motor processing time respectively.[6] Further, by measuring S-LRP and LRP-R as a function of the sequence of events in a traditional 2AFC, the relative contribution of motor and pre-motor processing stages towards sequential effects was estimated (see in Figure 3.4).

It is only natural to suppose that the pre-motor stage reflects the processing of stimuli and the motor stage the processing of responses. In fact, there is some empirical support for this association from experiments which attempt to selectively remove the influence of stimuli on the one hand, and of responses on the other hand, from sequential effects. In an experiment where the effect of responses is removed a pattern of reaction time results is obtained which is very similar to S-LRP, the pre-motor processing component (Maloney et al., 2005). Conversely, in an experiment where the effect of the stimuli is removed, a pattern similar to LRP-R is obtained (M. H. Wilder et al., 2013). That selectively removing the influence of stimuli and of responses produces reaction time results similar to LRP-R and S-LRP in isolation provides support for an association between the pre-motor stage and the processing of stimuli on the one hand, and the motor stage and the processing of responses on the other hand.

If there are separate processing stages involved in sequential effects it is likely that these play

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6 Note that S-LRP + LRP-R = RT for each trial. This is not necessarily true for the average S-LRP and LRP-R calculated from multiple subjects and shown in Figure 3.4, but in practice a sum of the average S-LRP and LRP-R is very close to the average RT pattern.

a role in the differences observed in sequential effects. The hypothesis is that changes in the relative contribution of stimulus and response processing are responsible for different patterns of sequential effects. However, it is not clear whether the pattern of pre-motor and motor processing, as measured by S-LRP and LRP-R, will always be the same either for different experimental conditions or across individuals for fixed experimental parameters. First of all, the experiment in which S-LRP and LRP-R were originally measured was conducted with a long RSI - 700 ms - with baseline problems preventing the same measurements when a short RSI was used. Secondly, there is no data regarding individual differences in S-LRP and LRP-R for different individuals. In short, these questions amount to asking whether it is only the *magnitude* of patterns such as S-LRP and LRP-R that changes or of if their *patterns* shown in Figure 3.4 change as well.

Here as attempt will be made to infer what discrete factors may be contributing towards sequential effects from the patterns of covariance present in a large dataset consisting of 158 participants performing different variations of a 2AFC. In light of the discussion above, the latent structure obtained will be related to separate processing stages involved in sequential effects. Finally, the potential of the latent structure encountered in explaining different patterns of sequential effects will be explored.

3.2 Method

3.2.1 Participants

158 participants performed several experiments which differed in the stimuli used and the response scheme. For each experimental design, four values of RSI were tested - 50, 250, 500 and 800 ms - except in the case of Experiment 2 with a 50 ms RSI due to a technical error. The majority of participants (149/158) were undergraduate students from the University of Adelaide and were

TABLE 3.1: Number of participants per experiment and RSI

Experiment	50ms	250ms	500ms	800ms	Total
1	4	5	6	7	22
2	0	6	6	5	17
3	10	5	5	10	30
4	5	5	5	5	20
5	5	4	6	8	23
6	8	5	5	5	23
7	8	5	5	5	23
Total	40	35	38	45	158

awarded course credit for performing the experiment. A few participants were recruited among University staff and graduate students as well as the surrounding community (9/158). All participants gave their informed consent to taking part in the experiments. Table 3.1 shows the number of participants used per each experiment and RSI. Three participants performed two different experiments. Two of the authors (DG and AP) are among the participants, having taken part in only one experiment each.

3.2.2 Experiments

Data from nine different variations of a 2AFC was used in this work. These variants were originally designed in order to test the impact of different experimental factors on sequential effects. Throughout, we will refer to the variants simply as ‘experiments’, despite the fact that they share a common procedure (described below). The experiments differ in two main respects: stimuli used and response scheme. Stimuli consisted either of two separate dots (aligned vertically or horizontally), a lower- or upper-case ‘o’, or two abstract geometric figures (such as a square, circle or triangle). Response schemes consisted of using the index and middle finger of the dominant hand

TABLE 3.2: Summary of experimental designs

Experiment	Stimuli	Response	Notes
1	$\{o, O\}$	Index and middle finger	
2	$\{o, O\}$	Both index fingers	
3	Two horizontal dots	Both index fingers	
4	Two horizontal dots	Both index fingers	Incompatible mapping
5	Two horizontal dots	Index and middle finger	
6	Two vertical dots	Both index fingers	
7	Two vertical dots	Both index fingers	Stimulus flashes for 50ms

or using both index fingers. Other minor aspects in which experiments differed were response-stimulus compatibility, i.e. whether response buttons were assigned to the stimuli on the same side or on the opposite side, and whether a stimulus was quickly flashed or if it remained on screen until the moment a response was made. Table 2 summarises the differences between the experiments; for each set-up, different group of subjects performed the experiment with a particular RSI value, which was held fixed throughout the experiment.

3.2.3 Procedure

Subjects sat approximately 60cm away from the computer screen, inside a darkened room. The stimuli were white, approximately 3 cm tall, and displayed against a gray background using Psychophysics Toolbox 3 and Matlab r2008a on a 17” Macintosh MacBook Pro. For responses, a Cedrus RT-530 response time box was used, which has one central round button surrounded by four rectangular buttons. The RT box was placed to the right of the screen if the subject was right-handed, to the left if left-handed, and in front of it if the subject was using both hands. In experiments where only one hand was used the subject was asked to use the dominant hand. Responses were made by pressing one of two buttons, each corresponding to a particular stimulus. Subjects were instructed to respond as quickly and accurately as possible by pressing the button assigned to the stimulus shown. If the stimuli differed in shape (Experiments 1, 2) rather than position on the screen (Experiments 3 through 7) the assignment of response button to stimulus was alternated with each new subject. Experiment 4 differed from the rest in that an incompatible mapping was used, i.e. the left side button was used to respond to the right side dot and vice-versa.

Stimuli remained on the screen until a response was made, except for Experiment 7 in which stimuli flashed for 50 ms and then disappeared. In either case, once a response was made the RT and the button pressed were recorded; after a fixed period of time termed the response-stimulus interval (RSI), the next stimulus appeared. The only feedback was a beep whenever a button was pressed. The accuracy and precision of RT measurements were both estimated to be on the order of one millisecond. Only trials where subjects responded correctly were included in the analysis.

All experiments consisted of 13 blocks of 120 trials each, with a short (approximately 1 min) break in between each block and a longer break (5 to 10 min) after the seventh block. Subjects were given one block of training before beginning except in Experiment 4 where the added difficulty required two such blocks. Data from training blocks was not used in the analysis. Sequences were generated randomly for each subject, with the constraint that the frequency of both stimuli be equal.

3.2.4 Data analysis

Variables

For each participant, the RT at each point in the sequence was recorded; trials where an error was made were discarded, as were the first four trials in each block. The RT values were then grouped according to preceding sequence of 5 stimuli, including the one being responded to, for a total of sixteen groups. The logarithm of all values in each bin was taken in order to de-skew the RT distributions, well known to be asymmetric (Ratcliff, 1993). Outliers beyond three standard deviations of the mean were removed. The mean of each group was then transformed back to linear scale by taking its exponential. Each individual is therefore represented in the analysis by a 16-long array of mean RT values, each corresponding to one of sixteen possible five-long sequences of stimuli or, equivalently, four-long histories of repetitions and alternations.

Inferring latent structure

Principal component analysis (PCA) was used in order to identify latent structure. The choice of PCA over factor analysis is due in part to the unusual nature of the variables used in this work, which are the product of a decomposition of the same variable - reaction time - according to the history of stimuli. One effect of this decomposition is that the mean of all sixteen variables will tend to add to a constant value equal to the overall mean reaction time taken across all trials, with a resulting loss in one degree of freedom.[7] In short, the sixteen variables are not independent,[8] a necessary condition for factor analysis to be performed (Gorsuch, 1983). In any case, the total variance explained by the four first components which will be retained is 96.6%, under which conditions PCA and factor analysis are expected to yield the same results (Gorsuch, 1983).

The latent variables identified will be referred to simply as ‘components’. Each component is attributed a set of *coefficients*, one for each of the sixteen observed variables, which can be interpreted as correlation coefficients or as the variance of each observed variable explained by the corresponding latent component. The set of coefficients for a particular component will be referred to as its ‘coefficient pattern’, by analogy with factor patterns in factor analysis. In addition, each individual participant is attributed a set of *scores*, one for each component retained, which reflect the relative contribution of each component for each individual’s results. Finally, the latent

[7] In order to understand why this is true, consider the shortest stimulus history possible, i.e. one-back: we are left with two possible sequences - A and R - which, save for random fluctuations in their count, will tend to add to a constant value equal to the mean for all trials. Extending this reasoning to longer histories of stimuli such as five-back we can see that the last variable will be to a large extent predictable from the previous fifteen.

[8] The determinant of the covariance matrix for all data is in fact close to zero (1e-48), indicating a matrix very close to singular and therefore inappropriate for factor analysis.

components will be denoted as C1, C2, C3 and so on, the numbering referring to the order in terms of variance explained. Before rotation, the first four components we chose to retain explain 78%, 12%, 4.9% and 1.25% of variance.

Component rotations

One of the main objectives here is to relate the latent structure encountered to separate stages of processing of sequential effects. Fortunately, data is available regarding the expected contributions of the two processing stages towards sequential effects, in the form of S-LRP and LRP-R (see above). Therefore, targeted procrustes rotations will be used in an attempt to find the best match possible between latent components and empirical data, instead of more traditional methods such as varimax which result in a relatively arbitrary solution (Gorsuch, 1983).

Oblique rotations will be used throughout except when estimating the variance explained by each component, in which case an orthogonal rotation will be used instead. The reason for this is that estimating the variance explained by correlated components is analytically intractable. In any case, the orthogonal solution is very similar to the oblique one and so the variance estimates are expected to be close to the truth.

Four components were retained and so four targets will be necessary for the procrustes rotation, one for each component - C1 to C4. The targets were were, in order: a constant vector; S-LRP; LRP-R; and the reaction time results of the second experiment performed by Jentzsch and Sommer (2002) shown in Figure 3.2. All targets were z-scored in order to scale and centre them. The rationale for the choice of target patterns is detailed below.

Recalculating component scores

When discussing individual differences it is be important to know the relative contribution of each latent component in determining the results of each participant, something which is usually achieved by analysing component scores. These scores are usually calculated relative to the grand mean, i.e. the vector of means for all variables across all subjects, which in the present case looks like a typical cost-benefit pattern. However, and again due to the nature of the variables used here as a decomposition of reaction time according to the history of stimuli, it would be more informative to analyse the contribution of each component relative to a baseline equal to each participant's overall RT for all trials. Subject scores obtained from PCA will therefore be recalculated in order to reflect deviations from a constant vector (see supplementary information for details). The recalculated scores are more readily interpretable as the coefficients of a linear combination of the form $s_1C1 + s_2C2 + s_3C3 + s_4C4$ where s_1-s_4 are scalars and $C1$-$C4$ vectors equal to the coefficient patterns of each of the first four latent variables.

Choice of number of components to retain

The choice of number of components to retain was made largely based on the interpretability of the coefficient patterns extracted with PCA before rotation, both with respect to the presence of a clear dependence on the sequence as well as their relationship with previous empirical results. Specifically, in the unrotated solution (see supplementary information), C1 shows a constant coefficient pattern, clearly indicating that this component is a consequence of overall differences in reaction time across subjects. The next two (unrotated) components - C2 and C3 - show a marked dependence on that last (C2) and second-to-last (C3) events, and are therefore unmistakably part of sequential effects. Finally, the fourth component - C4 - displays a coefficient pattern very similar to experimental results falling outside the usual spectrum of results obtained with a long RSI and shown in Figure 3.2. Post-hoc analysis of individual results (below) reveals C4 to be essential in explaining results obtained when the RSI is short, both in isolation and in combination with C2 and C3.

Traditional methods for choosing the number of components to retain lead to the rejection of C3 and C4, both clearly related to sequential effects. The reason for this is the very large proportion of the variance explained by C1 (78%) which, given the fact that it reflects differences in individual overall reaction time, is not related to sequential effects. The effects of the large share of variance taken by C1 could in fact lead to a underestimation of the total number of relevant components of sequential effects, so the possibility cannot be excluded that there are additional relevant components beyond the four studied here.

Estimating the proportion of sequential effects explained by each component

The first component identified is responsible for a considerable proportion of the variance in the data - 78%. However, this component reflects differences in overall RT, and is therefore not part of sequential effects. In order to estimate the relative contribution of the remaining components - C2, C3 and C4 - towards sequential effects, excluding overall RT effects, a separate PCA was conducted on a dataset where the individual overall reaction time - mean RT for all trials - was subtracted from each individual's results. The variance thus estimated was 38.2%, 35.5% and 11.5% respectively for C2, C3 and C4 after an orthogonal rotation with the same targets as used for the main PCA.

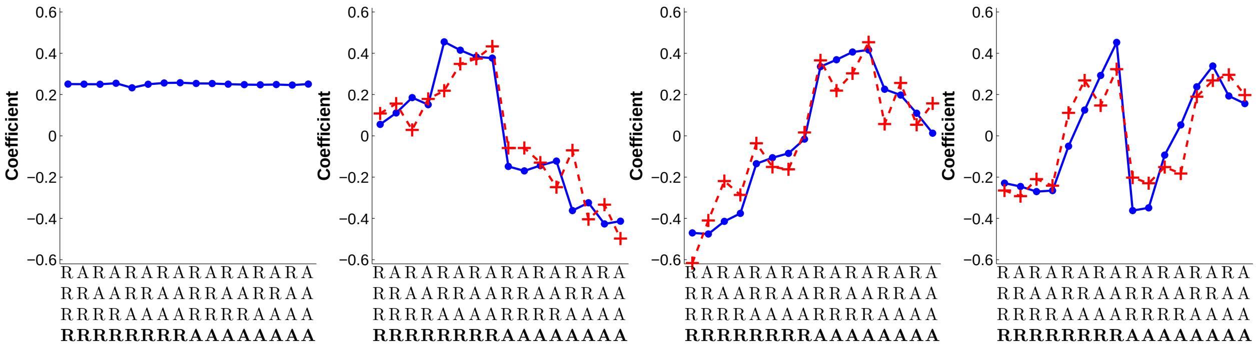

FIGURE 3.5: Latent structure of sequential effects. Solid blue lines - coefficient patterns of the first four components identified with PCA after an oblique targeted rotation (see main text for details). Dashed red lines - targets used for the rotation of the latent components shown linearly fit ($a + bx$) to the coefficient patterns for easy comparison. The target used for C1 was a constant vector and is not shown; this component is not considered to be a part of sequential effects. The components are ordered from left to right by amount of variance explained; this was estimated using an orthogonal rotation to be 78%, 8.1%, 8% and 1.9% respectively for C1 to C4.

3.3 Results

The presentation of results will be separated into two sections: firstly, the latent structure itself will be discussed by analysing the coefficient patterns of each component; secondly, it will be explained how changes in the relative contribution of C2, C3 and C4 explain both the dependence of sequential effects on the RSI, as well as individual differences, by analysing individual component scores.

3.3.1 Coefficient patterns

Figure 3.5 shows the coefficient patterns of the first four components identified with PCA after rotation, together with the respective targets. The first component - C1 - displays a constant coefficient pattern, indicating that it influences all variables in the same proportion. C1 therefore reflects individual differences in overall reaction time, and so it should not be considered part of sequential effects. Differences in overall RT are nevertheless considerable and so C1 is responsible for 78% of variance in the data.

The next two components retained - C2 and C3 - display coefficient patterns which are approximately left-right symmetric when plotted, implying that the two components have similar roles except that one is acting on alternations and the other on repetitions.[9] This symmetry is also observed to some extent in S-LRP and LRP-R, though arguably less clearly in this case. The variance explained by C2 and C3 was estimated by performing an orthogonal rotation as 8.1%, 8% respectively.

The last component retained - C4 - exhibits a pattern compatible with a pronounced dependence on the second-to-last event independently of the last one, and is similar in this respect to the results of Melis et al. (2002) and Jentzsch and Sommer (2002) shown in Figure 3.2. In fact, the choice to retain this component was largely based on the similarity between C4 and the said patterns of results, already visible before rotation. The variance explained by C4 was estimated with an orthogonal rotation to be 1.9%.

3.3.2 Discussion

The results presented here show that there are two main components related to sequential effects - C2 and C3 - and a third minor component - C4. The proportion of variance explained by these components is relatively small: 8.1%, 8% and 1.9% respectively. However, these values are to a large extent a consequence of the large proportion of the variance taken up by C1 - 78% - which means that the total variance explained by sequential effects is relatively small. Therefore, in order to more accurately estimate the proportion of sequential effects explained by C2, C3 and C4 a separate PCA was conducted in a transformed dataset with overall mean reaction time subtracted from each participant (see Method section). This resulted in variance estimates of 38.2%, 35.5% and 11.5% respectively for C2, C3 and C4. These numbers are a better representation of the

[9] Note that switching the A and R labels in the five-long sequences as usually ordered results in a mirroring of the plot across an imaginary vertical axis separating the sequences in the left and right halves.

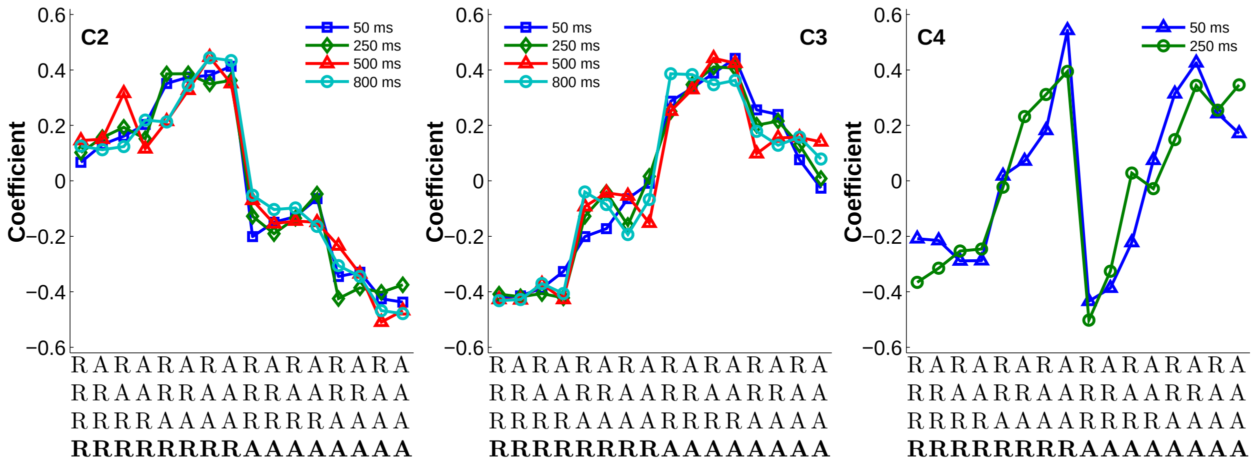

FIGURE 3.6: Invariance of the latent structure of sequential effects with RSI. Each panel shows the coefficient patterns of the equivalent component (from left to right: C2, C3 and C4) identified in four separate RSI subgroups - 50, 250, 500 and 800 ms - each including all subject which performed a task with a particular RSI irrespective of other experimental differences. Components significantly similar to C2 and C3 were identified in all all RSI subgroups, whereas a C4 anlogue was only identified in the 50 and 250 ms subgroups. The analysis performed in each subgroup was equal in every respect to that performed on the global pool of participants, i.e. PCA followed by an oblique rotation with the same targets.

contribution of each component towards sequential effects.

An attempt was made to give the latent components a more meaningful psychological interpretation by performing targeted rotations. In the case of C2 and C3 the targets chosen for the rotation were S-LRP and LRP-R, two EEG measures meant to reflect the relative contributions of the separate processing of stimuli and responses. S-LRP and LRP-R were nevertheless measured in an experiment with a long RSI, whereas C2 and C3 were identified in a dataset where a range of different RSI values was used. One possibility is that C2 and C3 are only present when the RSI is long, in which case the results of this work would be an artefact of grouping different experimental conditions. More generally, given that different experiments were grouped together, it becomes important to analyse how the latent structure varies across different experiments and RSI values.

Invariance of latent structure with RSI

Assessing if and how the latent structure of sequential effects changes with RSI is of particular importance since, as classical theories would have it, sequential effects observed when the RSI is short - i.e. the benefit-only pattern - are fundamentally different from those observed with a long RSI - i.e. the cost-benefit pattern (Soetens et al., 1985). In order to assess how the latent structure varies subjects were separated according to RSI, irrespective of experiment. Separate PCAs, similar in every respect to the global analysis, i.e. using oblique rotations and the same targets, were then conducted on each subgroup.

Figure 3.6 shows the coefficient patterns of the relevant components identified in the four RSI subgroups - 50, 250, 500 and 800 ms - excluding C1 which displays a constant coefficient pattern in all cases. In order to obtain a quantitative estimate of how similar these components are to the ones obtained globally the coefficient of congruence (Gorsuch, 1983) was calculated between equivalent components obtained in each subgroup and the global ones shown in Figure 3.5. The significance of the coefficient of congruence values obtained was then estimated (see supplementary information for methodology). Components significantly similar to the global C2 and C3 ($\alpha = 0.001$;$p < 0.001$) were obtained in all RSI subgroups (see Figure 3.6, left and centre panel), whereas a component significantly similar to C4 was only found in the 50 and 250 ms subgroups.[10]

In light of classical theories of sequential effects it is perhaps surprising to find the same two main components in all RSI subgroups. Moreover, the variance explained by these two components together remains largely the same across RSI: 70.1%, 73.4%, 58.8% and 76.9% respectively for the 50, 250, 500 and 800 ms subgroups. This begs the question: what is the reason for the differences observed for a short and long RSI, i.e. the benefit-only and cost-only patterns? At first sight it might seem that this is due to the presence of an extra component - C4 - when the RSI is short.

[10] The 500 ms subgroup did produce a significant C4 for a $\alpha = 0.05$ significance level, but this component showed a coefficient pattern visibly distorted relative to the global C4.

As we will see when discussing individual scores, C4 does play a role in the differences observed when the RSI is varied, but these differences are also the consequence of changes in the relative contributions of C2 and C3.

In short, the PCA results indicate that the latent structure of sequential effects consists of two main components which are present irrespective of RSI - C2 and C3 - with a minor component - C4 - present only when the RSI is short. An analysis of the latent structure for different experiments - collapsing across RSI - was performed with similar conclusions (see supplementary information). The analysis of different experiments is nevertheless less relevant than that of different RSI values, the reason being that experimental differences in the type of stimuli and response scheme are know to have relatively small effects when compared to the RSI.

What is C4?

C4 exhibits a coefficient pattern similar to previous experimental results shown in Figure 3.2, displaying approximately equal left and right halves when plotted, an indication of a degree of independence of the last event,[11] with reaction times depending to a large extent on whether the second-to-last event was a repetition or an alternation. One possibility is that C4 is the product of an activation taking place at $t-1$ influencing reaction times at time t directly. In order to evaluate this hypothesis, we must consider what the influence of an activation taking place at $t-1$ would look like if the event at time t had never happened. This can be achieved in the following manner: sequences are grouped two-by-two, discarding the first event at $t-3$, leaving us with eight sequences corresponding to the last three events (or four stimuli). These new sequences must now be reordered as if the event at time t is now the one at time $t-1$. Note that this process will necessarily always result in a plot with equal left and right halves.

[11] Note that the left and right halves of the plot, also known as the repetition and alternation curves, differ only with respect to the last event and whether it was a repetition or an alternation.

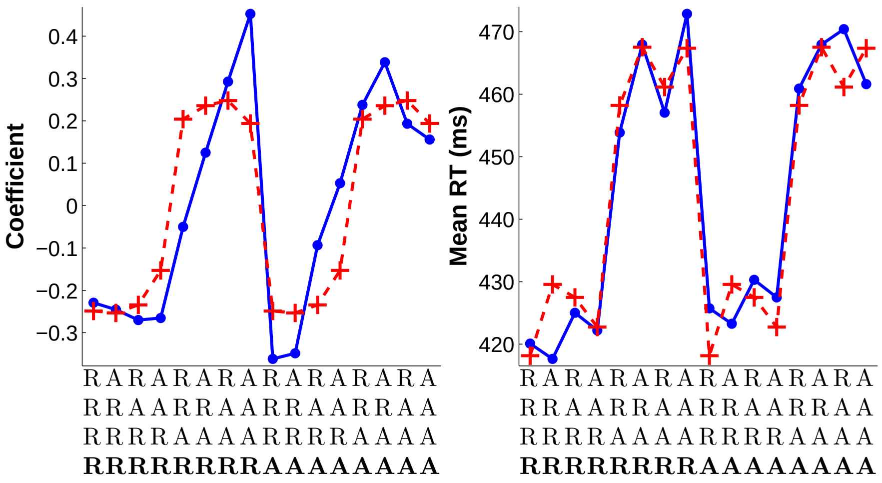

FIGURE 3.7: C4 as the consequence of delayed activation of C2 and C3. Solid blue lines - coefficient pattern of C4 (left panel) and reaction time data from the second experiment performed by Jentzsch and Sommer (2002). Dashed red lines - Left panel: best fitting linear combination of the form $k + w_1C2^* + w_2C3^*$, where C2* and C3* are reshuffled versions of C2 and C3 meant to illustrate the direct effect of activation at time $t-1$ on time t (see main text); best fitting parameter values were $k = 3.7 \times 10^{-4}$, $w_1 = -0.405$ and $w_2 = 0.448$. Right panel: best fitting linear combination $k + w_1SLRP^* + w_2LRPR*$ where SLRP* and LRPR* are reshuffled versions of S-LRP and LRP-R; best fitting parameter values were $k = 526.3$, $w1 = -0.551$ and $w2 = 0.646$. Note that in both cases the best fit is obtained with $w_2 \approx -w_1$.

The natural candidates for activation occurring at $t-1$ are C2 and C3 themselves. If we apply the procedure described above to C2 and C3 we obtain two new patterns - denoted here as C2* and C3* - neither of which looks very similar to C4 in isolation (not shown). However, if we take a linear combination of the form $w_1C2^* + w_2C3^*$, with $w_1 \approx -w_2$, this produces a pattern similar to C4 (Figure 3.7, left panel). As we will see below in the context of the analysis of component scores, this is exactly what is expected since, for a short RSI, individual scores on C2 and C3 are on average equal in magnitude and opposite in sign. We applied the same transformation to S-LRP and LRP-R - resulting in S-LRP* and LRP-R* - and fitted a combination of these to the results of the second experiment of Jentzsch and Sommer (2002) (see Figure 3.7, right panel). As with the latent components, the best fit of a linear combination $w_1SLRP^* + w_2LRPR^*$ was obtained with $w_1 \approx -w_2$.

If we assume for the time being that C4 is a combination of C2* and C3*, then why do these two influences show up in the PCA as a single component? One possibility is that C2 and C3

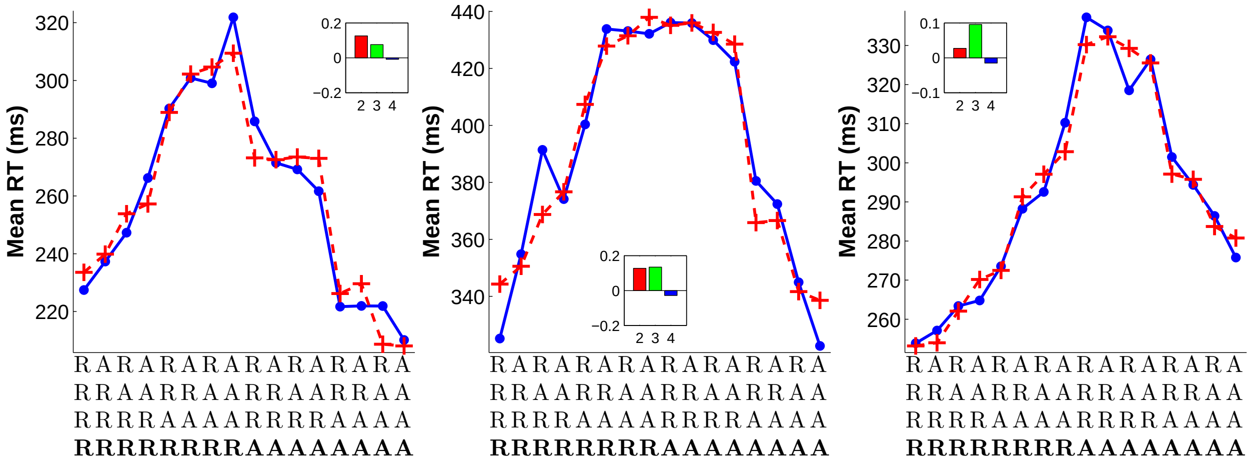

FIGURE 3.8: Effect of varying the relative balance of C2 and C3, both with positive scores and when C4 is absent. Solid blue lines - individual participants included in our experiments chosen for illustration purposes. Dashed red lines - best fitting linear combinations of the form $s_1C1+s_2C2+s_3C3+s_4C4$ where the s_i are linear coefficients which we refer to as 'scores' (see main text); the C_i are the coefficient patterns of first four components. The range of results shown is characteristic of individual differences observed in experiments conducted with a relatively long RSI. Inset plots show the scores on C2, C3 and C4.

integrate at time step $t-1$, showing up as a combined non-separable influence at time t. Finally, while the explanation for C4 is preliminary at this stage, if found to be true it would imply that sequential effects are in fact the product of two fundamental components rather than three. Next we turn to the analysis of how the relative contributions of the three components - C2, C3 and C4 - change both as a function of RSI as well as for different individuals.

3.3.3 Component scores

In this section the role of component scores in explaining individual differences, as well as the dependence of sequential effects on the RSI, will be discussed.

Individual Differences

Individual differences in sequential effects are now revealed more clearly to be a consequence of different contributions by C2, C3 and C4. Recall that scores were recalculated so as to make the model equivalent to a simple linear combination of the form $s_1C1 + s_2C2 + s_3C3 + s_4C4$, where the s_{1-4} are linear coefficients - referred to here simply as 'scores' - and $C1 - 4$ are the coefficient patterns of the four first components identified with PCA. Different individuals have different scores on the three components, and these determine the overall pattern of sequential effects.[12] In other words, any individual is now represented by a point in 'sequential effects space', the axes of which correspond to the scores on C2, C3 and C4. This three-dimensional space has eight octants corresponding to all possible combinations of sign of the three components. However, only half of the space is used, since C4 is always positive or close to zero. In order to clarify the effect of the three components we will analyse three cases separately: firstly we will look at the effects of varying C2 and C3 both with positive scores, and C4 held at zero; secondly, we will see the effect of allowing negative scores on C2 and C3 while still holding C4 at zero; finally, the effect of C4 will be illustrated.

A balanced score on both C2 and C3 produces results similar to the cost-benefit pattern of sequential effects (see Figure 3.8, central panel). A higher score on C2 relative to C3 induces a preference for alternations (see Figure 3.8, left panel), whereas a higher score on C3 induces a repetition bias (see Figure 3.8, right panel). Results from experiments conducted with a long RSI tend to fall along this range of scenarios (Soetens et al., 1985; Cho et al., 2002; A. D. Jones et al., 2002; Gokaydin et al., 2011), which can be described as a cost-benefit pattern with either a repetition or alternation bias.

Allowing the sign of the score on either C2 or C3 to go negative - while still holding C4 at zero - produces results no longer recognisable as a cost-benefit pattern. In particular, when the

[12] See supplementary information for a plot of all individual scores.

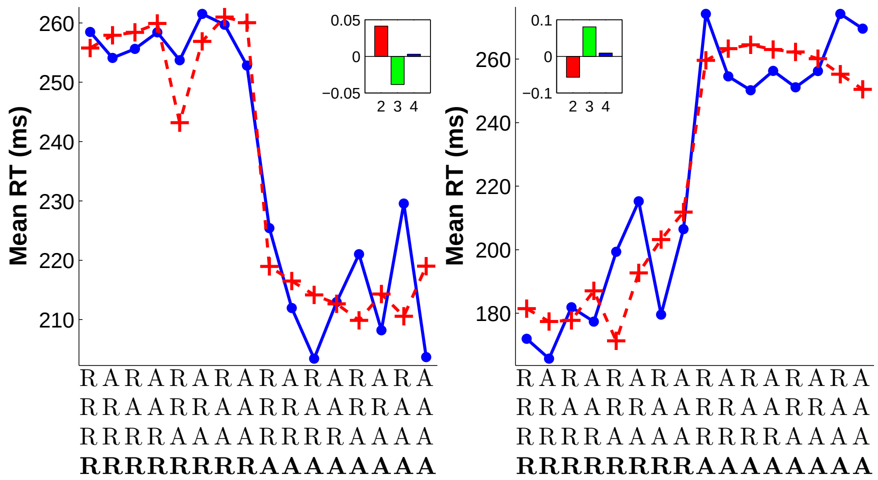

FIGURE 3.9: Effect of allowing negative scores on C2 and C3, with C4 absent. Solid blue lines - individual participants included in our experiments chosen for illustration purposes. Dashed red lines - best fitting linear combinations of the form $s_1C1 + s_2C2 + s_3C3 + s_4C4$ where the s_i are are linear coefficients which we refer to as 'scores' (see main text) and the C_i are the coefficient patterns of the four first components. Note how scores on C2 and C3 similar in magnitude and opposite in sign tend to produce patterns which may be mistaken for a two-tiered dependence on the last event and whether it was a repetition or an alternation; whether alternations or repetitions are faster depends on which component has a negative (or positive) score. Inset plots show scores on C2, C3 and C4.

scores on C2 and C3 are approximately equal in magnitude but opposite in sign the resulting pattern resembles a two-tiered dependence on the last event (see Figure 3.9), with faster reaction time to repetitions or alternations depending on which component - C2 or C3 - has a higher score. It is interesting to note that, if viewed in isolation, the patterns shown in Figure 3.9 would likely be interpreted as a trivial dependence on the last event, when in fact they are the product of a combination of two complex looking patterns. Only one individual participant was found to have a strong negative score on both C2 and C3, which may point to constraints on the allowed combinations of the two components. However, the good qualitative fit to the said subject indicates that it may be possible - yet rare - for both C2 and C3 to be negative. Finally it is worth mentioning that the contribution of a component with a negative score, when looked at in terms of its coefficient pattern, is an 'upside-down' version of what can be seen in Figure 3.5; this raises some conceptual issues which are discussed in detail below.

The influence of C4 is only felt when the RSI is short, under which conditions it sometimes

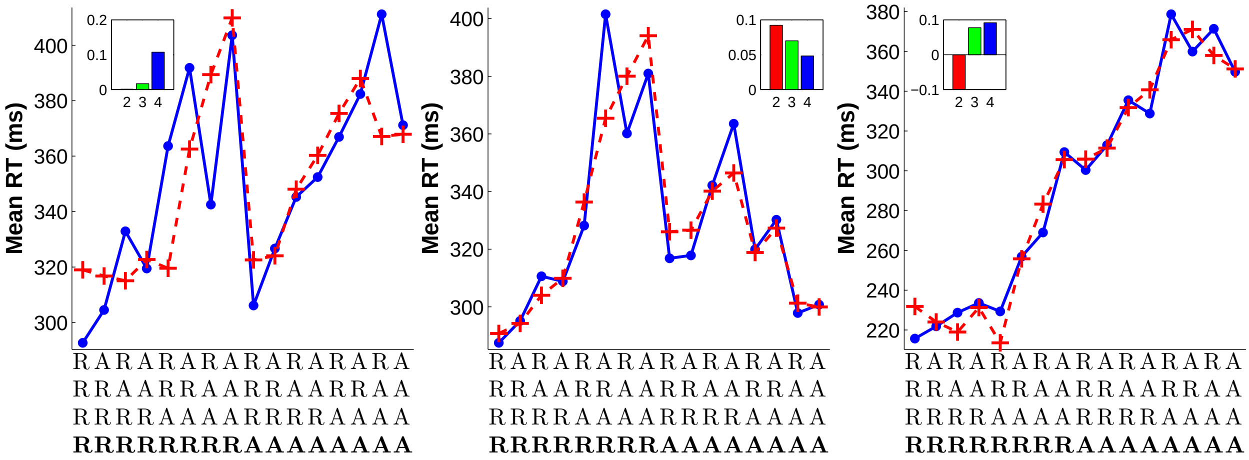

FIGURE 3.10: The effect of C4. Solid blue lines - individual participants included in our experiments chosen for illustration purposes. Dashed red lines - best fitting linear combinations of the form $s_1C1 + s_2C2 + s_3C3 + s_4C4$ where the s_i are are linear coefficients which we refer to as ‘scores’ for short (see main text); the C_i are the coefficient patterns of the first four latent components. The left panel shows an individual displaying results similar to C4 in isolation; the middle panel shows a pattern of sequential effects not described before but which is displayed by several individuals; the right panel shows an individual exhibiting a typical benefit-only type of result. Inset plots show scores on C2, C3 and C4.

manifests itself in relative isolation, i.e. with little contribution from either C2 or C3 (Figure 3.10, left panel). However, C4 is also necessary, in combination with C2 and C3, to explain other patterns of results, some of which were not described before in the literature (Figure 3.10, middle panel). Perhaps more importantly, C4 is also necessary in order to explain the the benefit-only pattern of sequential effects, often observed at an individual level when the RSI short (Figure 3.10, right panel). Note that all three individuals shown in Figure 3.10 come from experiments conducted with a 50 ms RSI. Finally, no single individual exhibited a strong negative score on C4 so it is not known whether this component can change sign in the same way as C2 and C3 can.

Dependence on RSI

Just like individuals can display differences in sequential effects as a consequence of different scores on C2, C3 and C4, the mean RT pattern of a group of individuals depends on the mean score on each of the three components. This is illustrated in Figure 3.11 which shows the mean

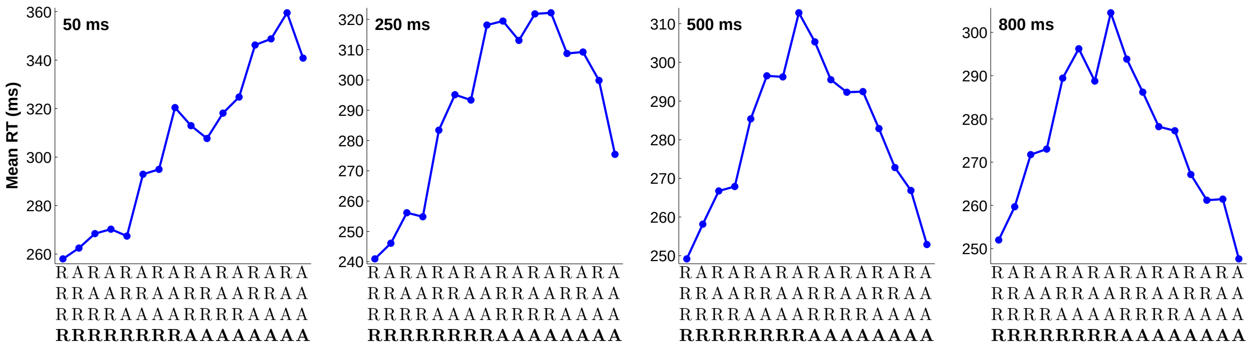

FIGURE 3.11: Reaction times averaged across all subjects performing a task with a particular RSI - 50, 250, 500 and 800 ms - irrespective of experimental design. Note how reaction time results change from a cost-benefit pattern when the RSI is long - 500 and 800 ms subgroups - to a benefit-only pattern when the RSI is very short - 50 ms. With a 250 ms RSI an intermediate pattern of results is observed. These different results can now be understood in terms of the mean scores on C2, C3 and C4 shown in Figure 3.12.

RT patterns of the four main RSI subgroups - 50, 250, 500 and 800 ms - irrespective of experiment performed, with corresponding mean scores as a function of RSI shown in Figure 3.12. Note the similarities between the evolution of the RT pattern as the RSI is varied and the results obtained previously by Soetens et al. (1985) (Figure 3.1): for a short RSI - 50 ms - a benefit-only pattern of results is observed; when the RSI is long - 500 and 800 ms - a cost-benefit pattern is observed instead; an intermediate pattern of results is observed with a 250 ms RSI.

Figure 3.12 shows mean scores on all components for four RSI subgroups - 50, 250, 500 and 800 ms - which are effectively the mean scores which produced the patterns shown in Figure 3.6. Two main trends are observed: firstly, the mean score on C2 decreases as the RSI is shortened, switching from a positive value for long RSI to a negative value for a short RSI; secondly, the mean score on C4 increases as the RSI is decreased. The mean score on C3 is always positive, increasing a little when the RSI is 250 ms only to decrease again at 50 ms. Note that in all cases the distribution of scores is fairly wide, except for C4 when the RSI is 500 or 800 ms, reflecting the fact that this component is absent when the RSI is long.

Putting things into context with the classical view, the cost-benefit pattern observed when the RSI is long is the product of a balanced mean score on C2 and C3 with no contribution from C4

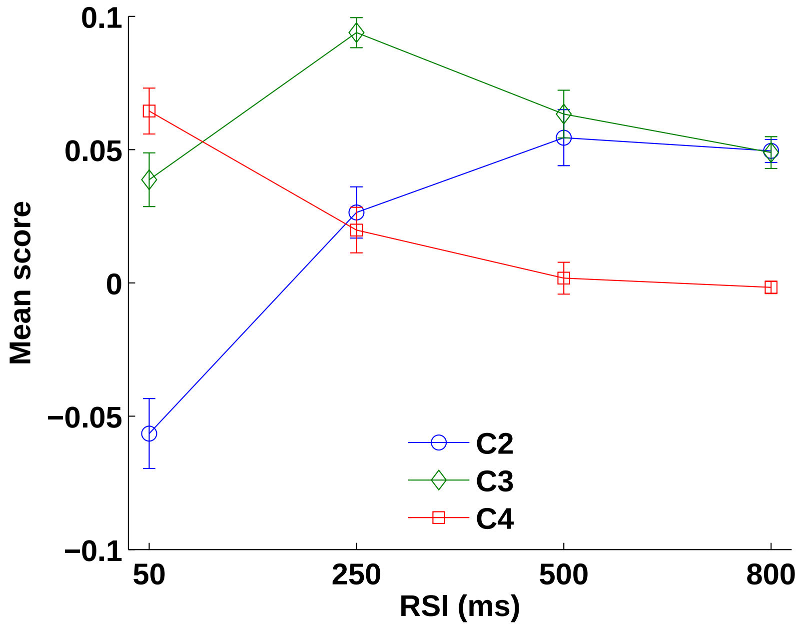

FIGURE 3.12: Mean scores on C2, C3 and C4 as a function of RSI. The means are taken across all participants performing a task with a given RSI irrespective of experimental design. Error bars show the standard error of the mean. Recall that the component 'scores' are not the original scores obtained directly from PCA but linear coefficients estimated in order to reflect deviations to a zero baseline (see main text and supplementary information). Also, the coefficient patterns used in estimating component scores were those of the global analysis shown in Figure 3.5 and not the ones obtained from PCA conducted on separate RSI subgroups shown in Figure 3.6.

(Figure 3.6, rightmost two panels). The benefit-only pattern can now be seen to be the consequence of a shift in sign on C2, together with the emergence of C4, when the RSI is short. As for the unusual results discussed above and shown in Figure 3.2, these can now be seen to be the consequence of C4 occurring in relative isolation. The fact that C4 is already manifest in the benefit-only type of result shows that what were seemingly disparate results are in fact closely related. More generally, all the different kinds of sequential effects are now seen fall along the same continuum.

3.3.4 Discussion

This work firmly establishes individual differences as structured and not just the product of noise. Moreover, the similarities between individual differences and the way in which average results depend on the RSI are now explained by the fact that both phenomena are the product of variation in the scores on C2, C3 and C4. It is only natural to suppose that the results of a single individual will also depend on the RSI, although this has not yet been observed empirically. Conversely, the RSI is not the sole determinant of the component scores as there are significant individual differences for a fixed RSI value. Together this evidence points to a common mechanism underlying all sequential effects which is not only sensitive to the RSI but also exhibits considerable variation across individuals.

That all types of sequential effects lie on the same continuous space means that the differences observed are quantitative rather than qualitative in nature. This raises questions regarding the classical view of sequential effects as the product of two fundamentally different mechanisms, one operating at a long RSI - subjective expectancy - and the other for a short RSI - automatic facilitation (Soetens et al., 1985). The mapping, if any, between the two mechanisms and the components identified here is unclear. The results presented here provide only partial support for a qualitative transition, inasmuch as one of the sequential effects components - C4 - is only present at short RSI. However, the greatest portion of the variance is explained by the same two components irrespective of RSI: together, C2 and C3 explain 70 and 73% of the variance due to sequential effects in the 50 and 250 ms groups respectively. Finally, if C4 is confirmed to be the product of residual C2 and C3 activation, this would mean that no qualitative difference exists between short and long RSI sequential effects.

The analysis of component scores has consequences for the hypothesis that C4 is the product of residual activation of C2 and C3 at time $t-1$ influencing reaction times at time t. It was found that C4 was best explained by a combination of the activation of C2* and C3* - the expected patterns of C2 and C3 as if the last event did not happen - with weights similar in magnitude but a negative sign on C2*. These values match the mean scores on C2 and C3 when the RSI is 50 ms - the same conditions which tend to produce C4 - which are also approximately equal in magnitude with a negative score on C2 (see Figure 3.12). However, a C4 component is also found in the

250 ms subgroup, but in this case the mean scores on C2 and C3 are different, the mean score on C2 being positive. The explanation for this apparent incoherence may be that it is only in those subjects within the 250 ms subgroup which have a negative score on C2 that C4 is observed. In fact, significant negative correlations between scores on C2 and C4 were found for both the 50 and 250 ms subjects ($r = -0.64, p < 1e-3$; $r = -0.44, p = 0.008$). This interpretation is compatible with the view of C4 as the consequence of processing constraints discussed in detail below.

3.4 General Discussion

3.4.1 The nature of sequential effects - C2 and C3

The PCA results indicate the presence of two main components responsible for sequential effects. On the other hand, previous empirical evidence points to the existence of two separate processing stages involved in sequential effects, one pre-motor in origin and related to stimuli, and the other motoric and related to responses. It stands to reason that latent components and processing stages might be related, and in this spirit an attempt was made to map the coefficient patterns of the two latent variables - C2 and C3 - to the best evidence available about the relative contributions of the two processing stages - S-LRP and LRP-R (see Figure 3.5). The similarities encountered are consistent with the proposed relationship but fall short of providing conclusive evidence that the latent variables C2 and C3 do in fact reflect the separate processing of stimuli and responses. With this in mind, the implications of such a relationship, if it did indeed hold true, will be discussed.

Recall that S-LRP and LRP-R are time measurements. One possibility then is that C2 and C3 simply reflect the time that it takes to process stimuli and responses in a serial fashion. However, this view clashes with the fact that the C2 and C3 can have a negative score, implying a negative processing time. A more nuanced view would be to consider that C2 and C3 reflect different signals related the separate processing of stimuli and responses. Note that while S-LRP and LRP-R have time units, they were measured with respect to a point defined by a threshold amplitude of the LRP. What this means is that S-LRP and LRP-R might reflect the *amplitude* of different signals rather than simply processing times. If we take this view it becomes easier to accept that C2 and C3 might become negative, as this may possibly reflect a negative contribution of corresponding neurological signals towards reaction times.

One recent suggestion in the literature is that S-LRP and LRP-R reflect the tracking of different statistics about the environment (M. Jones et al., 2013) and, inasmuch a relationship between S-LRP/LRP-R and C2/C3 holds, this would imply the latent components also reflect different statistics. However, we again stumble on the fact that C2 and C3 can have a negative sign, implying that under some circumstances subjects would be tracking some form of inverse statistics, something which makes little sense from a computational point of view. When the RSI is long, both C2 and C3 are almost always positive, in which case a computational interpretation of the latent components may be possible. In fact, it has been argued that the cost-benefit pattern of sequential effects approximates the computations of an ideal observer (Yu & Cohen, 2008). However, it seems that the full range of different sequential effects, and in particular those observed with a short RSI, might only be explainable at a process level. The possible role of processing constraints in the results observed when the RSI is short, and in particular the emergence of C4, is discussed below.

A final possibility is that C2 and C3 play the role of separate detectors of repeating and alternating patterns in the sequence. Several authors have argued in the past for the need to postulate independent mechanisms in charge of detecting repetitions and alternations (D. Hale, 1969; Maloney et al., 2005). This theory fits with the symmetry observed in the coefficient patterns of C2 and C3, which implies that C2 and C3 have similar roles but applied to alternations and repetitions respectively. Finally, since higher coefficient values imply slower reaction times, in this view C2 would play the role of an alternation detector and C3 that of a repetition detector.

Irrespective of computational interpretations, there are some clues as to the mechanisms underlying sequential effects. In particular, there is considerable evidence for the role of a geometric weighted average of the previous sequence, also known as exponential filter, in sequential effects (e.g. Laming, 1969; Yu & Cohen, 2008; M. Jones et al., 2013). Interestingly, an exponential filter of the sequence of stimuli produces a pattern of results with a remarkable degree of similarity with LRP-R (not shown), and it has in fact been argued that the two are related (M. Jones et al., 2013). By proposing a correspondence between LRP-R and C3, it is further implied that this component might reflect an exponential filter of the sequence.

The mechanism behind S-LRP is less well understood, partly because it displays a strong alternation bias - i.e. faster RT to alternations - a seemingly simple feature which has nevertheless been notoriously hard to reproduce (M. Wilder et al., 2009; M. Jones et al., 2013). Some authors suggest that S-LRP corresponds to a second type of exponential filter, one which is applied to the sequence of repetitions and alternations rather than individual stimuli (M. Jones et al., 2013). However, such a filter produces a pattern of results with no alternation or repetition bias (not shown), and in order to reproduce the alternation bias of S-LRP it has been necessary to postulate an additional mechanism (M. Jones et al., 2013). So while the role of an exponential filter in generating a component looking like LRP-R seems well established, the mechanism behind S-LRP is not as well understood, and the same conclusions apply to the latent components C2 and C3 insofar as they correspond to S-LRP and LRP-R respectively.

3.4.2 The role of processing speed - C4

Crucial to understanding the nature of C4 is a study which contrasted sequential effects in young (ages between 19 and 25 years) and elderly (ages between 60 and 75 years) subjects (Melis et al., 2002). When performing a 2AFC with a long RSI - 1000 ms - there was little difference between the two groups except that the elderly participants were slower overall. However, when performing the same task with a 50 ms RSI, results were markedly different: the young group produced a benefit-only pattern characteristic of experiments with a short RSI (see Figure 3.11, left panel); in contrast, the elderly group displayed a dependence on the second-to-last event (see Figure 3.2) which we now know is related to C4. Melis et al. (2002) suggest that the underlying variable responsible for the differences observed between young and elderly groups is speed of processing, of which age is only a close correlate.

Speed of processing as a factor in sequential effects is compatible with the view of C4 as residual activation of C2 and C3 from the previous time step discussed above: a processing delay could in principle lead to a greater overlap between the activation at time step $t-1$ and that at time t. This interference between adjacent events would only be felt with a short RSI, when pressure on processing capacity is maximal. Regrettably, information on the age of participants was not collected for this study, something which would have allowed for a possible correlation between C4 score and age to be investigated. Nevertheless, the vast majority of the subjects in this study were first year psychology students with a modal age of 18 years. It is therefore highly unlikely that processing speed limitations due to age played a role in the experiments reported here.

Finally, age might not be the only factor influencing processing speed. A second experiment which produced a pattern similar to C4 was conducted in subjects with a mean age of 27.4 years (Jentzsch & Sommer, 2002). It is unlikely that processing speed would have been limiting at such a relatively young age. However, the experiment in question was an unusually long version of a 2AFC both in terms of total number of trials (3960) and number of trials per block (330), more than twice the length of most 2AFCs (Soetens et al., 1985; Cho et al., 2002; A. D. Jones et al., 2002) as well as of our own experiments. Fatigue could therefore also play a role, possibly via a saturation effect and resulting decrease in processing speed. More work is necessary in order to establish the role of processing speed and/or fatigue in sequential effects. Just as the trajectory of the mean scores on C2, C3 and C4 as the RSI is varied was investigated here, it is important to do the same when varying age and the length of the experiment.

3.4.3 Conclusion

This work uncovers a structure which may in the future provide a unified framework for understanding sequential effects. A latent structure was identified consisting of two main components and a minor one. A mapping was proposed between the two main components of sequential effects and the separate processing of stimuli and of responses. Further, the possibility was discussed that the minor component is a consequence of processing constraints when the response-stimulus interval is very short. Irrespective of the interpretation of the latent components, this work provides a unified descriptive model of a wide range of types of sequential effects, allowing for a clearer contextualization of past and future experimental results. Finally, the results presented here may carry more general implications for the mechanisms underlying human pattern detection, both at a psychological as well as neurophysiological level.

4

An oscillator-based modelling framework for sequential effects

In this section an entirely different approach to modelling sequential effects is proposed. To begin with the need for a different modelling framework is motivated by analysing some difficulties with classical models, followed by a list of aspects of sequential effects a complete model of sequential effects must explain, several of which cannot be captured by any previous model. Next the formal details of the framework, which is based on the physics of oscillatory harmonic motion, are introduced. Finally, the way in which the framework could be of use in tackling different aspects of sequential effects will be discussed. In general a complete model of sequential effects will not be revealed here, but the overall framework will be argued to have a great deal of potential in modelling sequential effects. Some early successes of the framework include being able to replicate key aspects of the latent structure of sequential effects discussed in Chapter 3, a feat which is largely due to the unprecedented capacity of the model to meaningfully parameterise individual differences.

4.1 The difficulties with modelling sequential effects

At the heart of sequential effects modelling lies one crucial assumption: that a particular event increases the expectation of seeing another event of the same type. An 'event' can be defined as a particular stimulus or, as is often the case, a repetition or alternation of stimuli. Usually this 'priming' is assumed to decay exponentially with time. From the point of view of the current event, this means that the influence past events have on predictions about the future decays exponentially into the past. This is sometimes referred to in memory research as the exponential law of forgetting (Wixted & Ebbesen, 1991). One of the consequences of this exponential decay is that only a limited set of recent events will influence behaviour. In the specific case of sequential effects in a 2AFC only the past five stimuli [1] seem to have a significant impact on reaction time (Remington, 1969).

4.1.1 A tale of two filters

The simplest way to implement an exponentially decaying memory is to use an exponentially weighted moving average, otherwise known as exponential filter (Abraham, 2005). In discrete

[1] As mentioned in Chapter 2, the exponential decay of human memory should be taken with a grain of salt, since it only applies when the sequence of events is random.

time an exponential filter can be written recursively as

$$p(t+1) = (1-\alpha)x_t + \alpha p(t) \tag{4.1}$$

where x_t is the event at time t coded as 0 or 1; $p(t)$ is the probability that the event x_t is a 1; and α is a parameter varying between 0 and 1 which determines how quickly the past is forgotten. Equation (4.1) can also be written non-recursively by explicitly assigning weights to past events according to an exponential function and taking their weighted mean (see Chapter 2). The exponential filter is inbuilt more or less explicitly in all sequential effects models proposed in the literature so far (Falmagne, 1965; Laming, 1969; Cho et al., 2002; Yu & Cohen, 2008; M. Wilder et al., 2009; Gokaydin et al., 2011; M. Jones et al., 2013), and so it is useful to consider it in some detail.

Equation 4.1 gives us our first model of sequential effects. When fitting model predictions to reaction time (RT) data, it is customary to assume that the higher the probability attributed to the next event the shorter the reaction time should be, or put simply that $RT \propto 1 - p(t)$. Figure 4.1 (left panel) shows predictions made by an the exponential filter as a function of the last five stimuli plotted in the manner customary in the sequential effects literature (Vervaeck & Boer, 1980), where sequences are shown in terms of repetitions and alternations of stimuli. One might be forgiven for thinking that such a simple model stands no chance of approximating human behaviour, and yet it does a very good job of describing the results of some experiments where reaction times were averaged across multiple subjects (A. D. Jones et al., 2002; M. H. Wilder et al., 2013), as well as some individual subjects included in groups which do not display the same pattern on average (see Figure 4.2).

An exponential filter can be thought of as being sensitive to repetitions of stimuli only, since its predictions are a function of how many stimuli of the same type are present in the preceding sequence, as well as how recently they occurred. Writing the sequence in terms of repetitions

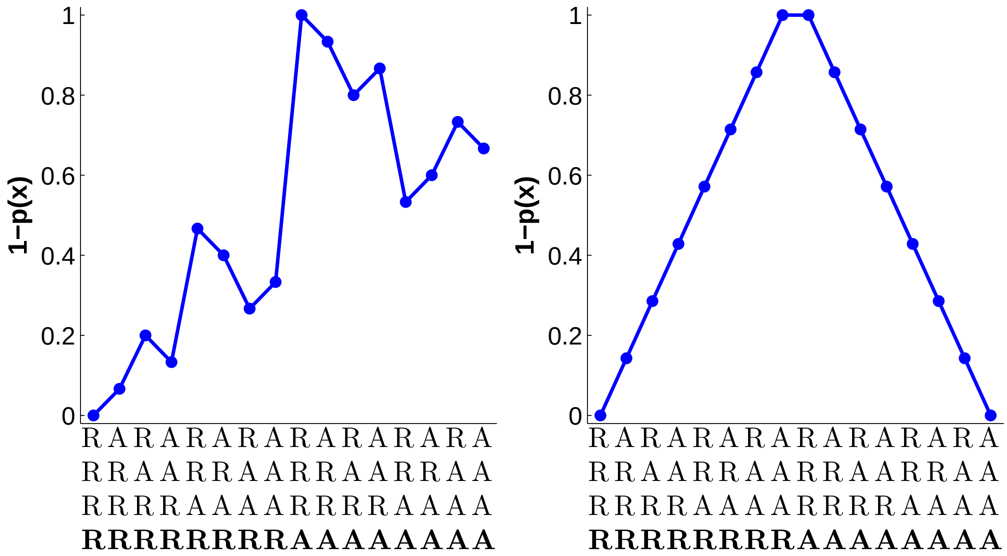

FIGURE 4.1: Two types of exponential filter. Left panel - simple exponential filter applied to a sequence encoding the stimuli themselves. Right panel - exponential filter applied to a sequence encoding repetitions and alternations of stimuli, i.e. an A/R filter. Note that in both cases the sequence effectively consists of 0' and 1's, except that these have different meanings depending on the filter. For instance, in the simple exponential filter the sequences 01001/10110 represent XYXXY - where X/Y stand for two different stimuli - whereas in the A/R filter case the same sequence would be written as 0010 if an 1 was chosen to represent a repetition and 0 an alternation. $\alpha = 0.5$ in both cases.

and alternations highlights this fact: note how predictions depend on the number of repetitions before the last event and how recently they occurred (see Figure 4.1, left panel). Conversely, an exponential filter is insensitive to alternations, a fact best illustrated by contrasting predictions for the sequences XYXYX (AAAA) and XYXYY (AAAR): although the sequence has been perfectly alternating so far, the exponential filter attributes a greater probability to a repetition occurring next.

Despite evidence that humans sometimes behave like an exponential filter, we are also clearly sensitive to alternating patterns (Laming, 1968; D. Hale, 1969) and this is reflected in the most common type of sequential effects, several examples of which are shown in Figure 4.3. This type of result - termed cost-benefit in the psychology literature - displays relatively fast reaction times to sequences such as RRRR and AAAA, and slow to sequences such as AAAR and RRRA, thereby revealing a sensitivity to both repeating and alternating patterns. Notwithstanding this

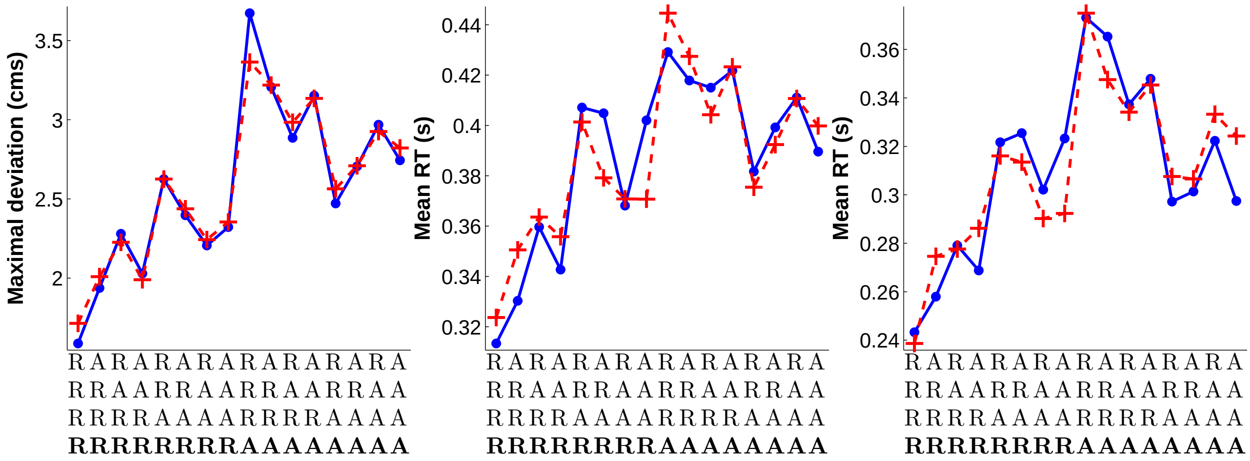

FIGURE 4.2: Humans sometimes behave like a simple exponential filter, both collectively and individually, and across multiple types of tasks. Solid blue lines show empirical data and dashed red lines show the best fit of a simple exponential filter with a linear transformation to adjust for scale. Best fitting exponential decay rate λ values were, from left to right: -0.65, -0.69 and -0.56. Left panel - data from a reaching task designed by M. H. Wilder et al. (2013). Middle panel - data from a modified 2AFC task by JA. D. Jones et al. (2002) where subjects had to respond to one of many stimuli with one finger and to any other stimulus with another finger. Right panel - single individual performing Experiment 1 in Chapter 3.

dual sensitivity, humans often display a degree of preference for one or the other type of pattern as measured by faster reaction times overall to either repetitions or alternations. If reaction times are faster overall to repetitions we speak of a repetition bias or simply a preference for repetitions, and similarly for the case of alternations.[2] Figure 4.3 (left panel) show an example of a pattern of results with an alternation bias, which nevertheless can still be considered to fall within the cost-benefit type of result.

In order to capture the sensitivity to alternations displayed by humans another type of exponential filter is often incorporated into models of sequential effects, one which acts on the sequence of repetitions and alternations rather than the individual stimuli.[3] The predictions of this filter - which we will refer to as A/R filter - display a perfectly balanced preference for repetitions and alternations (Figure 4.1, right panel), and in this respect they are reminiscent of the cost-benefit patterns shown in Figure 4.3. However, by virtue of the fact that the sequence itself now consists

[2] A preference for either repetitions or alternations can easily be visualised as differences between the two halves of traditional sequential plots, as the left half contains all sequences ending with R and the right half ending with A.

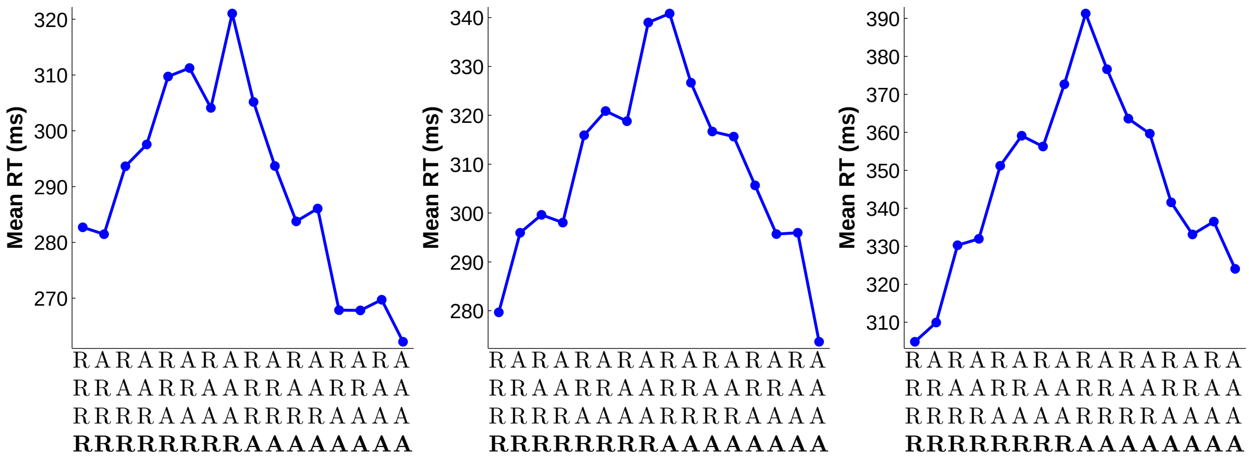

FIGURE 4.3: Typical patterns of sequential effects obtained with a long RSI (i.e. above 500 ms). Left panel - results from a 2AFC with a 1000 ms RSI included in Soetens et al. (1985); Middle panel - results from the first experiment reported by Jentzsch and Sommer (2002) which used a 700 ms RSI; Right panel - results from a 2AFC with an 800 ms RSI reported by Cho et al (2002). Note how all three experiments display a typical cost-benefit pattern with some variation in preference for - i.e. faster RT to - repetitions or alternations.

of repetitions and alternations, the A/R filter is insensitive to the relative frequencies of the stimuli themselves.

The exponential filter applied to the sequence of stimuli is sensitive to the frequencies of the stimuli but insensitive to alternations; conversely, an A/R filter is sensitive to both repetitions and alternations but blind to the frequencies of the stimuli. Since humans have been found to be sensitive to the base frequencies while at the same time detecting both repeating and alternating patterns (Laming, 1968; M. Wilder et al., 2009), the natural solution is to combine both types of exponential filter (shown in Figure 4.1). This solution has been adopted many times in the theoretical literature since Laming (1969) first suggested it, with practically all models proposed so far depending more or less explicitly on one or both types of filter (see Chapter 1). In fact, simply summing the two filters - shown in Figure 4.1 - provides an excellent approximation to reaction time data from some experiments such as that conducted by Cho et al. (2002) (shown in

[3] In a sequence coded in terms of repetitions and alternations the sequence elements themselves have a different meaning. For instance, at the level of stimuli the sequence ARAR is an average of the sequences 01100 and 10011; in a sequence coded in terms of repetitions and alternations ARAR is simply represented by 0101 if it was decided that 0 should stand for an alternation and 1 for a repetition.

Figure 4.3, right panel).

The combination of two filters has been very successful in replicating several aspects of sequential effects. However, there is no combination of the two filters that produces one recurrent observation in empirical data: an alternation bias. A repetition bias can can easily be replicated since it is a natural feature of the simple exponential filter at the the sequence level, but a preference for alternations has been notoriously difficult to capture. Over the years, a few solutions have been proposed to this problem, all of which have problems of its own. Yu and Cohen (2008) proposed a model which is effectively equivalent to an A/R with an added prior bias which can introduce a preference for repetitions or alternations. However, much like the A/R filter, the model by Yu and Cohen (2008) is unable to capture the individual stimulus frequencies.

M. Jones et al. (2013) proposed a model that again combines both types of exponential filter and which is able at the same time to reproduce an alternation bias. The key to the model is a 'cue competition' mechanism, effectively an interaction between the two types of exponential filter. However, and despite its success in capturing an alternation bias while still remaining sensitive to the frequency of the stimuli, the model suffers from a problem common to most sequential effects models proposed so far (Laming, 1969; Cho et al., 2002; M. Wilder et al., 2009; Gokaydin et al., 2011; M. Jones et al., 2013): they rely on hard-coding the detection of alternations. In other words, either alternations are made part of the raw data or otherwise different sub-models must be used in order to detect repetitions and alternations separately, with the two predictions subsequently combined. Ideally however one would have the detection of alternations arise in a natural way, rather than being explicitly coded.

To the modelling difficulties discussed so far one could add a host of empirical phenomena which are in need of explanation, many of which are hard to capture with any of the models proposed so far. For instance, the dependence of sequential effects on the RSI seems to call for a continuous time model when in fact almost all models of sequential effects are discrete time in nature (but see Cho et al., 2002; Gao et al., 2009). Additional aspects which influence sequential effects such as response-stimulus compatibility require a model capable of differentiating between stimuli and responses, which none does so far. The main objective here is to suggest a modelling framework which, while still incomplete, will not be limited in the sense that it carries the potential to explain all aspects of sequential effects.

There are two core ideas behind the new framework proposed here: firstly, that exponential filtering plays a central role in sequential effects; secondly, that sequential effects are the product of two separate components. We have reviewed exponential filtering above, and will now turn to the idea of two separate processes involved in sequential effects. This diversion will be brief as it has been discussed extensively in Chapter 3 and a more detailed discussion of this idea is presented in Chapter 5.

4.1.2 Two separate processing stages

A long-standing question in the literature has been whether sequential effects are caused by the sequence of stimuli or that of the corresponding responses (Bertelson, 1963; Soetens, 1998). A closely related but more general question is whether there are any separate mechanisms contributing towards sequential effects. Overall, the picture is starting to emerge that there are indeed two discrete contributions towards sequential effects, and further that these are associated with the processing of stimuli and responses respectively. Three lines of evidence support this: firstly, electrophysiological studies where pre-motor (i.e. stimulus) and motor (i.e. response) processing times were measured (Jentzsch & Sommer, 2002); secondly, the latent variable analysis reported in Chapter 3; thirdly, the results of experiments which attempt to selectively remove the effects of stimuli and responses separately (Maloney et al., 2005; M. H. Wilder et al., 2013). The results produced by all three approaches shows a remarkable degree of agreement (see Figure 4.4)

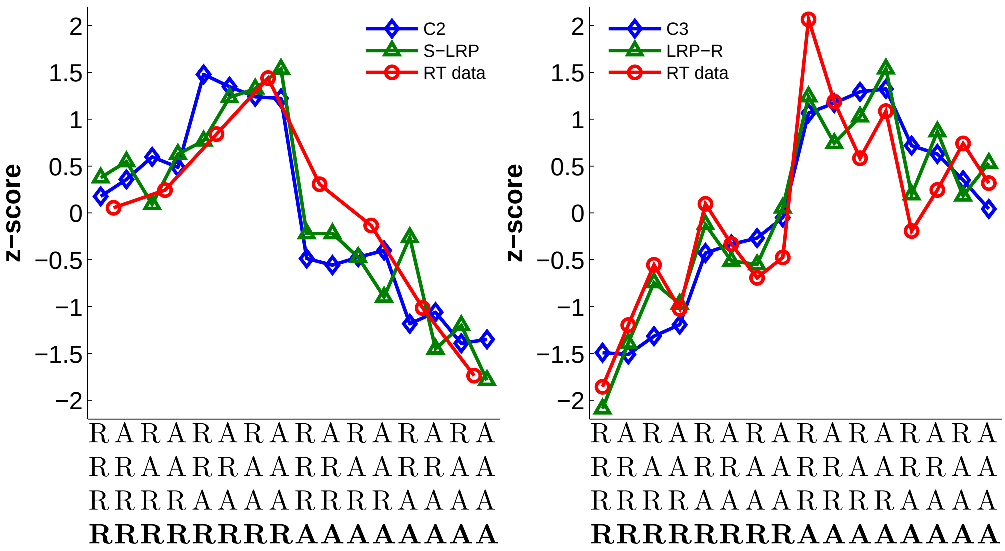

FIGURE 4.4: Evidence for two processing stages involved in sequential effects. Three sources of evidence are shown: pre-motor and motor processing times measured with EEG (S-LRP and LRP-R); the two main latent variables of sequential effects (C2 and C3); and the results of two experiments which isolate the contribution of stimuli and responses towards sequential effects. Left panel shows S-LRP, C2 and RT data from Maloney et al. (2005) reflecting the contribution of stimuli only; note that RT data in this case consists of only eight data points since the authors presented as a function of the last four stimuli. Right panel shows LRP-R, C3 and RT data from M. H. Wilder et al. (2013) reflecting the contribution of responses only.

and together point to the existence of two components with approximately symmetrical contributions towards sequential effects: the pre-motor or stimulus-associated stage seems to have an alternations bias, whereas the motor or response-associated stage has a repetition bias. The view of sequential effects as the product of two discrete and independent components is discussed in more detail in Chapters 3 and 5.

In light of our previous discussion on exponential filtering, it is tempting to speculate that the motor or response processing stage corresponds to an exponential filter of the sequence of stimuli. In fact, such a filter shows a remarkable degree of similarity with the results shown in the right panel of Figure 4.4. It is not so clear what the mechanism behind the pre-motor or stimulus processing stage might be. Some authors have recently proposed an association between an A/R filter and the stimulus associated component of sequential effects (M. Jones et al., 2013). However, the A/R filter (Figure 4.1, right panel) is unable to produce the strong alternation bias displayed by the pre-motor or stimulus processing stage (Figure 4.4, left panel). As discussed above, a separate mechanism was therefore proposed by M. Jones et al. (2013) to account for as alternation bias, the merits of which are discussed elsewhere in this work (Chapters 1 and 5).

Given the prevalence of the two exponential filter approach in the theoretical literature, it is perhaps unsurprising that a mapping has been sought between the two filters and the two processing stages, particularly since one of the filters - the one at the sequence level - shows an almost perfect correspondence with one of the processing stages - the motor or response-associated stage. Part of the reason why this association was proposed may have been the absence of a mechanism which produces an alternation bias similar to that displayed by stimulus processing stage. One of the main objectives here is to propose such a mechanism, but before this we will review several empirical aspects of sequential effects which a complete model must be able to capture. While the answers to all these questions will not be provided here, it is important to ensure that the overarching modelling framework is not limited in the sense of being able to represent all relevant experimental parameters.

4.1.3 Ingredients for a complete model

There is a great deal of variation in sequential effects both when experimental conditions vary as well as for different individuals performing the same experiment (see Chapter 3). For instance, when the RSI is long a pattern referred to as cost-benefit - shaped like an inverted ‘v’ - is usually obtained; conversely, when the RSI is short, a different pattern of results termed benefit-only is often observed. In some cases an entirely different pattern of sequential effects is observed (Melis et al., 2002; Jentzsch & Sommer, 2002), discussed in Chapter 3 as the possible product of processing delays. The first task for a complete model of sequential effects is therefore to explain this wealth of different results, which seems to call for such a model to be able to parameterise both the RSI as well as individual differences. Finally, by making assumptions with respect to the distribution of individual parameters, it should be possible to replicate a latent structure similar to that which is observed in empirical data.

In addition to the RSI there are a host of other experimental factors which influence sequential effects, such as whether stimuli are spatially overlapping or not (Bertelson, 1963), different response schemes (Hannes, 1968; Gokaydin et al., 2011) and stimulus-response compatibility (Soetens et al., 1985). These tend to result in changes which fall along the continuum of results described by varying the RSI. A clear example of this is S-R spatial mapping with, when made incompatible, results in a shift of the entire spectrum between benefit-only and cost-benefit patterns of results towards higher values of RSI (Soetens et al., 1985). While seemingly not producing qualitatively different results, the reason why these variables influence sequential effects is in need of explanation. We will refer to the effects of changing the stimuli, the responses or the compatibility between them as 'stimulus and response effects' for the sake of brevity.

It is useful at this point to summarise the different empirical aspects of sequential effects that are in need of explanation, before turning to a discussion of the minimum requirements for a model to be able to explain these different phenomena.

- The cost-benefit and benefit-only patterns of sequential effects.
- The dependence of sequential effects - and of their latent structure - on the RSI.
- Individual differences and associated latent structure.
- Stimuli and response effects.
- The possible role of processing delays in sequential effects.

Incorporating the RSI as a parameter calls for a continuous-time modeling framework, when in fact most models proposed so far are of a discrete-time nature (e.g. Laming, 1969; Yu & Cohen, 2008; M. Jones et al., 2013). Two continuous-time models have been proposed so far (Cho et al., 2002; Gao et al., 2009), both building on the leaky accumulator modelling framework (Usher, 2001). However, as discussed above, these models depend on hard-coding the detection of repeating and alternating patterns via 'biasing' mechanisms, instead of letting this emerge naturally. Moreover, the model by (Gao et al., 2009) proposes an additional three biasing mechanisms in order to account for different patterns of sequential effects, making this a complex and difficult to interpret model. Finally, as discussed in Chapter 3, the different patterns of sequential effects are far more parsimoniously explained through different contributions by the two processing stages shown in Figure 4.4.

Another requirement of a complete model is for it to be able to provide a meaningful parameterisation of individual differences. Variation in such parameters should be able to capture not only the individual patterns of sequential effects, but also the covariance structure characteristic of empirical data. Moreover, it is also necessary to explain how the dependence on the RSI is related to individual differences, as the two phenomena seem to be closely related (see Chapter 3). Previous models sometimes incorporate parameters regulating some form of preference for alternations and repetitions (e.g. Yu & Cohen, 2008). In one case a mixture parameter for the two types of exponential filter shown in Figure 4.1 is included (M. Wilder et al., 2009). However, it is unclear whether any of these models would be able to parsimoniously explain individual differences as well as why these are related to the dependence of sequential effects on the RSI.

Stimuli and response effects suggest the need for some form of representation of space in the model. This does not necessarily imply continuous two- or even three-dimensional space. For instance, in order to capture the effects of stimulus-response compatibility it may be enough to have four different points of entry into the model, two for the stimuli and another two for the corresponding responses. In order to capture effects such as the difference between separate and overlapping stimuli a more detailed representation of space may become necessary. No model so far provides a representation of space, although two models based on the leaky accumulator

[3] Please refer to Chapter 1 for a discussion of these models.

framework do conceptualise a separation between the different stimuli. Nevertheless, even in this case it is unclear how a a differentiation between stimuli and responses could be represented.

Finally, if processing delays are confirmed to play a part in sequential effects this may add further requirements to a complete model. Depending on what is meant by 'processing delays' it may become necessary to take into consideration more complex aspects of the decision making process. Some speculations regarding this topic are given below and in Chapter 5.

Previous models of sequential effects target almost exclusively the cost-benefit pattern of sequential effects, and so should strictly speaking be considered models of sequential effects observed when the RSI is long (but see Gao et al., 2009). In contrast, the aim here is to outline a modelling framework with the potential to explain all aspects of sequential effects, and to begin to explain some of these. In addition to continuous time, space will also be incorporated, first in a discrete way but with the possibility of extension to continuous space (see Chapter 5). The system upon which the framework is built allows for a conceptualisation of individual differences with recourse to parameters also related to the the dependence on the RSI. Finally, in light of the previous discussion on the difficulties faced by previous models, producing an alternation bias will be as natural as a repetition bias, using exactly the same mechanism. In fact, the two fundamental types of behaviour of the system considered below show symmetrical preferences for repetitions and alternations, much like the putative processing stages involved in sequential effects and shown in Figure 4.4.

4.2 A new framework for sequential effects

4.2.1 Representing the stimuli in continuous time

In order to build a continuous time model[4] of sequential effects, we must first of all represent the stimuli in continuous time. Whereas before stimuli corresponded to a particular value - 0 or 1 - in a discrete sequence, they will now be represented by a function of time $F(t)$.[5] The presence of a stimulus at a particular time t will be marked by a constant value of $F(t)$ - set at 1 throughout - and its absence by a value of 0. The time course of the presentation of a single stimulus will look like a square pulse as illustrated in Figure 4.5. At the time of stimulus onset - t_1 - the function $F(t)$ takes a constant value and when the stimulus disappears - t_2 - it returns to a value of 0. This scheme allows for the representation of the duration of the stimulus presentation, as well as the interval between successive stimuli. In a typical 2AFC the stimuli remain on screen until the moment a response is made, in which case the duration of stimulus presentation is equal to the reaction time and like it is a random variable. Here the simplifying assumption is made that the stimulus presentation time is constant. In addition, stimuli and responses will not be distinguished on a first approach, so each square pulse can be taken to represent a stimulus-response pair.

A square pulse can be constructed formally as the difference between two shifted Heaviside step functions $\mathcal{H}(t)$ as

$$f(t) = \mathcal{H}(t - t_1) - \mathcal{H}(t - t_2) \tag{4.2}$$

where t_1 is the beginning of the pulse and t_2 the end.

[4] Throughout, the modelling framework proposed here will often be referred to as 'model' for short, even though a complete model of sequential effects is not intended here.

[5] The use of an upper-case 'F' here is intended to avoid any ambiguity with 'f', used throughout to denote frequency, and can be taken to mean 'force' as it will eventually be applied to an oscillator as a forcing function.

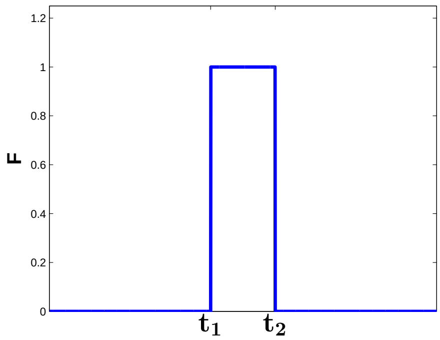

FIGURE 4.5: A square pulse function representing a single stimulus. t_1 is the time of stimulus onset and t_2 the moment the stimulus disappears which, in a typical 2AFC, corresponds to the moment a response is made.

The next step is to represent the difference between the two alternative stimuli in the model. To begin with this will be done by attributing opposite signs to pulses representing different stimuli, a seemingly arbitrary distinction but one which will later emerge naturally from a more realistic model. Mathematically, a negative sign pulse can be constructed just as easily as one with a positive sign by switching the sign of the two terms in (4.2). Figure 4.6 shows examples of how $F(t)$ looks like for the regular sequences RRRR and AAAA.

This way of constructing sequences of stimuli has one important practical consequence: it allows for a distinction between repetitions and alternations based on their *frequency*. Figure 4.6 illustrates this fact by showing two sinusoidal functions of frequencies 0.5 Hz and 1 Hz overlaid on the sequences AAAA and RRRR respectively. Throughout, we will refer to the repetition frequency as f_{REP} and to the alternation frequency as f_{ALT}, while keeping in mind that $f_{REP} = 2f_{ALT}$ by definition.

4.2.2 The physics of oscillatory motion

Once stimuli - or stimulus/response pairs - are defined in continuous time, the more general nature of the model must be decided upon. The choice of core modelling unit will lie with the damped

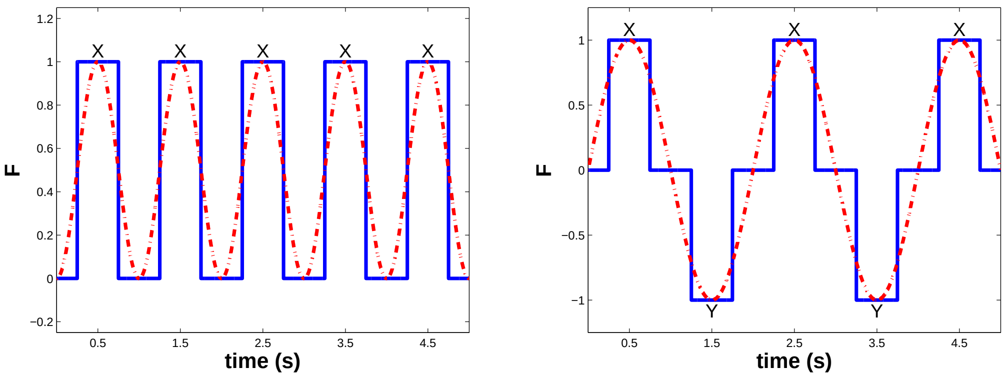

FIGURE 4.6: Functions of time representing RRRR (left) and AAAA (right) in terms of square pulses. Solid blue lines - functions representing the sequence RRRR (left panel) and AAAA (right panel). Individual pulses represent stimuli - either X or Y - the sign of the pulse determining which stimulus is being shown. Dashed red lines - superimposed sinusoidal functions of frequencies 1 Hz for RRRR and 0.5 Hz for AAAA. The sinusoids are meant to facilitate visualisation of the fact that the repetition frequency f_{REP} is twice the alternation frequency f_{ALT} by definition.

linear harmonic oscillator. Part of the reason for this choice is that a damped oscillator acts as an exponential filter of its natural - or resonating - frequency[6] (Horowitz, 1989), and exponential filtering in general has been shown to be of central importance in sequential effects.

To begin with the motion of a linear damped oscillator will be studied in some detail, with particular emphasis of the properties of relevance towards sequential effects, such as its behaviour as an exponential filer. Following this exposition an interpretation of the different model components in psychological terms is given.

A function describing the motion of a damped harmonic oscillator - $y(t)$ - must satisfy the following second order linear differential equation[7]

$$\ddot{y} + \gamma \dot{y} + \omega_0^2 y = F(t) \tag{4.3}$$

[6] The equation for the damped oscillator is essentially the same as that which describes an RLC circuit, well known from electronics to act as a band-pass filter.

[7] For a detailed discussion of this type of system please refer to any introductory physics of oscillations (Main, 1993; French, 2003) or linear differential equations (Boyce & DiPrima, 2005) textbook, depending on whether a more applied or more mathematical approach is preferred.

where y represents the position of the oscillator; ω_0 is the natural frequency of the oscillator in radians; and γ is the damping coefficient determining how much energy is lost per unit time due to friction. We start by considering what happens if there is no forcing, in which case $F(t) = 0$. If γ is small enough - relative to ω_0 - so that the system is under-damped, $y(t)$ will describe an oscillatory trajectory around an equilibrium position according to

$$y(t) = e^{-\gamma t} A cos \omega t + \delta \tag{4.4}$$

where A is a constant related to the maximum amplitude of the motion; ω is the frequency of the oscillation;[8] and δ is the phase. $e^{-\gamma t}$ represents the exponential decrease in amplitude of motion with time: the higher γ is the steeper the exponential decay. Note that the constants A and δ are not free but determined by the initial conditions, i.e. the position and velocity at $t = 0$.

Next we consider what happens when the oscillator is forced. In the context of our model $F(t)$ will consist of a train of five pulses as defined above but we will start by analysing the simpler case of a sinusoidal force such as

$$F(t) = F_0 cos(\omega_f t + \phi) \tag{4.5}$$

where F_0 is the maximum amplitude of the force; ω_f its frequency; and ϕ its phase. The motion of a forced oscillator will never die out, but rather settle to a long-term steady-state motion described by

$$y(t) = B cos(\omega t + \delta) \tag{4.6}$$

[8] Note that in general ω is *not* equal to ω_0. The only case in which the two are equal is if there is no damping, i.e. $\gamma = 0$. However, for $\gamma << \omega_0$, as in most cases of practical relevance, the difference between ω and ω_0 is negligible.

where B is the amplitude of motion; δ is a constant phase difference between the force and the motion of the oscillator.[9] Note that in this case $\omega = \omega_f$, i.e. the oscillator will settle to a motion with the same frequency as the driver. Unlike in the case of free motion, the two constants B and δ are no longer determined by the initial conditions but rather by the parameters of the forcing function $F(t)$ together with ω_0, and in particular the difference between ω_f and ω_0, the natural frequency of the oscillator. Of particular relevance for our purposes is the amplitude of motion B as a function of ω_f, which is given by

$$B(\omega_f) = \frac{F_0}{\sqrt{(\omega_0^2 + \omega_f^2)^2 + \gamma^2\omega_f^2}} \tag{4.7}$$

Intuitively, it would seem that the maximum amplitude of motion corresponds to the case where ω_f is equal to ω_0, in which case the driving frequency matches the natural frequency of the oscillator. However, due to damping, $B(\omega_f)$ actually peaks *below* ω_0, implying that in order to maximize the amplitude of motion one must drive the oscillator with a frequency slightly below the natural frequency of the oscillator. This difference is nevertheless small for most cases of practical relevance.

Resonance and filtering

At steady state, the phase difference δ implies that the driving force is sometimes going with and at other times against the motion of the oscillator. So the driving force is sometimes adding and other times removing energy from the system. The average power - energy per unit time - supplied to the oscillator over many cycles is given by

9 In order to keep this discussion simple the transient behaviour of the oscillator will be ignored. Nevertheless, the solutions in all calculations throughout all include this transient behaviour as all consist of the most general solution of the differential equations.

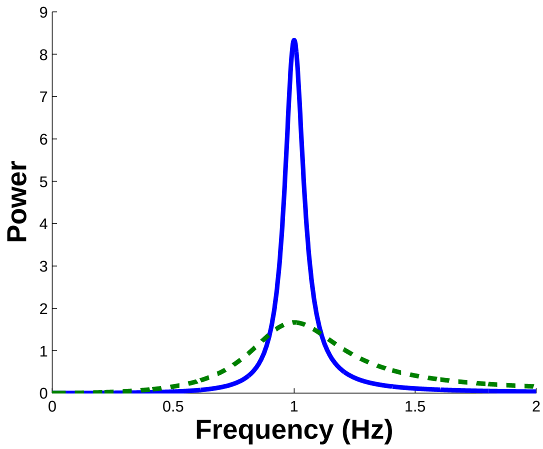

FIGURE 4.7: Average power absorbed by a damped oscillator as a function of the sinusoidal driving frequency. Two separate curves for different values of γ: the tall an narrow curve was produced with $\gamma = 0.6$; the short and wide curve was produced with $\gamma = 3$. Remaining parameters were equal for both curves: $\omega_0 = 2\pi$ (1Hz) and $F_0 = 1$. Note that the width of the curves has consequences for the range of frequencies which are amplified through resonance, and therefore for the band of frequencies filtered by an oscillator.

$$\langle P(\omega) \rangle = \frac{F_0^2}{2\gamma} \frac{\gamma^2 \omega^2}{\sqrt{(\omega_0^2 + \omega^2)^2 + \gamma^2 \omega^2}} \tag{4.8}$$

which results in a curve displaying a peak at ω_0 (see Figure 4.7), implying that the maximum energy input into the oscillator happens when the driving frequency equals ω_0 - the natural frequency of the oscillator. Conversely, the energy absorbed by an oscillator being driven with a frequency far from ω_0 is close to 0. This is the phenomenon know as *resonance*: driving forces with frequency close to ω_0 lead to increasing amplitude of motion with each cycle, whereas those far form ω_0 do not. The damping coefficient γ plays a crucial role by determining the width of the resonance curve: the lower γ is the narrower and taller the resonance peak (see Figure 4.7).

The usefulness of considering a simple sinusoidal input lies in the fact that any function can be approximated - via Fourier series - by a weighted sum of sinusoids with different frequencies. In addition, the principle of superposition of linear systems states that if $y_1(t)$ is the solution to (4.3) for $F_1(t)$; and $y_2(t)$ is the solution for $F_2(t)$; then the solution for $F_1(t) + F_2(t)$ is given by

$y(t) = y_1(t) + y_2(t)$. In other words, the response to a sum is equal to the sum of the responses to the individual terms, and we can consider separately the effects on the oscillator of the individual frequencies present in the driving force separately (see below for a more detailed discussion of this point).

When the driving force $F(t)$ is a function composed of many different frequency components, an oscillator will amplify - through resonance - those frequencies which are close to the natural frequency ω_0, while at the same time attenuating those frequencies far from ω_0. How close a frequency must be to ω_0 in order to be amplified - or *filtered* - depends on the width of the resonance peak which, as we have seen, depends on γ. A small γ will determine a narrow range of frequencies - or *band* - filtered by the oscillator; conversely, a high value of γ will determine a wide frequency band.

So far we have seen why an oscillator works as a filter. We must now understand why it acts as an *exponential* filter. It was shown above that, in the case of free motion, the maximum amplitude decreases exponentially due to damping. This happens because the damping force $F_d = -\gamma v$, where v is the velocity, always opposes the motion and therefore leads to a dissipation of any energy stored in the oscillator. Similarly, and excluding small transient fluctuations, the average energy per cycle in a forced oscillator will dissipate according to (Main, 1993)

$$\langle E \rangle = \frac{1}{2}\omega_0^2 A^2 e^{-\gamma t} \tag{4.9}$$

where A is a constant. As we will see, the consequence of this is that a damped forced oscillator will ‘forget’ its input in an exponential fashion, thereby providing a very good approximation to the discrete exponential filter.

So it has been shown how stimuli will be represented by continuous functions and explained why the physics of a damped oscillator dictates that it will behave like an exponential filter. Next the more general relationship between the oscillator parameters and the psychological constructs it is meant to represent is discussed, with particular emphasis on what the dependent measure will be which correlates with reaction time. Two parameters of the model are particularly relevant: the velocity of the oscillator, given by $\dot{y}(t)$, representing the expectation state; and the damping coefficient γ which determines the exponential decay of memory, and plays a role analogous to α in the discrete time filer in (4.1).

Given the discontinuous nature of the pulses applied to the oscillator the differential equations will be solved via the Laplace transform method (Boyce & DiPrima, 2005). The solutions presented throughout are all analytic given the linear nature of the systems considered here. Appendices D E contain the technical details of how the equations were solved.

4.2.3 The relationship between physics and psychology

Because of the unprecedented use of oscillators to model sequential effects it becomes particularly important to establish how the different components of the model map onto the usual concepts considered in the psychology literature. Three aspects in particular will be focussed on: expectation state, memory decay and reaction time. But before we go on it is useful to briefly review how traditional models handle these same concepts. Discrete-time probabilistic models take the probability of the next event to correspond to the expectation about what the next stimulus will be. The higher the probability the stronger the expectation and the shorter the reaction time should be, so that $RT \propto 1 - p(t)$. Memory of past events is assumed to decay exponentially, how quickly depending on parameters such to α in Equation (4.1). In models based on the leaky accumulator framework, reaction times are calculated as the mean time taken to reach a fixed threshold; expectations are introduced via biasing terms - alternation and repetition ‘detectors’ - and an exponential decay is implemented by a constant rate leaking of decision units involved (e.g. Cho et al., 2002).

Here the velocity of the oscillator will be taken to represent expectations in the following way: an oscillator with positive velocity will be considered to be ‘expecting’ a stimulus with a positive sign, and one with negative velocity to be expecting a stimulus with negative sign. Reaction time should therefore be a function of the magnitude of the velocity at the moment a stimulus is displayed as well as whether the stimulus is that which was expected. One possibility is to think about this problem in terms of the the energy input into the system by a particular pulse. Consider the work performed by a force $F(t)$

$$W = \int F(t)v(t)dt \tag{4.10}$$

It follows from (4.10) that W depends on the magnitudes of $v(t)$ and $F(t)$ and whether they have have the same sign. If W is positive energy is added to the system and if it is negative energy is removed from the system. So a stimulus which is expected will tend to increase the amplitude of motion and one which is not expected will tend to decrease it. If $F(t)$ happens to be a constant function such as a pulse, W is greatly simplified as

$$W = F_0 \int v(t)dt \tag{4.11}$$

where F_0 is a constant equal to the amplitude of the pulse. If we further accept the duration of the pulse to be short, the instantaneous energy input is equal to $F_0 v_0$ where v_0 is the velocity at the exact moment the pulse is initiated. Finally, since the effects of stimulus intensity will not be considered, the exact value of F_0 is irrelevant (it is set at 1 throughout) and we can simply take the quantity $v_e = \mathrm{sgn}(F_0)v_0$, which has units of velocity, as a measure of how much the system was expecting a particular pulse. As discussed above, the stronger the expectation the shorter the reaction time should be so we will take $RT \propto -v_e$.

In the case of a sequence of several stimuli, and in particular of five stimuli, the expectation state for the last stimulus - the velocity of the oscillator just before the last pulse - is a function of the four preceding stimuli and whether they added or removed energy from the system, as well as the phase of the oscillator. This information is all encoded in the velocity at the time the last pulse arrives and so v_e will be calculated at *the exact moment the last pulse is initiated*. Throughout, v_e will be referred to loosely as 'velocity' but it should be kept in mind that the sign the velocity assumes depends on the sign of the last pulse. This situation is analogous to discrete-time models, in which the choice of $p(t)$ or $1 - p(t)$ depends on the nature of the next stimulus and must be changed dynamically.

Due to energy dissipation, any pulse applied to the oscillator will eventually be 'forgotten' by the system, and this is meant to represent the decay of memory about past events. In the particular case of sequential effects in a 2AFC, it has been shown that stimuli beyond the last five do not have any significant influence on reaction time (Remington, 1969).

Now that all the ingredients have been collected, in the next section the simplest model possible - a single damped oscillator - will be analysed.

4.2.4 A single oscillator model

Behaviour at resonance - REP and ALT

Results of the single oscillator model will be calculated by forcing the oscillator with each of the usual sixteen sequences of stimuli in isolation. As always, sequences come in pairs, so for instance XYYXY represents the same pattern as YXXYX, and both can be written ARAA. In practice only results for those sequences starting with the same stimulus - say X - will be calculated since we are assured results for the sister sequences will be the same. We start by setting the natural frequency

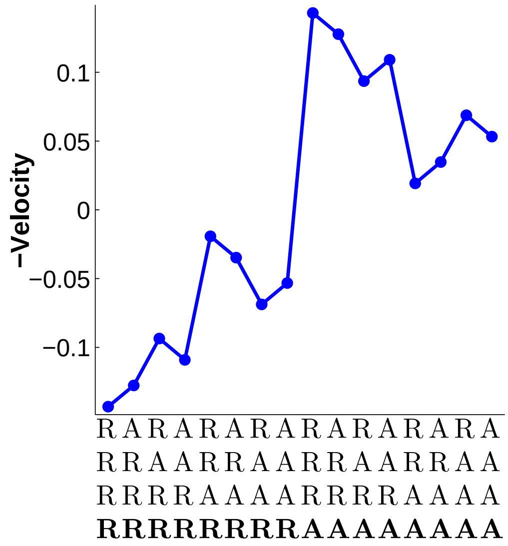

FIGURE 4.8: Results of an oscillator set to resonate with repetitions (REP). The curve shows velocity measurements taken just before the last pulse and take into account whether the last stimulus was expected or not (see main text). The natural frequency of the oscillator is set to match the repetition frequency so $\omega_0 = f_{REP} = 1$ Hz; damping was set at $\gamma = 1.5$ for this example. Results are shown in terms of the negative of the velocity in order to facilitate comparison with empirical reaction time results since $RT \propto -v$.

of the oscillator ω_0 equal to f_{REP} and both equal to 1 Hz, with $\gamma = 1.5$, producing the results shown in Figure 4.8.

When ω_0 is set to match f_{REP} - the repetition frequency - the system is set to resonate with repetitions producing a pattern of results nearly identical to those of the exponential filter defined in Equation 4.1 (see Figure 4.1, left panel). However, these similarities belie what is a considerably more complex system in the case of an oscillator. The approximation to the simple exponential filter is only valid when $\omega_0 \simeq f_{REP}$, i.e. when the oscillator is *resonating with repetitions*. When ω_0 is shifted away from f_{REP} the system will filter frequency components other than f_{REP}, which are unrelated to repetitions. One particular frequency that may by of significance is f_{ALT}. Figure 4.9 shows what happens if ω_0 is now set to f_{ALT}.

In much the same way as a damped oscillator tuned to f_{REP} can be thought of as detecting repetitions, an oscillator tuned to f_{ALT} works as a detector of alternations. Note how the pattern of results in Figure 4.9 is an almost perfect symmetrical copy of that shown in Figure 4.8. The

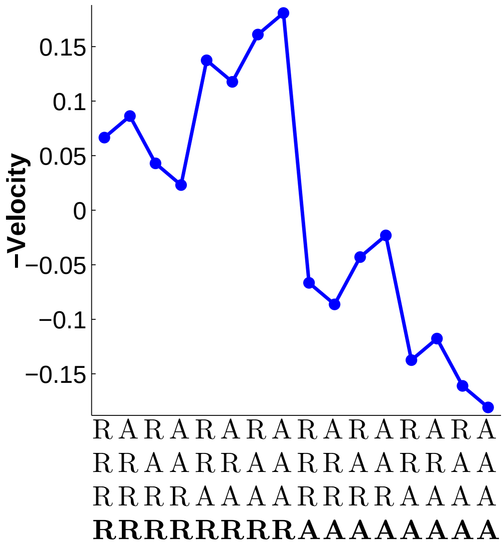

FIGURE 4.9: Results of an oscillator set to resonate with alternations (ALT). The curve shows velocity measurements taken just before the last pulse and take into account whether the last stimulus was expected or not (see main text). The natural frequency of the oscillator is set to match the alternation frequency so $\omega_0 = f_{REP} = 0.5$ Hz; damping was set at $\gamma = 1.5$ for this example. Note how results are almost perfectly symmetrical with those of an oscillator tuned to f_{REP} shown in Figure 4.8, except for a slightly greater amplitude of ALT compared to REP (see main text).

oscillator tuned to f_{ALT} is in fact operating as an *exponential filter of alternations*. From now, the pattern of results of an oscillator tuned to repetitions will be referred to as REP for short, and the pattern obtained when it is tuned to alternations as ALT, while keeping in mind that these resonance patterns are not fixed but depend on γ (see below). Figure 4.10 shows an illustration of the parameters used to produce REP and ALT respectively.

When looked at in terms of motion of the oscillator, resonance with repetitions and with alternations correspond to somewhat different phenomena. When the system is tuned to repetitions, the oscillator is forced with one pulse per cycle; when it is tuned to repetitions it sees two pulses each cycle (see Figure 4.11). This gives the oscillator tuned to f_{ALT} a slightly greater amplitude of motion which translates into a small difference in the scale of REP and ALT.

In order to discuss how model results depend on damping we will consider the effect of varying γ on REP, the effect on ALT being analogous in every respect. Recall that γ regulates the rate

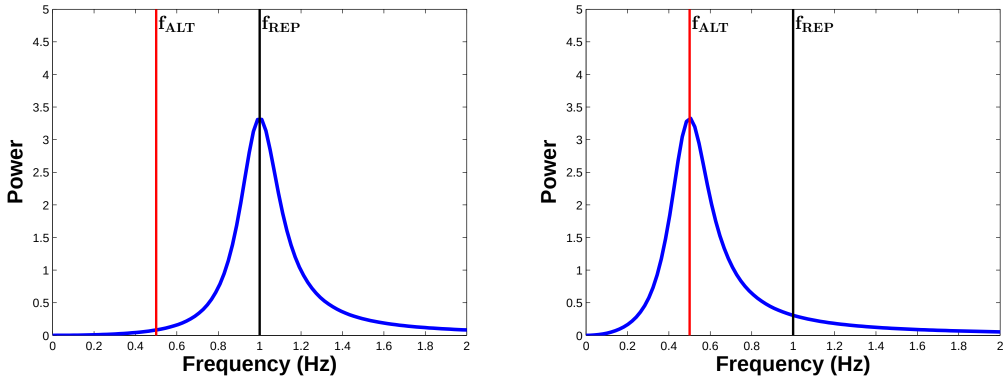

FIGURE 4.10: Illustration of the parameters which produce REP (left panel) and ALT (right panel). Blue lines - mean power absorbed by the oscillator as a function of sinusoidal driving frequency with $\omega_0 = 1$ Hz (left panel) and $\omega_0 = 0.5$ Hz (right panel); $\gamma = 1.5$, the same value used to produce Figures 4.8 and 4.9. Red and black vertical lines mark the positions of f_{ALT} and f_{REP} respectively. Note that the peak of the power curves, determined by ω_0, is situated either at f_{REP} to produce REP (left panel) or f_{ALT} to produce ALT (right panel). Recall that $f_{REP} = 2f_{ALT}$ by definition.

of exponential decay, and so higher values of γ determine a shorter 'memory' span and vice-versa. The result is that, as damping is increased, model results will depend progressively less on more distant events, until they reduce to a two-tiered dependence on whether the last event was a repetition or an alternation. Figure 4.12 shows an illustration of this principle for three values of γ, where a progressive loss of sensitivity to older events can be observed. Note that this progression is perfectly analogous to what happens in the case a discrete-time exponential filter as the parameters regulating the exponential decay are varied (not shown).

Figures 4.8 and 4.9 show the results of setting ω_0 equal to f_{REP} and f_{ALT} respectively, the two dominant frequency components in the square-pulse functions representing sequences of stimuli. Outside these values the oscillator will filter out other frequency components. In order to understand what determines the behaviour under these circumstances, and what frequency components other than f_{REP} and f_{ALT} might be present, we must turn to a frequency domain view of the model. As we will see, there is a complex relationship between the frequency content of the sixteen different functions representing the sequences of stimuli and the behaviour of the oscillator. Switching to a frequency domain view of the input functions will also allow for a better understanding of how γ affects the model.

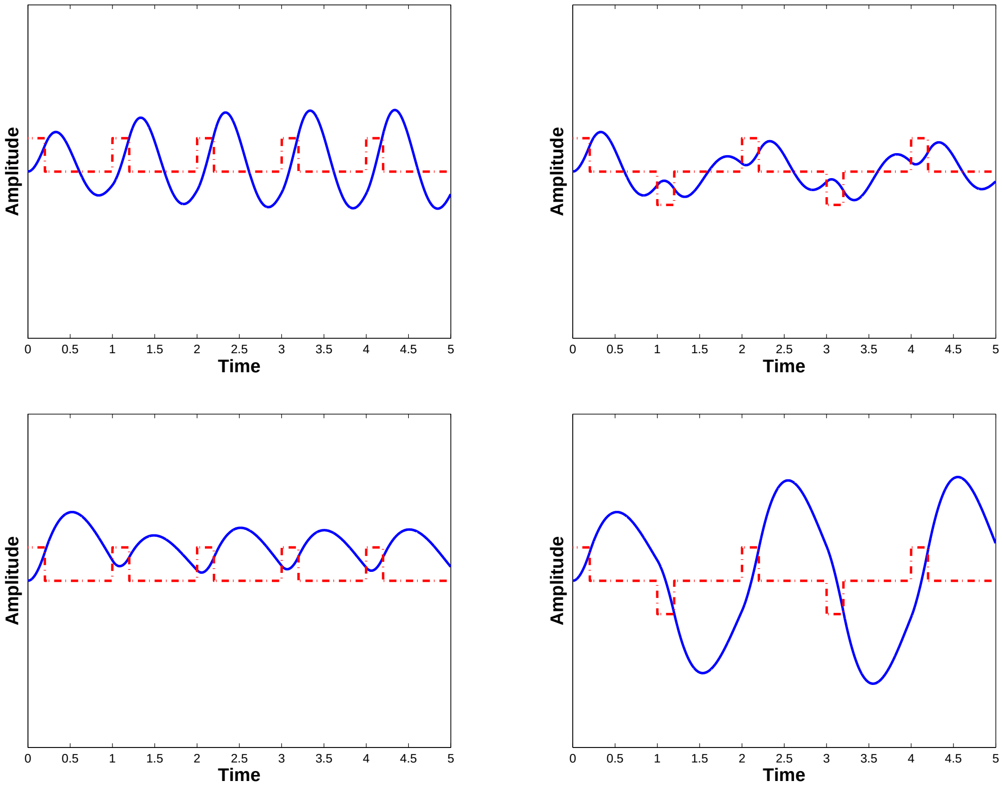

FIGURE 4.11: Behaviour of an oscillator when resonating with repetitions and with alternations. Blue lines - position of the oscillator. Red dashed lines - force applied to the oscillator (not to scale). Top two panels - oscillator tuned to f_{REP}; Bottom two panels - oscillator tuned to f_{ALT}. When an oscillator tuned to f_{REP} is forced with RRRR (top left panel) each new pulse builds upon the last to increase the velocity of the oscillator, the same being true of an oscillator tuned to f_{ALT} when it is forced with AAAA (bottom right panel). Conversely, when the oscillators are forced with a frequency to which they are not tuned - top right and bottom left panels - each pulse does little to increase the velocity. Notice how resonance with alternations induces a greater amplitude of motion when compared with with repetitions, the reason being that in the former case the oscillator is forced twice during each cycle.

Representing sequences in the frequency domain

In order to understand how a function looks like in the frequency domain we start by calculating its Fourier transform

$$x(f) = \int_{-\infty}^{\infty} e^{2\pi i f t} x(t) dt \tag{4.12}$$

where $x(t)$[10] can be taken as any of the five-pulse functions discussed above. The spectral energy

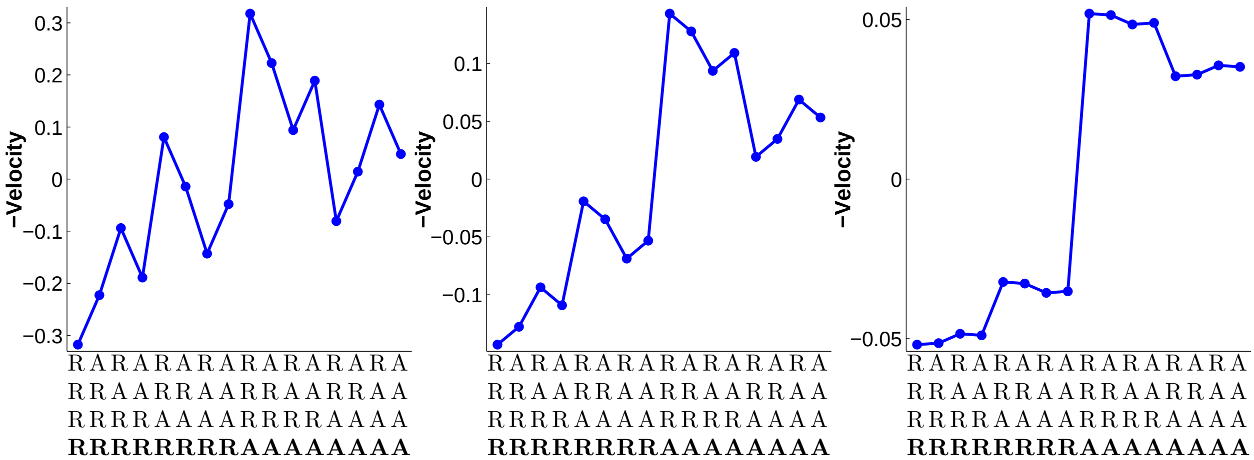

FIGURE 4.12: Effect of varying the damping coefficient on model results. From left to right, γ equals 0.6, 1.5 and 3 respectively. Note how the pattern of results depends progressively less on older events until, for $\gamma = 3$, only the last two events have any significant influence. The reason for the changes observed is the steeper exponential decay of memory determined by higher values of γ. Ultimately, by increasing γ enough, results will reduce to a two-tiered dependence on the last event and whether it was a repetition or an alternation. Only the effect of varying γ on REP is shown but the effect on ALT is analogous. The behaviour of a discrete time exponential filter as the α parameter is varied is qualitatively the same (see main text).

density can then be calculated according to

$$S(f) = |x(f)|^2 \tag{4.13}$$

For the discrete case the integrals in (4.12) and (4.13) are replaced by sums. Also, note that the energy as defined in (4.13) is not equivalent to the mechanical energy discussed above in the context of an oscillatory motion, but rather a measure of the content of a function in terms of individual frequency components which make it up.

Let us start by considering how the functions representing RRRR and AAAA look like in the frequency domain (see Figure 4.13).[11] As expected RRRR has a strong peak around the repetition frequency f_{REP} and AAAA around the alternation frequency f_{ALT}. An interesting observation can be made about these spectra: AAAA presents harmonic peaks every odd-integer multiple of the

[10] Note the slight change in notation here in order to avoid the letter f which now stands for frequency.

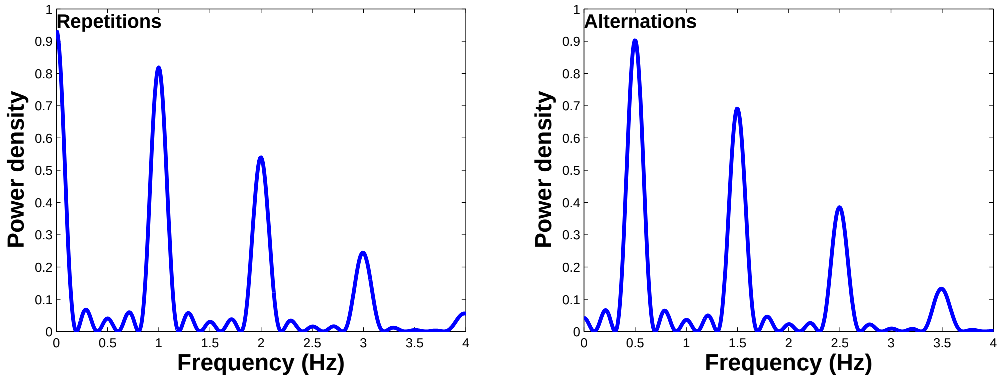

FIGURE 4.13: Spectral energy density - i.e. spectra - of a perfectly repeating (left panel) and perfectly alternating (right panel) sequences. Note how RRRR shows harmonic peaks for all integer multiples of f_{REP} whereas AAAA shows only harmonic peaks at odd-integer multiples of f_{ALT}. Also, notwithstanding a decrease in power as the frequency increases, the overall pattern visible in the interval [0 1] Hz is repeated for higher frequencies.

base frequency f_{ALT}, a general property of square waves (Main, 1993); in contrast, and by virtue of being 'lifted' onto the positive domain, RRRR shows harmonic peaks every integer multiple of f_{REP}, as well as a peak at frequency 0. The spectra of RRRR and AAAA, as well as any of the other sixteen possible sequences, repeats itself - with some attenuation - every integer multiple of the base frequency of pulses, which is equal to f_{REP}. If f_{REP} is set to 1 Hz, as will be the case throughout, this means that we can restrict ourselves to the interval [0 1] Hz when discussing sequences in the frequency domain.

The functions considered here are all composed of five pulses, with any differences due solely to the sign of the pulses. An important consequence of this is that the total spectral energy content as defined by

$$S = \int_{-\infty}^{\infty} |x(f)|^2 df \tag{4.14}$$

is *constant* for any sequence of stimuli of equal length, with only the energy distribution changing. This has consequences in that an important trade-off is observed across the sixteen sequences:

[11] The set of all frequency spectra for all sixteen sequences is shown in Appendix F.

spectral energy is either fully concentrated in f_{REP}, in f_{ALT}, or it is distributed across other frequencies lying in the intervals between f_{REP} and f_{ALT}. An illustration of this principle is shown in Figure 4.14 where the spectra of four different sequences is shown: RRRR, AAAA, ARAR/RARA (the spectrum of the last two sequences is equal). Note how the sequence RRRR has - other than a trivial peak at $f = 0$ - only one dominant peak at f_{REP} and similarly that AAAA has only one main peak at f_{ALT}. On the contrary, the sequences ARAR/RARA have their energy fully concentrated on intermediate frequency components. The fact that only the sequences RRRR and AAAA display a single dominant peak reflects the fact that these are the only two perfectly regular sequences.

In order to make the statement about the trade-off in spectral energy content above clearer, we will denote the spectral energy in and around f_{REP} and f_{ALT} as E_{REP} and E_{ALT} respectively. Further, let us denote the spectral energy content of the *intermediate frequencies* between f_{REP} and f_{ALT} as E_{INT}. E_{REP}, E_{ALT} and E_{INT} define three regions which partition the frequency spectrum; the boundaries between these regions will not be specified, but are taken to lie approximately where the spectral peaks at f_{REP} and f_{ALT} end and secondary frequency peaks begin. Since the total spectral energy is constant $E_{REP} + E_{ALT} + E_{INT} = k$, with three important corollaries of this relation: firstly $E_{INT} \propto -(E_{REP} + E_{ALT})$; secondly $E_{ALT} + E_{INT} \propto -E_{REP}$; and finally that $E_{REP} + E_{INT} \propto -E_{ALT}$. The relevance of these relations is made clear next.

It is one thing for a trade-off to be observed in the frequency content of the sequences, and another whether this trade-off will manifest itself in terms of model results. Some sequences have the same spectrum, and yet have very different results when applied to the oscillator. The reason for this is damping, which makes velocity measurements - taken at the end of the train of pulses - more dependent on recent events.[12] For instance, consider the case where $\omega_0 = f_{REP}$: the sequence AARR will result in a larger velocity measurement than the sequence RRAA, despite

[12] By making use of the principle of superposition of linear systems it is possible to calculate the contribution of each frequency component towards the velocity of the oscillator, but this was omitted here for the sake of keeping this discussion as simple as possible.

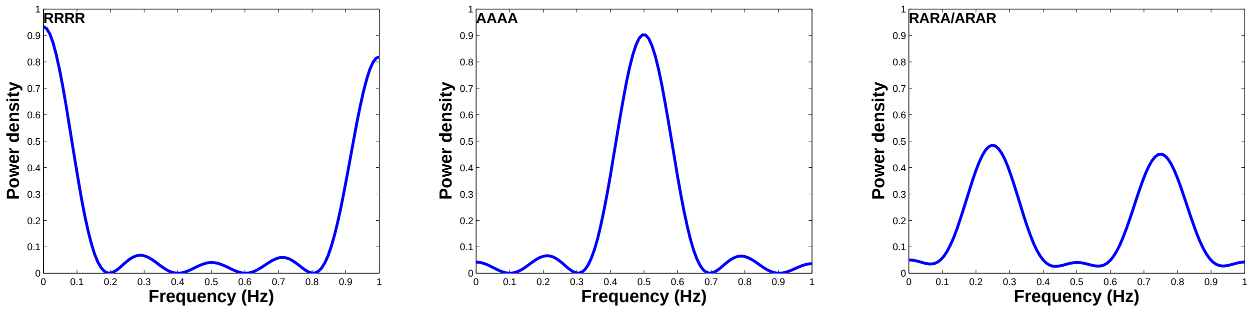

FIGURE 4.14: Power spectra RRRR, AAAA and ARAR(RARA) in the interval [0 1] Hz. Note that the sequences ARAR and RARA have the same frequency spectrum. The spectral energy of the sequences RRRR and AAAA is concentrated almost exclusively around f_{REP} and f_{ALT} respectively, whereas for ARAR and RARA it is distributed among intermediate components.

both having the same spectrum, because the repetitions in AARR occurred more recently. Yet despite this caveat, the trade-off in spectral energy manifests itself at the level of oscillator velocity measured at the end of the pulse train.[13] For instance, $E_{ALT} + E_{INT} \propto -E_{REP}$ seems to imply that, if an oscillator had a wide enough frequency band - i.e. high enough γ - so as to encompass E_{ALT} and E_{INT}, then the pattern of results should be similar to the negative of those observed when only E_{REP} is filtered. In other words, filtering all frequencies other than f_{REP} should result in a pattern approximating -REP, and this is in fact the case as shown below, in particular for the case of a heavily damped system.

In order to complete the discussion of a single oscillator system two separate cases will be studied separately: one in which the single oscillator is heavily damped (though not over-damped) and another where it is lightly damped. As we will see, the discussion of the way in which oscillator velocity depends on the frequency content of the sequences will be of crucial importance in understanding model results, as well as later in anchoring the discussion of artificial latent structures.

[13] The reason for this is that the trade-off discussed for five-long sequences of pulses would also be verified for sequences of any length, or in fact for any weighting of the sequences, so long as it was the same for all.

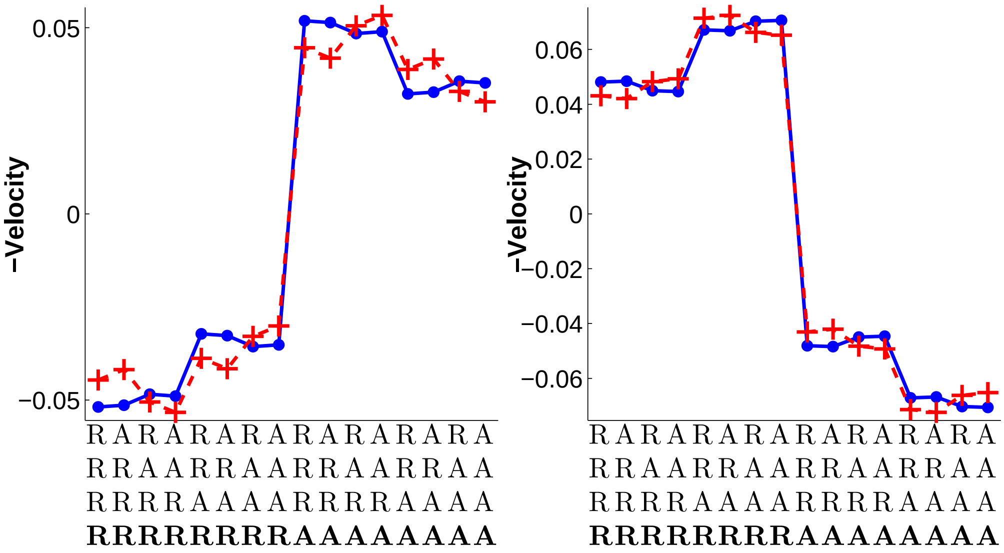

FIGURE 4.15: Patterns similar to -REP and -ALT produced by a heavily damped oscillator. Solid blue lines - REP (left panel) and ALT (right panel) calculated with $\gamma = 3$ and results shown in terms of the negative of velocity as usual. Dashed red lines - model results in terms of velocity - not its negative - for $\omega_0 = 0.63$ in Hz (left panel) and $\omega_0 = 0.73$ (right panel), both with $\gamma = 3$. Note that in both panels the two patterns are really inverted copies of each other, but are shown with the same orientation in order to facilitate comparison.

A heavily damped system

While perhaps of lesser psychological relevance when compared to a lightly damped system - for reasons that will soon be made clear - the study of a heavily damped system is useful for several reasons. Firstly, it will complete our understanding of the range of possible types of behaviour of a single oscillator; secondly it will illustrate clearly the trade-off in spectral energy discussed in the previous section; thirdly, it will eventually be useful in understanding how the behaviour of an oscillator is related to its latent structure.

As discussed above, a high value of γ determines a relatively wide frequency band filtered by the oscillator. This will make it possible to observe directly the trade-off relations in spectral energy discussed above, but this time in terms of oscillator velocity. Specifically, it is possible to position the frequency band - by varying ω_0 - in such a way as to obtain patterns of results similar to -REP as well as -ALT (see Figure 4.15). An illustration of the model parameters which make

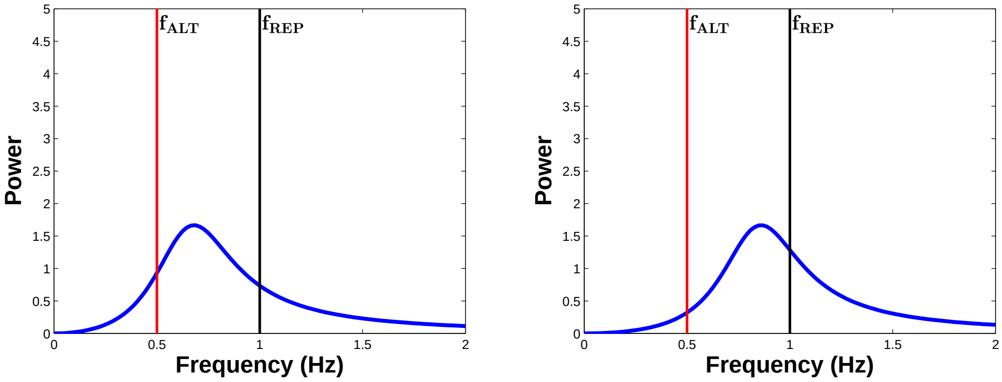

FIGURE 4.16: Illustration of the model parameters which produce patterns resembling -REP (left panel) and -ALT (right panel). Solid blue lines - resonance curves with $\gamma = 3$. The values of ω_0 which produce -REP and -ALT respectively are 0.63 Hz (left panel) and 0.73 Hz (right panel) respectively. Note the wide resonance peaks which determine fairly wide frequency bands filtered by the oscillator. This allow the oscillator to filter out both the frequencies around either f_{REP} or f_{ALT} - depending on the case - as well as the intermediate frequencies (see main text).

these ‘inversions’ possible is shown in Figure 4.16.

The full range of qualitative types of behaviour displayed by a single heavily damped oscillator is shown in Figure 4.17. All these patterns have one thing in common: because of the heavy damping, and resulting steep exponential decay, no appreciable dependence on events beyond the last two is observed. In amongst this catalogue of different types of behaviour is a pattern resembling -(REP+ALT), something what was predicted from the discussion of the trade-off in spectral energy content (see above). Also shown in Figure 4.17 are results similar to -REP and -ALT but with a clearer dependence on the two last events, as well as patterns similar to a two-tiered dependence on the last event. Overlaid on all the different types of result is a best fitting linear combination of REP and ALT, i.e. the the patterns obtained at resonance with f_{REP} or f_{ALT} respectively. The fairly good qualitative fits highlight one essential fact about a heavily damped oscillator: it behaves like a linear combination of the form $c_1 ALT + c_2 REP$ where c_1 and c_2 are coefficients that can be positive or negative. This is true despite the fact that the patterns resembling -REP and -ALT are not inverted copies of REP and ALT in any real sense, but rather a consequence of the trade-off in spectral energy. Next we turn to the more psychologically relevant case of a lightly damped oscillator.

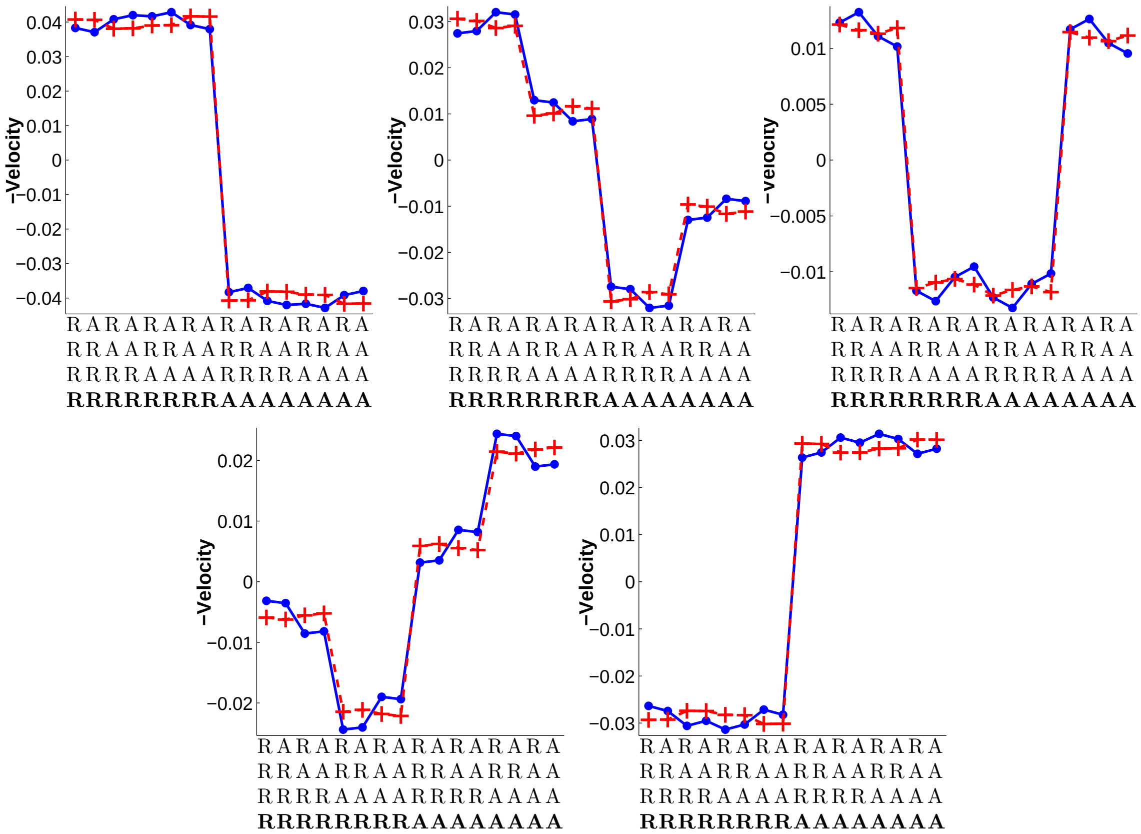

FIGURE 4.17: Selected patterns of results for a single heavily damped oscillator. Solid blue lines - results in terms of velocity of a single oscillator as ω_0 is varied from 0.5 to 1 Hz with $\gamma = 3$. From left to right and top to bottom ω_0 is: 0.67, 0.75, 0.82, 0.87 and 0.93 Hz. These values were chosen as representative of different qualitative patterns and are otherwise not significant. Panels (a) and (e) show patterns with an apparent dependence on the last event only, with either a repetition or alternation bias; panels (b) and (d) show patterns similar to -REP and -ALT but with a clearer dependence on the last two events; finally, panel (d) shows a concave pattern similar to -(REP+ALT). Dashed red lines - best fitting linear combination of ALT and REP of the form $c + (aALT + bREP)$, illustrating how a heavily damped oscillator is well approximated by a linear combination of REP and ALT (see main text).

A lightly damped system

Our best estimate of the human 'damping coefficient' comes from experiments which resulted in a pattern of results similar to what is expected of a simple exponential filter. Fitting the results of a single oscillator to the patterns shown in Figure 4.2 yields the following estimates for the human γ: 0.66, 0.82 and 0.68. By contrast, a value $\gamma = 3$ was used in the analysis of the heavily damped system, so it seems that a lightly damped oscillator might be a better model for human behaviour.

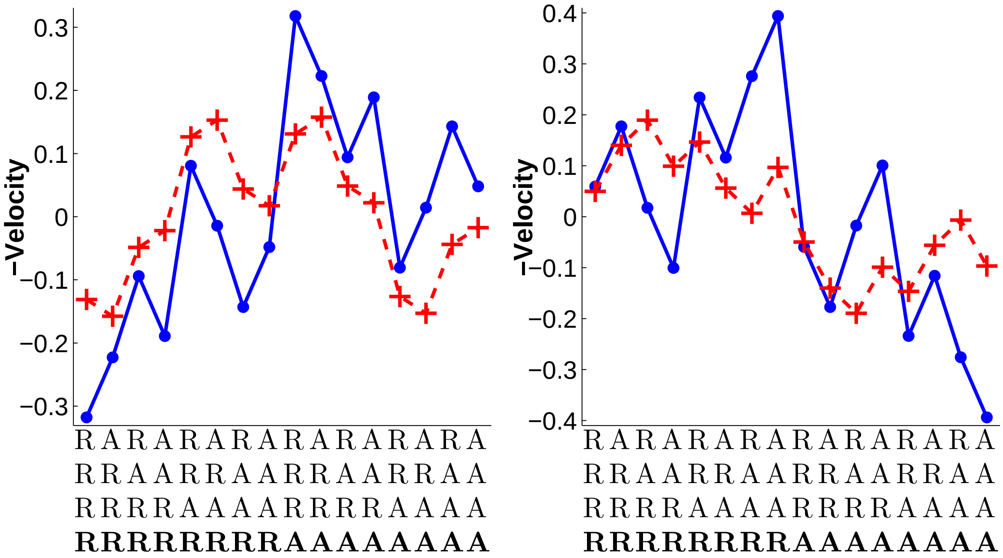

FIGURE 4.18: Demonstration that obtaining results similar to -REP and -ALT is no longer possible with a lightly damped oscillator. Solid blue lines - REP (left panel) and ALT (right panel) obtained with $\gamma = 0.6$. Dashed red lines - best fitting oscillator results fit to REP and ALT by linearly transforming results according to $a + bx$ where b was constrained to be negative.

Based on the estimates above, $\gamma = 0.6$ will be the choice for the analysis of a lightly damped oscillator.

When the damping coefficient γ is low the oscillator filters a narrow frequency band. Under these conditions, it is unlikely that patterns such as -REP and -ALT will occur, since these require a wide filter band. Take the relation $E_{ALT} + E_{INT} \propto -E_{REP}$ discussed above: a narrow frequency band is unlikely to encompass both E_{ALT} and E_{INT}, i.e. both f_{ALT} and all intermediate frequency components. Moreover, such a narrow band is unlikely even to cover the set of all the intermediate frequencies, in which case we might have to consider individual frequencies included in E_{INT} in isolation. In general the lightly damped oscillator is expected to show a wider range of different types of qualitative behaviour as ω_0 is varied, when compared to the heavily damped oscillator.

Figure 4.18 illustrates the fact that with a lightly damped oscillator it is no longer possible to find a value of ω_0 which results in a behaviour similar to -REP and -ALT. Moreover, Figure 4.19 show results for the lightly damped oscillator for the same values of ω_0 used when illustrating the

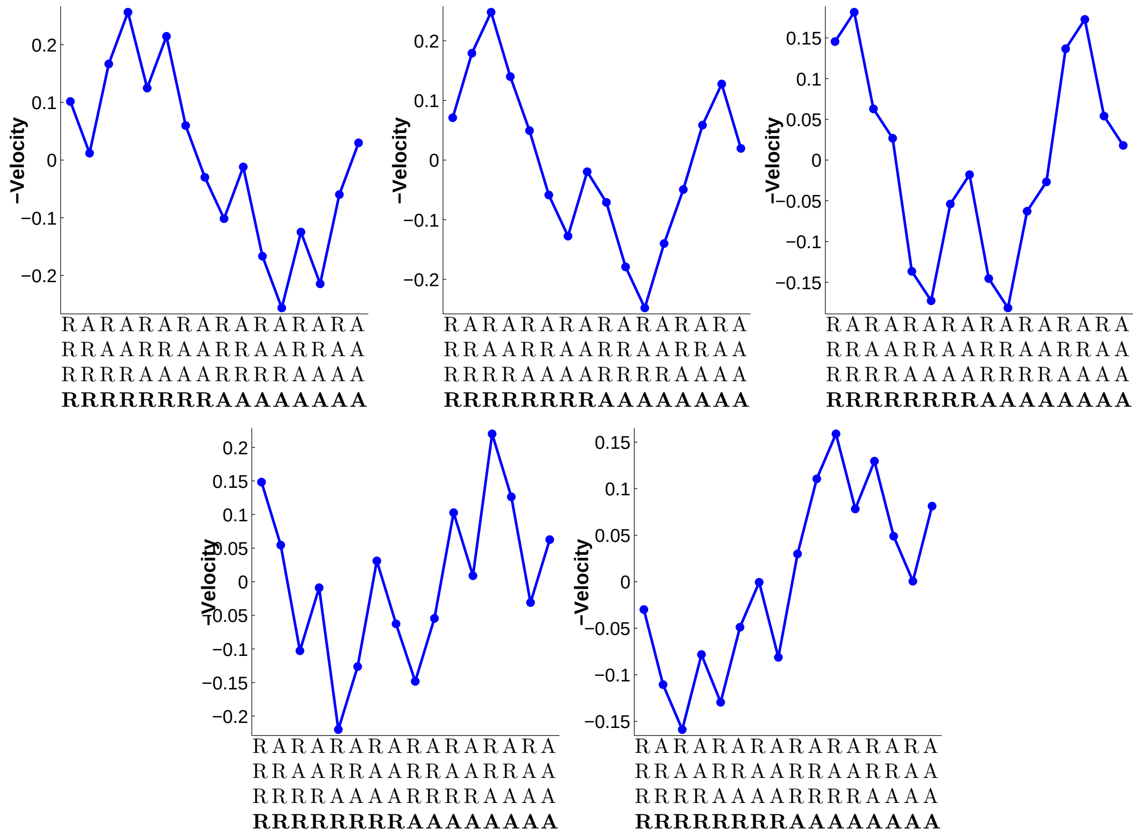

FIGURE 4.19: Selected patterns of results for a single lightly damped oscillator. The different panels show results for the same values of ω_0 used when illustrating the behaviour of a heavily damped oscillator - 0.67, 0.75, 0.82, 0.87 and 0.93 Hz - but this time with $\gamma = 0.6$. Note the qualitative differences relative to the results shown in Figure 4.17 and the fact that in the lightly damped case results are no longer well described by a linear combination of the resonance patterns REP and ALT.

more heavily damped system and show in Figure 4.17, illustrating not only the more diverse nature of results obtained when γ is low but also that under these circumstances the model is no longer well approximated by a simple combination of REP and ALT.

The single oscillator model is highly unrealistic in that an arbitrary distinction is made between the stimuli based on their sign, which further limits the number of possible alternatives to two. Before turning to a discussion of the possible uses of the model in capturing different aspects of sequential effects, it is important to first present a more realistic version of the model, one which will include two rather than just one oscillator. This is also important in that it will begin to lay the groundwork for an entire family of models which among other things can include any number of oscillators as well as alternative stimuli. Foreshadowing some of the conclusions of this work, the behaviour of a system of two oscillators will be found to be effectively equivalent to that of the single oscillator model discussed so far. So despite the importance of the next section the discussion that will follow after it can be read with the single oscillator model in mind.

4.2.5 Two coupled oscillators

The way to relax the distinction between the stimuli based on their sign is to begin to conceptualize space in the model. From now on, different stimuli will *always have a positive sign*, and will be distinguished based on the oscillator to which they are applied. While different oscillators can be considered to represent spatially separate locations, no specific mapping to any real locations such as neurological loci is implied, nor are the oscillations meant to represent any particular type or scale of neural activity. For now the model, while harbouring a great deal of potential to make concrete predictions, should be considered to be abstract in nature. Discretizing the system provides not only a natural way to remove the artificiality of the sign assumption, but is useful in that is allows a description of the the system in terms of its normal modes, defined as those states of a system in which all parts move with the same frequency, whether in phase or not (French, 2003).

We start by considering two coupled oscillators, described by the following pair of second order differential equations

$$\begin{aligned} \ddot{y}_1 + \gamma\dot{y}_1 + \omega_0^2 y_1 + k(y_1 - y_2) &= F_1(t) \\ \ddot{y}_2 + \gamma\dot{y}_2 + \omega_0^2 y_2 + k(y_2 - y_1) &= F_2(t) \end{aligned} \tag{4.15}$$

where the subscripts refer to different oscillators and k represents the strength of the coupling between them. In the system as set up in (4.15) the coupling is proportional to the difference

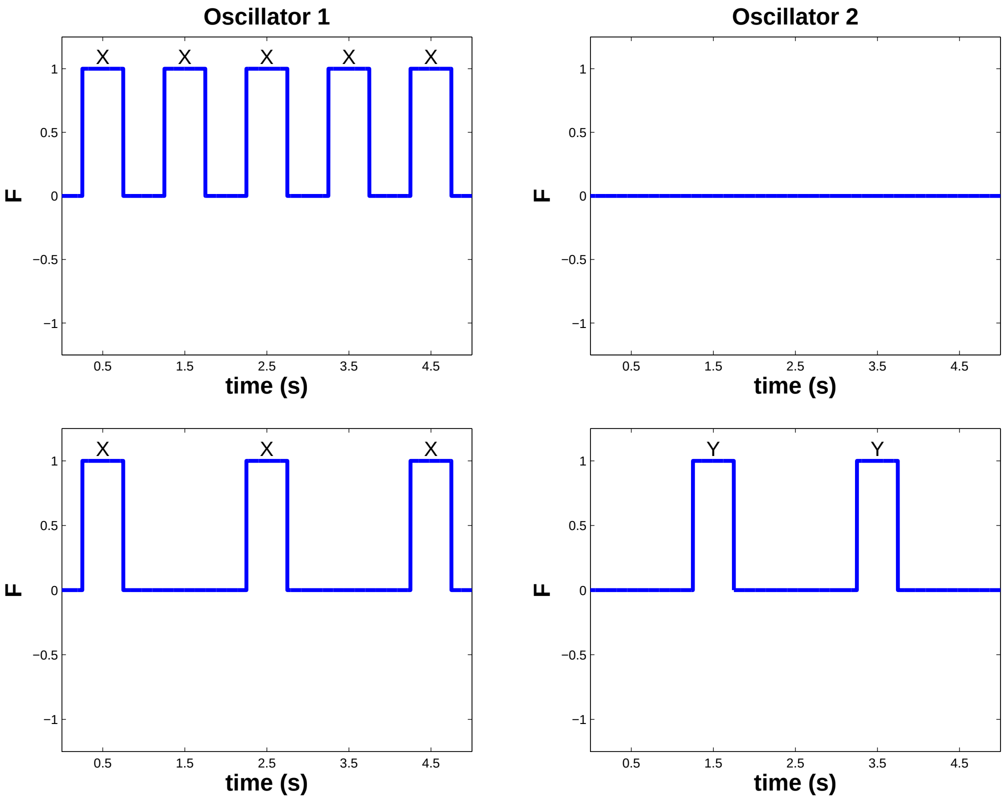

FIGURE 4.20: Example forcing functions for two coupled oscillators in standard coordinates. Top two panels show the sequences RRRR and bottom two AAAA. Left panels represent the forces applied to one oscillator - $F_1(t)$ - and right panels the other oscillator - $F_2(t)$. Notice how, in contrast with the case of a single oscillator, the distinction between stimuli is now made by applying the respective pulse to a particular oscillator.

between the amplitude of the two oscillators at any given moment. This way of handling the coupling has its roots in the case of two masses connected by a spring, and is unlikely to be physically realistic for our purposes. However, it is equivalent to a coupling proportional to the amplitude of the other oscillator, and is convenient for reasons made clear below. The functions $F_1(t)$ and $F_2(t)$ represent the sequences of stimuli in a manner different from that which was shown for the case of a single oscillator. Figure 4.20 shows how $F_1(t)$ and $F_2(t)$ look like for the sequences RRRR and AAAA. Note how all pulses have a positive sign: in the case of RRRR all pulses are applied to the same oscillator; for AAAA the oscillator which is forced alternates.

The system in (4.15) is coupled since y_1 depends on y_2 and vice-versa. A more useful description of the system can be achieved by switching to canonical or normal mode coordinates. A system with two oscillators has two normal modes: the first is revealed by adding the two equations in (4.15), and the second by subtracting them. We then define new coordinates $\{q_1, q_2\}$ such that $q_1 = \frac{y_1+y_2}{\sqrt{2}}$ and $q_2 = \frac{y_1-y_2}{\sqrt{2}}$. Equations (4.15) are thus transformed into

$$\begin{aligned} \ddot{q}_1 + \gamma\dot{q}_1 + \omega_0^2 q_1 &= F_1(t) + F_2(t) \\ \ddot{q}_2 + \gamma\dot{q}_2 + (\omega_0^2 + 2k)q_2 &= F_1(t) - F_2(t) \end{aligned} \tag{4.16}$$

where a normalising constant $\frac{1}{\sqrt{2}}$ is omitted. Note that the two equations in 4.16 are now *uncoupled*. In other words, the system now consists of a set of two completely independent oscillators, each describing the motion of a normal mode of the system. Here the usefulness of a coupling proportional to the difference in amplitude is revealed because one of the normal modes will always have a natural frequency equal to ω_0. The first normal mode is sometimes referred to as the pendulum mode, and physically it corresponds to the motion of both oscillators in phase with the same frequency ω_0; the second normal mode is referred to as the breathing mode and corresponds to the motion of both oscillators in anti-phase with frequency $\sqrt{\omega_0^2 + 2k}$.

Note that with the change in coordinates the functions on the right side of (4.16) change as well. Some interesting properties of the way the stimuli are represented now come into effect: firstly, because each pulse is applied to one oscillator only, $F_1(t) + F_2(t)$ is *always the same function*, and consists of a train of five equal amplitude pulses with the same sign; secondly $F_1(t) - F_2(t)$ results in a force applied to q_2 in which the stimuli are distinguished by their sign, much as in the case of a single oscillator discussed before. Figure 4.21 shows an illustration of how $F_1(t) + F_2(t)$ and $F_1(t) - F_2(t)$ look like for the case of the sequences RRRR and AAAA.

Given that $F_1(t) + F_2(t)$ is always the same function, the behaviour of q_1 is always the same no matter which sequence we are considering. This means that q_1 contributes a constant term to the behaviour of the system in terms of the original coordinates $\{y_1, y_2\}$ and can be ignored. Under

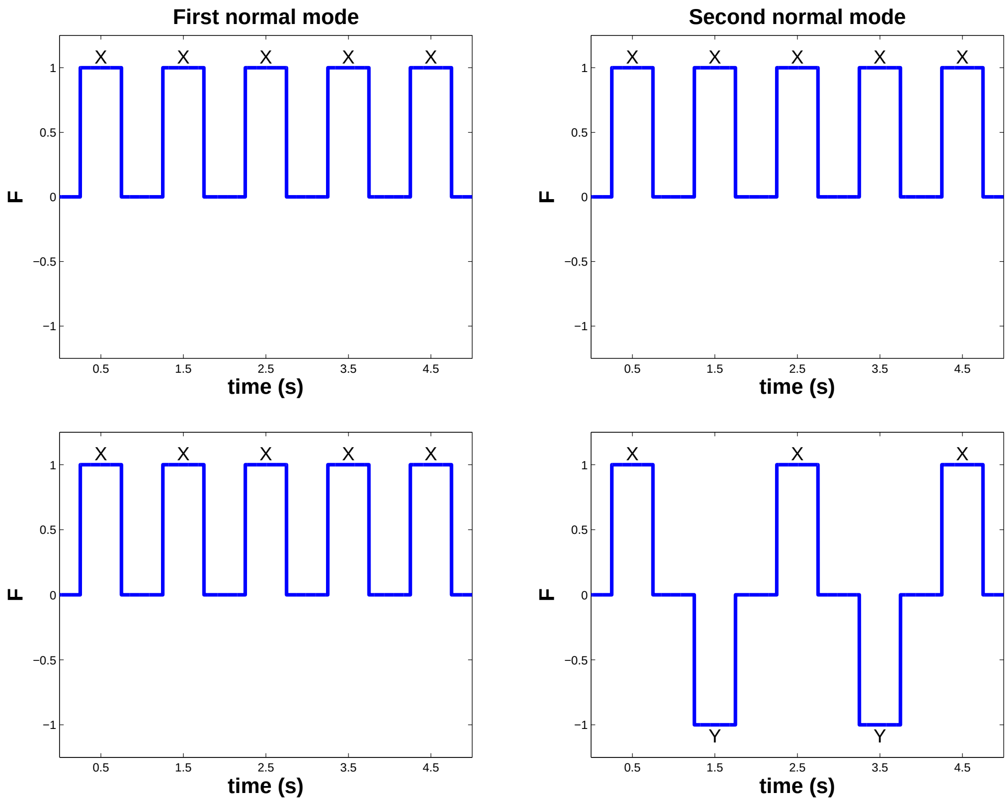

FIGURE 4.21: Example forcing functions for two oscillators in canonical (i.e. normal mode) coordinates. Top two panels show the sequences RRRR and bottom two AAAA. Left panels represent the forces applied to the first normal mode - q_1 - and the right panels to the second normal mode - q_2. These forces are consist of transformations of the original forces - $F_1(t)$ and $F_2(t)$ - applied to the original (physical) oscillators and shown in Figure 4.20. The force applied to q_1 is proportional to $F_1(t) + F_2(t)$ and that applied to q_2 proportional to $F_1(t) - F_2(t)$. Note how the force applied to the first normal mode is always the same irrespective of the sequence of stimuli chosen. On the other hand, the second normal mode experiences a string of pulses in which different stimuli are differentiated by sign.

the specific set-up described here, q_2 is driving the dynamics of the system and this is effectively a single oscillator with frequency $\sqrt{\omega_0^2 + 2k}$. In order to recover the same resonance patterns obtained with a single oscillator - REP and ALT - one must simply adjust $\sqrt{\omega_0^2 + 2k}$ so that it matches f_{REP} or f_{ALT}, the results of which are shown in Figure 4.22. In short, we can take the behaviour of a single oscillator where pulses have different signs as being equivalent - up to a constant - to the behaviour of a system with two degrees of freedom where pulses are applied to different oscillators and do so without loss of generality.

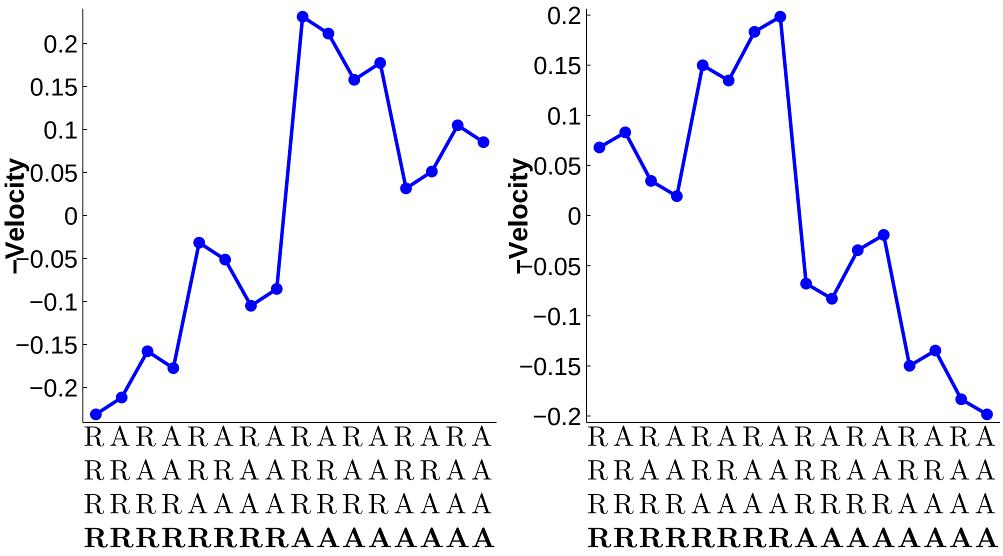

FIGURE 4.22: Resonance behaviour of a system of two coupled oscillators when its second normal mode is set to resonate with either f_{REP} (left panel) or f_{ALT} (right panel). As explained in the main text, the first normal mode contributes a constant value towards the velocity so the behaviour of the system is fully determined by the second normal mode. It is this second normal mode which was made to resonate with either repetitions or alternations by varying k and ω_0 so that $\sqrt{\omega_0^2 + k}$ matches either f_{REP} or f_{ALT}; $\gamma = 1.5$.

A system with two oscillators has two resonance peaks, a fact illustrated in Figure 4.23. However, the way in which the stimuli are represented means that one of the normal modes will make a constant contribution to results no matter what the sequence is. More generally, a system with n oscillators has, by definition, n normal modes with an equal number of corresponding natural frequencies.[14] One possible use of extending the model to include more oscillators would be to distinguish stimuli from the associated responses, which would require four oscillators - two for the stimuli and two for the responses. As the number of oscillators grows so does the number of possible ways in which they are coupled, i.e. the topology of the system. One can also introduce oscillators which are not directly forced but that are coupled to other oscillators which are, for a myriad possible configurations. Ultimately, by taking the infinite limit of the number of oscillators the model can be made continuous in space, in which case it will be described by a partial differential equation. Chapter 5 contains a more detailed discussion of how the model can be extended

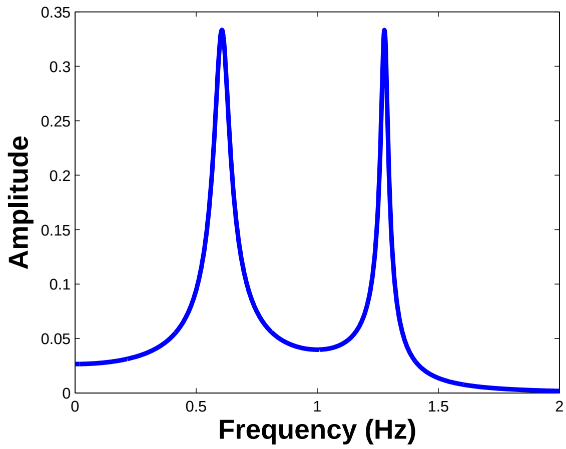

FIGURE 4.23: Amplitude response of a system of two coupled oscillators as a function of the frequency of a sinusoidal force applied to one of the oscillators. Parameters for the example were: $\omega_0 = 1$ Hz; $\gamma = 1.5$; $k = -25$. Note that neither of the two resonating frequency peaks is situated at ω_0 because a coupling proportional to the amplitude of the opposite oscillator was used in this case.

as well as the problems we expect to encounter.

4.3 Assessing the modelling framework

In this section several aspects of sequential effects will be discussed, as well as how the oscillator modelling framework can be used in order to tackle these phenomena. This picture is necessarily incomplete at this stage, and so will be most of the sections which will follow. In a few cases this may be due to limitations of the model itself, such as in the case of RSI dependence and the time-insensitivity of stimulus processing in sequential effects; in other cases, such as reaction time fits, this lack of completeness is largely due to missing empirical evidence. In general, it is argued here that the general framework shows a great deal of potential, particularly since the work presented here should be taken to be a first approximation to what is almost certainly a more complex model. Other limitations of the general approach, as well as possible extensions, are discussed in Chapter 5. Notwithstanding all these issues, the model is successful in capturing some crucial aspects of

[14] Because we only discuss a system with two oscillators, a mathematical treatment of the case of an arbitrary number of oscillators is not given here.

sequential effects such as the centrality of two elements with opposite first order biases as well as a covariance structure similar to that which is observed in reaction time data. But perhaps the greatest strength of the general approach is that that all relevant empirical aspects of sequential effects can be formalised in the the context of this framework, raising the prospect of a complete model of sequential effects based on the principles outlined below.

4.3.1 Covariance structures

In this section we discuss covariance structures obtained by performing a principal component analysis (PCA) on artificial datasets generated by a single oscillator based on simple assumptions about the nature of individual differences. While all efforts will be made to remind the reader of important facts, the following discussion hinges very strongly on Chapter 3 and on understanding the latent structure of sequential effects. Also of particular relevance is the discussion of a heavily and lightly damped oscillators above, and the differences between them. Where repeating facts would be too lengthy or summarising them too confusing, the reader will be referred to the previous chapter or to the sections above.

The objective of the ensuing discussion is not only to demonstrate that the model is successful in replicating key aspects of the latent structure of sequential effects, but also to discuss the relationship between a physical model and associated latent structure and to draw conclusions by analogy with the latent structure obtained from reaction time data. It is hoped that by doing so the meaning and the limits to the interpretation of the latent structure of sequential effects will be made clearer.

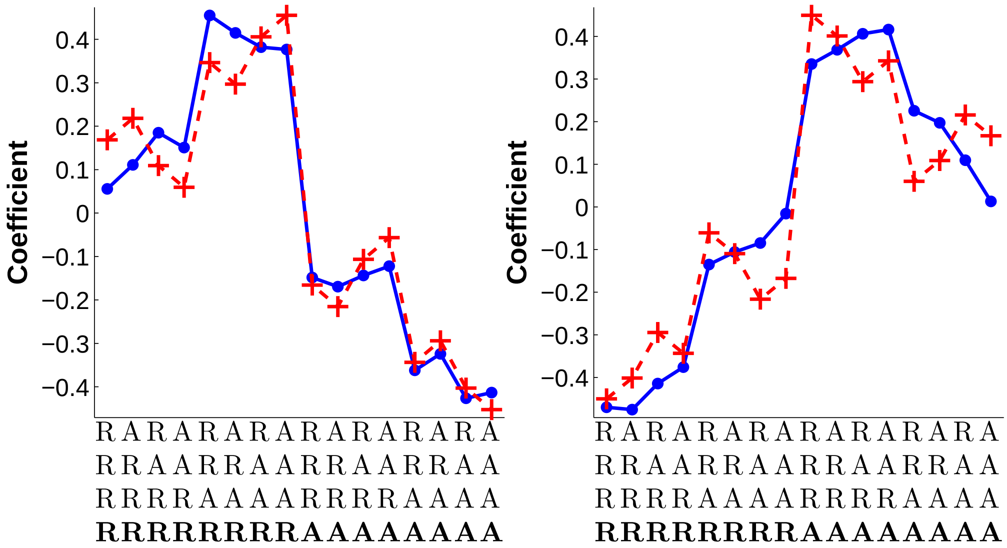

FIGURE 4.24: A comparison of the two main latent components of sequential effects with the two resonance patterns. Solid blue lines - the two main latent components of sequential effects: C3 (left panel) and C2 (right panel). Dashed red lines - REP (left panel) and ALT (right panel) calculated with $\gamma = 1$. The components C2 and C3 shown were rotated using ALT and REP as targets; this rotation also included C1 and C4, for which the same targets as used in Chapter 3 were used, i.e. a constant vector and the results of the second experiment included in Jentzsch and Sommer (2002) respectively. The results of this rotation are virtually the same as when S-LRP and LRP-R were used as targets.

REP and ALT as C2 and C3

Two main latent components responsible for sequential effects - C2 and C3 - were identified in Chapter 3. A psychologically meaningful interpretation for the latent variables encountered was sought by attempting to relate C2 and C3 to evidence available about two processing stages involved in sequential effects, one associated with the processing of stimuli and the other with the processing of responses. One key anchor point for the proposed relationship is the similarity between the patterns of C2 and C3 with the relative contributions of pre-motor processing - S-LRP - and motor processing - LRP-R - respectively towards sequential effects, as measured by EEG (Jentzsch & Sommer, 2002). Here the possibility of a relationship between C2/C3 and REP/ALT is discussed, which would further imply that the two types of resonance might also be related to S-LRP/LRP-R.

Figure 4.24 shows the results of performing a targeted rotation of the latent structure of sequential effects, but this time using REP and ALT as targets, instead of S-LRP and LRP-R as in Chapter 3.[15] Perhaps unsurprisingly, given the overall similarity between ALT/REP and S-LRP/LRP-R respectively, results differ little from those obtained in Chapter 3 (see Figure 4.24). A relationship is therefore proposed here between the two main latent components of sequential effects and the two types of resonance REP and ALT. If both this relation as well as the mapping of latent components and the separate processing of stimuli and responses proposed in Chapter 3 holds, this would further imply that the the two types of resonance might be associated with different neurological loci. As discussed below a relationship between latent components and types of resonance, if it exists, is more complex and nuanced than a simple direct correspondence.

The relationship between the latent structure of sequential effects and the underlying physical truth is hard to evaluate without access to additional empirical evidence about the two processing stages of sequential effects. It is particularly important to elucidate if the relative contribution of both stages - visible in S-LRP and LRP-R - changes with the RSI as well as across different individuals. In the mean time, some progress can be made by studying a physical system able to produce a latent structure similar to that of sequential effects in reaction times.

Individual differences in the context of an oscillator

In order to perform PCA on artificial data generated by an oscillator we must first decide on the way individual differences will be represented in the model. We start with a minimalistic set of assumptions: each individual has a different natural frequency and the corresponding values of ω_0 are distributed uniformly in the interval [0.25 1.25] Hz. In truth the interval [0.5 1] Hz would be enough since it captures all qualitatively different types of behaviour of a single oscillator, but a slightly wider interval will make it clear that the choice of interval is of no importance. Each case

[15] The targets used for C1 and C4 were the same as before. See Chapter 3 for details.

in the resulting artificial dataset will consist of a 16-long vector of velocities obtained by running a single oscillator model in the usual way for a particular value of ω_0.

Data from a heavily damped and a lightly damped oscillator, i.e. with $\gamma = 3$ and $\gamma = 0.6$ respectively, will be analysed in turn; these are the same values used in the discussion of both systems above. For each value of γ two hundred random values of ω_0 will be drawn. Latent structures will be obtained by performing PCA on the two resulting datasets. In some cases the latent structure will be left unrotated and in other cases a targeted procrustes rotations (see Chapter 3) will be performed with a variety of targets made clear as the discussion progresses. Finally, the latent variables resulting from PCA conducted on oscillator data will be designated as O1, O2, and so on, in order to avoid any possible confusion with the components obtained from reaction time data - C1, C2, C3 and C4.

The heavily damped case

Studying the latent structure of a heavily damped oscillator will serve as a form of control for the more psychologically relevant lightly damped oscillator. This control is necessary because both systems display very similar covariance structures when two components are retained, despite marked differences in the behaviour of the two systems (see above), and it is important to understand why this is the case.

We begin by analysing the latent structure obtained with PCA before any rotation. Only two relevant components explaining non-negligible amounts of variance were obtained - O1 and O2 - which explain 94.9% and 4.7% of the variance respectively for a total of 99.6%. These two components are shown in Figure 4.25 together with the two main components of sequential effects - C2 and C3 - also before any rotation. Much like in the case of empirical data, it would seem that the first component before rotation reflects first order effects - i.e. the effect of the last event

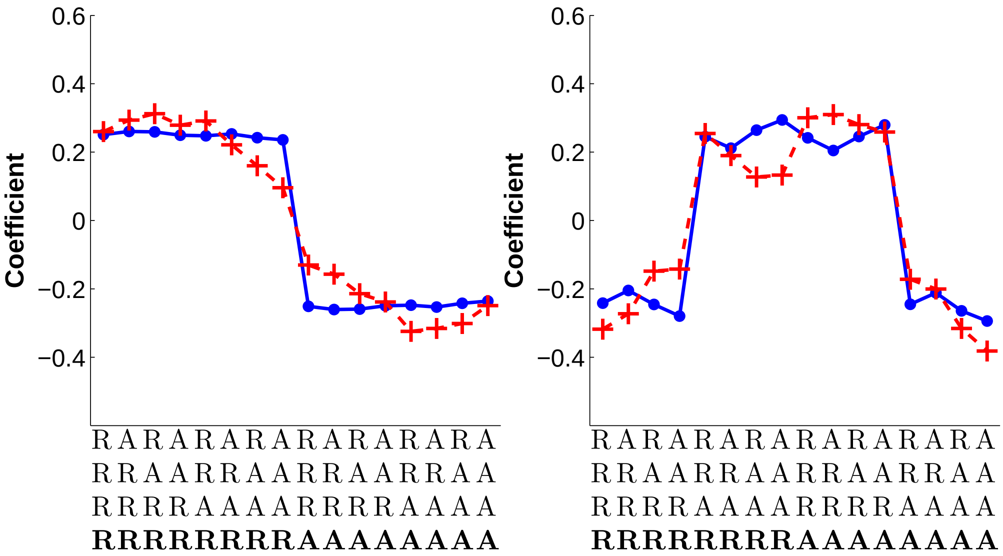

FIGURE 4.25: Latent structure of a heavily damped system before rotation. Solid blue lines - O1 (left panel) and O2 (right panel), the two first latent components identified with PCA run on an artificial dataset generated by drawing two hundred random values of ω_0 and calculating model results with $\gamma = 3$. Dashed red lines - the two main latent components of sequential effects identified in empirical reaction time data - C3 (left panel) and C2 (right panel) - before any rotation. O1 and O2 explain 94.9% and 4.7% of the variance present in the artificial data respectively, for a total of 99.6%. Note how O1 and O2 in this case can be understood as the effect of the last and second-to-last events respectively on model results.

and whether it is a repetition or an alternation - and the second component represents the effect of the second-to-last event. The significant difference in variance explained by O1 and O2 reflects the steep exponential decay determined by the large value of γ chosen in this case. That O1 and O2 together explain virtually all the variance in the data reflects the fact that the third-to-last event does not contribute in any significant manner towards the velocity of the oscillator under heavy damping conditions.

The point was made before in Chapter 3 that, while the unrotated C2 and C3 can be mapped onto two simple concepts such as first and second order effects, this is not necessarily the most relevant basis for the system. A similar point can be made about O1 and O2, the difference being that we now *know* the most relevant basis to consist of REP and ALT, not first and second order effects. Since we have access to this information we can use it to perform a targeted rotation on O1 and O2 using REP and ALT as targets, the results of which are shown in Figure 4.26. The almost

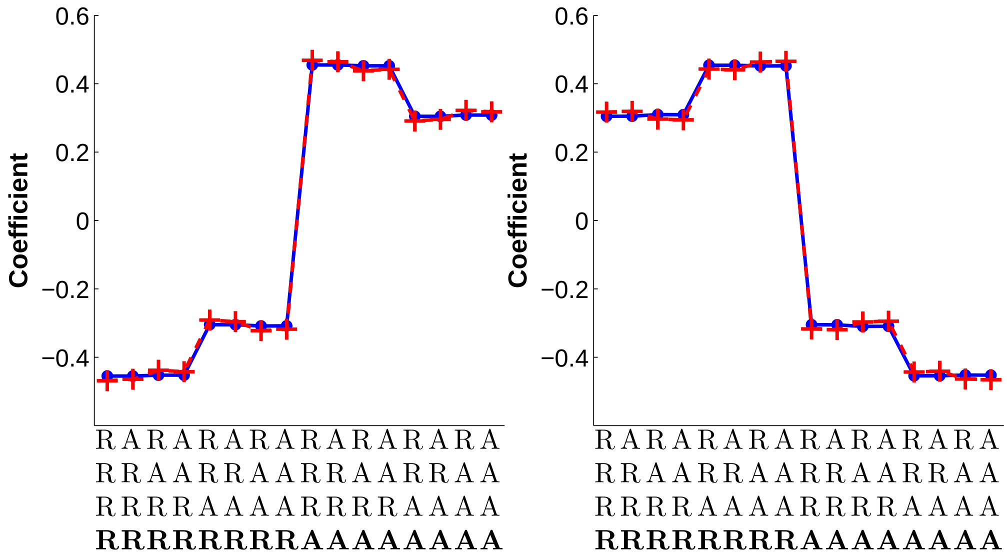

FIGURE 4.26: Latent structure of a heavily damped system after rotation. Solid blue lines - first two latent components - O1 (left panel) and O2 (right panel) - identified with PCA run on an artificial dataset generated by a single heavily damped oscillator with $\gamma = 3$. Dashed red lines - REP (left panel) and ALT (right panel) obtained with $\gamma = 3$. That the two latent components can be rotated to look almost exactly like the two types of resonance reflects the fact that a heavily damped system is well approximated by a linear combination of REP and ALT (see main text).

perfect fit between the latent components and the two types of resonance was expected since we knew the system behaved approximately like a linear combination of REP and ALT (see above). However, finding the most relevant rotation of O1 and O2 depended on knowing beforehand what the real basis of the system is, an obvious impossibility in the case of empirical data.

The lightly damped case

The first two components - O1 and O2 - identified in data from a lightly damped oscillator are effectively the same as those in the heavily damped case, and correspond to first and second order effects respectively. However, as expected from the longer memory span determined by a lower γ, two extra components - O3 and O4 - can be be found which explain non-negligible amounts of variance, and which can be understood as the effects of the third-to-last and fourth-to-last events[16] (see Figure 4.27). As expected, the share of the variance explained by the first two components

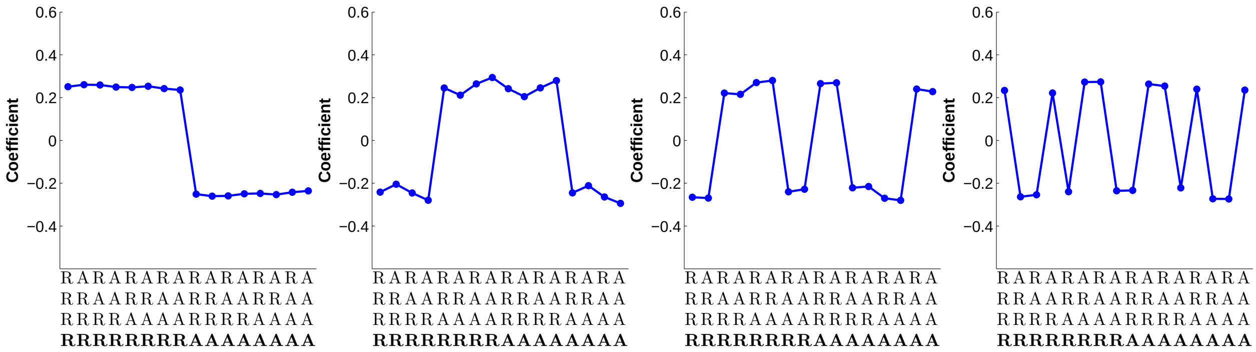

FIGURE 4.27: Latent structure of a lightly damped system before rotation. Panels show first four latent components identified with PCA on a dataset obtained by drawing two hundred random values of ω_0 and calculating model predictions in the usual way (see main text). The four components shown - O1, O2, O3 and O4 - are ordered from left to right in terms of decreasing variance explained, which is 52.1%, 24.1%, 15.4% and 8.2% respectively for a total of 99.8% of variance. Note how the unrotated components can be understood as the effects of the last, second-to-last, third-to-last and fourth-to-last events respectively for O1, O2, O3 and O4.

O1 and O2 is now reduced to 76.3%, with O3 and O4 explaining 15.4% and 8.2% of variance respectively for a total of 99.8% for all four components. Note that while in the case of a heavily damped oscillator the space of results was two dimensional, it is four-dimensional in the case of a lightly damped oscillator.

As with the case of a heavily damped oscillator we have no reason to believe that the best basis for the system in this case is the one shown in Figure 4.27. One the other hand, while we have a strong expectation that REP and ALT will be part of the real basis, it is not so clear what the remaining two elements will be. Presumably the two extra components reflect contributions by minor frequencies in the input, in which case they could perhaps be estimated by shifting ω_0 in such a way that the band filtered by the oscillator now lies in the intermediate area between f_{REP} and f_{ALT}. A visual inspection of the spectra of the different sequences of stimuli reveals several minor frequency peaks observed in between f_{REP} and f_{ALT}, mostly centred around 0.6 Hz and 0.8 Hz, or sometimes 0.75 Hz (see Appendix F). Fortunately, the width of the filtered band in this case is such that it is possible to *partition* the interval between f_{REP} and f_{ALT} in approximately equal

[16] One way to visualise this is to note that the 'jumps' in the magnitude of the coefficients occur between sequences which differ in the third-to-last event in the case of O3, and fourth-to-last event in the case of O4.

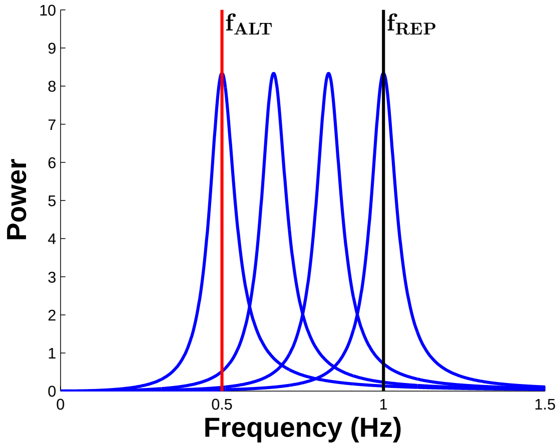

FIGURE 4.28: Illustration of the model parameters used in order to generate the targets for the rotation of the latent structure of a lightly damped oscillator. Solid blue lines - resonance curves for four oscillators with $\gamma = 0.6$ and the following values of ω_0: 0.5, 0.66, 0.83 and 1 Hz. Red and black vertical lines show the position of f_{ALT} and f_{REP} respectively. Note how this choice of parameters results in a partition of the interval between f_{ALT} and f_{REP} in terms of the frequency bands filtered by each value of ω_0.

stretches (see Figure 4.28 for an illustration of this point). So as targets for the rotation we will use the results obtained by setting ω_0 to 0.5 (f_{ALT}), 0.66, 0.83 and 1 Hz (f_{REP}), the intermediate values chosen in order to equally partition the interval between f_{REP} and f_{ALT}. The rotated latent structure, together with the respective targets, is shown in Figure 4.29. The excellent fit reflects the fact the four frequency bands used cover all the qualitative types of behaviour of the lightly damped system. By further decreasing γ it may be possible to generate a more complex latent structure with more than four components.

In general, it should come as no surprise that almost perfectly fitting bases were found for both the heavily and the lightly damped systems. In both cases the system is fully deterministic with no artificial noise introduced, under which conditions any component identified with PCA *must* be meaningful. Real data however is noisy and not every latent component is expected to have a valid interpretation. Next some of the implications of our study of latent structures of artificial datasets for the empirical latent structure of sequential effects are discussed.

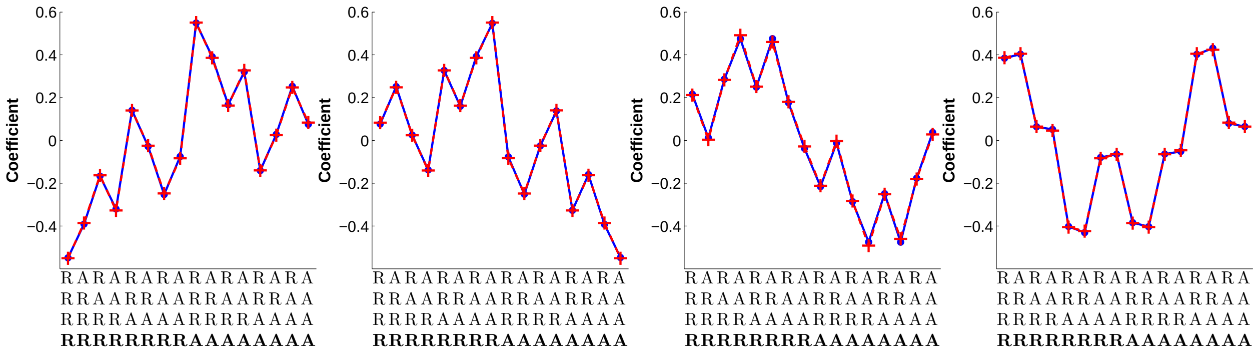

FIGURE 4.29: Latent structure of a lightly damped system after rotation. Solid blue lines - first four latent components of an artificial dataset generated from a lightly damped oscillator. Red dashed lines - targets used for rotation of the latent structure shown for comparison. The targets were the results of running the oscillator model with $\gamma = 0.6$ for the following values of ω_0 (from left to right): 0.5, 0.66, 0.83 and 1 Hz (see main text and Figure 4.28 for an explanation of this choice of values.)

Implications for the latent structure of sequential effects

We have seen in the previous section how the latent structure of oscillator generated data is related to damping, which determines the width of the band of frequencies filtered, as well as to the frequency content of the input. The natural question to pose is: since we expect humans to be closer to a lightly damped system, are minor frequency components also present in reaction time data? Recall that three components related to sequential effects were identified in Chapter 3 - C2, C3 and C4 - which explained, before rotation, 12%, 4.2% and 1.25% of variance. The reason for these relatively small values was that the first component - C1 - while unrelated to sequential effects, accounted for 78% of variance.

It is possible to estimate the variance explained by the four latent components of the lightly damped oscillator if a component analogous to C1 was present which explained a similar amount of variance as in the empirical case. A crude linear extrapolation results in estimates of variance explained by the four components - O1 through O4 - of 11.4%, 5.3%, 3.4% and 1.8% respectively, reflecting the fact that these are now forced to account for only 22% of the total variance. These values imply that meaningful components would explain minute amounts of variance, and would likely be rejected by any traditional method used to choose the number of components to retain such as scree plot inspection. In fact, in the analysis conducted in Chapter 3, theoretical arguments had to be invoked in order to justify the choice of retaining C4, and even C3 would be at risk of being rejected by scree plot analysis.

So if minor latent components corresponding to intermediate frequencies were present in empirical data, not only would they be expected to explain tiny amounts of variance, but could quite possibly be drowned in noise. One possibility would be to repeat the analysis performed in Chapter 3 by retaining an extra two components - for a total of six - and using the two extra targets used to rotate the later structure of the lightly damped oscillator (see Figure 4.29, two rightmost panels). However, this procedure is formally unsound as it could lead to over-fitting and is therefore not shown here.[17] In the future, with a greater reaction time dataset, it may be possible to validate the presence of relevant minor components in empirical data.

Closely related to whether there are extra components in reaction time data is the question of what would happen if meaningful components were left out. Consider the case of the lightly damped oscillator: what would happen if we chose to retain only two components when we actually know the underlying truth to be four? Let us assume for the sake of argument we did not know the origin of the oscillator data, and that our hypothesis was that there might be two meaningful components related to S-LRP and LRP-R, much as was the case in the analysis performed in Chapter 3. The result of choosing to retain two components - O1 and O2 - and using S-LRP and LRP-R as targets for the rotation of the two components is shown in Figure 4.30, together with the two latent components of sequential effects - C2 and C3 - rotated against the same targets.

One possible interpretation of the effects of discarding the two minor latent components O3 and O4 is that the two main components O1 and O2 - corresponding to ALT and REP - were reduced to their first and second order influences. This makes sense if we think that the two first

[17] This analysis was nevertheless conducted with a good qualitative fit to all targets. Some indication that this result might be sound is given by the fact that using random vectors as targets for a fifth and sixth components - or even just one of them - does not produce any appreciable fit. Nevertheless, this analysis should be considered to be highly preliminary.

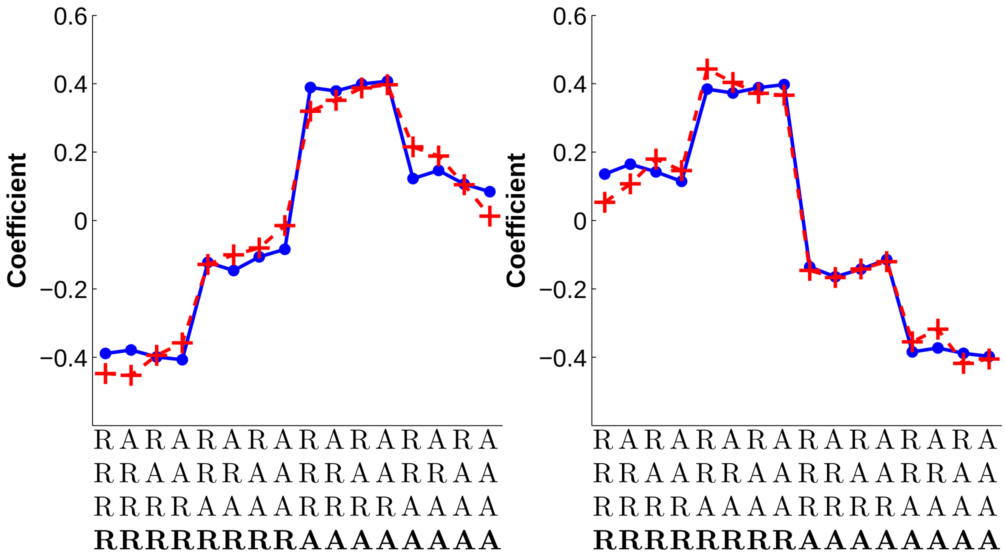

FIGURE 4.30: Effect of underestimating the number of latent components in the latent structure of a lightly damped oscillator. Solid blue lines - O1 (left panel) and O2 (right panel). Dashed red lines - the two main latent components of sequential effects - C3 (left panel) and C2 (right panel). Both O1/O2 and C2/C3 were rotated to match the same targets, i.e. S-LRP and LRP-R respectively. Note how similar O1 and O2 are to C2 and C3 when the number of components present in the oscillator data is underestimated.

components before rotation correspond to first and second order effects (see above): any attempt to rotate only the first two components - O1 and O2 - must stay within the same subspace. So rotating O1 and O2 to match REP and ALT while leaving out O3 and O4 results in the reduction of the two resonance patterns to their first and second order contributions.[18]Visually, the absence of third and fourth order contributions by REP and ALT results in four flat ‘plateaus’ depending on the four possible combinations of the last two events - RR, AR, RA and AA. A reduced dependence on three- and four-back events is also displayed by C2 and C3, shown in Figure 4.30 for comparison, which could possibly reflect the leaving out of extra meaningful components.

In the last few sections PCA was performed on artificial datasets generated by both a heavily damped and a lightly damped oscillator, and showed that in both cases it is possible to relate the latent structure obtained to the frequency content of the input. By extrapolating to the case of the empirical reaction time dataset analysed in Chapter 3 it was speculated that there may be other

[18] Mathematically this is the projection of REP and ALT when taken as vectors onto the subspace of first and second order effects.

meaningful latent variables contributing towards sequential effects, although it is preliminary at this stage to draw any clear conclusions. In general, it was shown that an oscillator system can display latent structures very similar to that which is observed in the empirical dataset, particularly if only two components are retained (see Figure 4.30). Next the possibility of fitting reaction time data directly is discussed.

4.3.2 Reaction time fits

No attempt will be made here to analyse in detail the fit of a single oscillator model to RT data, the main reason for this being the highly unrealistic assumption of a system with one degree of freedom. The fit of a system with several oscillators could be evaluated, but there are several complicating factors associated doing this. In this section the reasons why it is likely that the system underlying sequential effects has multiple degrees of freedom will be discussed, followed by a discussion of the difficulties associated with introducing additional oscillators. It should be kept in mind that a model can be found that fits reaction time data, but that this fit would be uninformative and possibly even misleading in the absence of a clear interpretation of the parameters involved. Moreover, while there are some fundamental properties of the class of systems considered here which will be argued to be useful in modelling sequential effects, there are still some fundamental gaps which must be filled before a full model is proposed; chief amongst these are the insensitivity of the pre-motor stage to the RSI and the difficulties encountered when attempting to reproduce the dependence of sequential effects on the RSI.

Some indirect evidence for a system with more than two degrees of freedom underlying sequential effects comes from the fact that humans often display a reaction time pattern best explained by a sum of two components with opposite first order effects, such as the cost-benefit - or inverted ‘v’ - pattern of results commonly observed when a long RSI is used. This type of result seems to imply that a component with a repetition bias and one with an alternation bias *co-exist*, and this is not possible with a single oscillator because such a system is either resonating with repetitions or alternations, never both at the same time. So a system with at least two degrees of freedom is necessary, one to detect repetitions and one for alternations. Loosely speaking, this discussion finds a striking parallel in the argument reiterated over the years by several authors that detecting repetitions and alternations at the same time requires two separate mechanisms (D. Hale, 1969; Maloney et al., 2005). The suggestion here - preliminary at this stage - is that the apparent separation between mechanisms takes root in the independence of the normal modes of dynamical systems.

Expanding the system to include any number of oscillators is straightforward, as demonstrated above for the case of two coupled oscillators. However, in order to design a system capable of resonating with both repetitions and alternations, at least three oscillators would be necessary. The reason for this is that irrespective of the number of coupled oscillators there will always be a normal mode - corresponding to the motion of all oscillators in phase - which does not make any contribution towards sequential effects, much like the pendulum mode of a two oscillator system (see above). If we consider three oscillators, and assume two to receive the input in the form of stimuli-response pairs, then the third oscillator must be left free, in which case a decision must be made about how it is coupled to the remaining oscillators, i.e whether it is coupled to both or just one. Another option would be to consider four oscillators, all of which receive input representing stimuli and responses separately, but then even more complicated decisions must be made about the topology of the network of coupled oscillators. In addition, note that so far the discussion about oscillators has been kept largely abstract and in terms of possible patterns of behaviour of one or two oscillators; expanding the system to include more oscillators would imply that additional aspects of the model must be made clear, such which oscillator's velocity correlates to reaction time.

Notwithstanding all the considerations above, a system with enough degrees of freedom to allow for resonance with repetitions and alternations at the same time is expected to display a

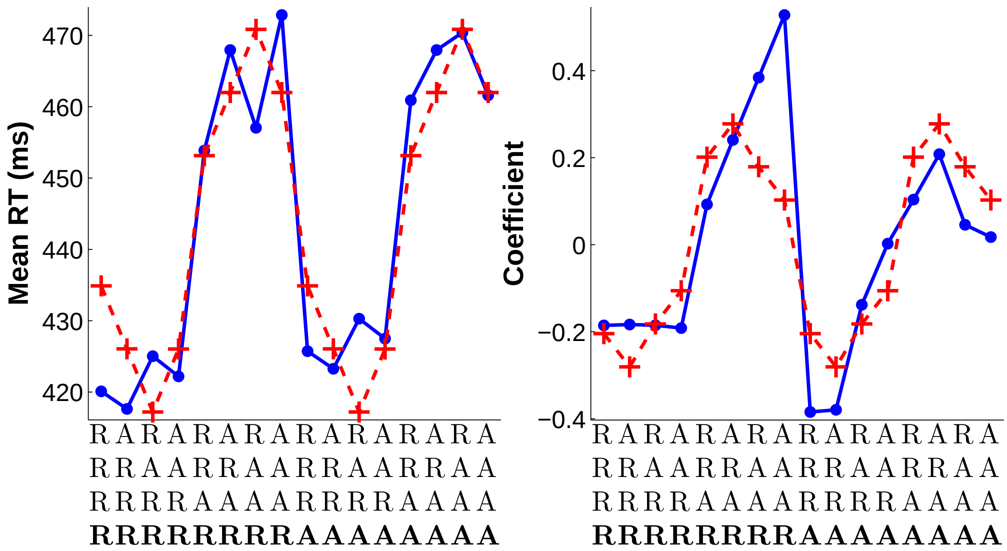

FIGURE 4.31: Fits of a single oscillator with the last pulse removed. Solid blue lines: C4, the minor latent component of sequential effects identified in Chapter 3 (left panel); RT data the second experiment conducted by Jentzsch and Sommer (2002) (right panel). Dashed red lines: best fitting results of a single damped oscillator with the last pulse (i.e. stimulus) removed. $\gamma = 0.6$ and $f_{REP} = 1$ in both cases. Best fitting values of ω_0 (in Hertz) were 1.14 and 0.4 for the data shown in the left and right panels.

reasonably good fit to reaction time, inasmuch as such a system is capable of producing pattern of results similar to REP, ALT or a combination of both, and even extending to the benefit-only pattern of sequential effects observed when a short RSI is used. Such an attempt at building a more complex system should nevertheless be done in a principled manner and with further empirical evidence.

We proceed with an analysis of how an oscillatory system may be of use in explaining the minor latent component of sequential effects - C4 - identified in Chapter 3, and associated empirical results displaying the same pattern.

4.3.3 Unusual patterns of sequential effects and C4

We have seen that a single damped oscillator can produce latent components similar to the two main components - C2 and C3 - of sequential effects in human RT data. However, there is one last component of sequential effects - C4 - in need of explanation, together with the empirical results which display a similar pattern (Jentzsch & Sommer, 2002; Melis et al., 2002). As discussed in Chapter 3, the pattern of C4 points to a dependence on the second-to-last event, irrespective of the last one. Also in Chapter 3 it was suggested that C4 was due to a combination of C2 and C3 stemming directly from the second-to-last event as if the last event had never happened. Since C4 occurs primarily when RSI is short, it was further suggested that its occurrence was due to processing delays only made evident when processing capacity was under greater pressure. In the ensuing discussion the possibility of a mapping between C2/C3 and ALT/REP respectively is implied.

In Chapter 3 the influence of C2 and C3 as if the last event never happened was estimated, which resulted in two new patterns C2* and C3*, a combination of which with $C3^* \propto -C2^*$ was shown to provide a good fit to C4. Estimating what REP and ALT would look like if the last event never happened is easy with an oscillator: all that is required is that we apply only the first four pulses of each sequence, leaving out the last one. Velocity measurements will again be taken at the beginning of the last pulse, which is now the fourth. By analogy to the case of C2 and C3, the equivalents of ALT and REP if the last event is left out will be referred to as ALT* and REP*. For now the question of what kind of mechanism can result in the absence of a significant influence of the last event will be ignored (see below for a discussion of this point).

Recall that the results of a heavily damped oscillator are well described by a combination of ALT and REP (see Figure 4.17). By analogy, leaving out the last pulse turns the system into a combination of ALT* and REP*. Under these circumstances, simply leaving out the last pulse and varying ω_0 might result in a combination of ALT* and REP* which resembles C4. It is perhaps not so intuitive that the same should be true of a lightly damped system since in this case results are no longer well approximated by a combination of ALT and REP (see Figure 4.19). Nevertheless, it turns out that it is possible to obtain a pattern resembling C4 with a single lightly damped oscillator, as shown in Figure 4.31. The fact that a pattern similar to C4 can be obtained with just a single oscillator is an interesting observation, but the real C4 is more likely to be generated by a system with more than one degree of freedom.

Next we turn to a discussion of what is perhaps the most complex aspect of sequential effects: their dependence on the RSI. As it turns out, this is also the most difficult aspect to capture within the oscillatory framework proposed here.

4.3.4 Dependence on RSI

The dependence of sequential effects - and associated latent structure - on the RSI is a complex issue. Recall that in a 2AFC stimuli usually remain on-screen until a response is made, and so $RSI + RT = ISI$ - where ISI stands for 'inter-stimulus interval'. On the other hand, the overall RT remains relatively constant as the RSI is varied, with only a slight increase when a very short RSI is used (Soetens et al., 1985). Therefore, reducing RSI implies not only shortening the ISI - thereby decreasing the average frequency of the stimuli - but also changing the proportion of time dedicated to the stimulus presentation. Assuming an average 300 ms reaction time, a 50 ms RSI would imply that the stimulus is on-screen 86% of the time; conversely, for an 800 ms RSI this would be 28% of the time. If anything, this effect is made worse by the fact that the average RT of subjects tends to increase to a small extent for low RSI values.

Both the changes in frequency as well as the relative proportion of the ISI taken up by the RT result in changes to the spectral content of the sequences of pulses and consequently for the behaviour of the oscillator. In broad terms, increasing the frequency of the stimuli widens the base frequency interval which repeats itself harmonically at higher frequencies (see Figure 4.6), whereas reducing the frequency narrows this interval. The overall effect of increasing the proportion of the ISI dedicated to the RSI - i.e. the relative width of the pulses - is to decrease the power of the harmonic repeats; conversely, narrowing the relative width of the pulses increases the power of

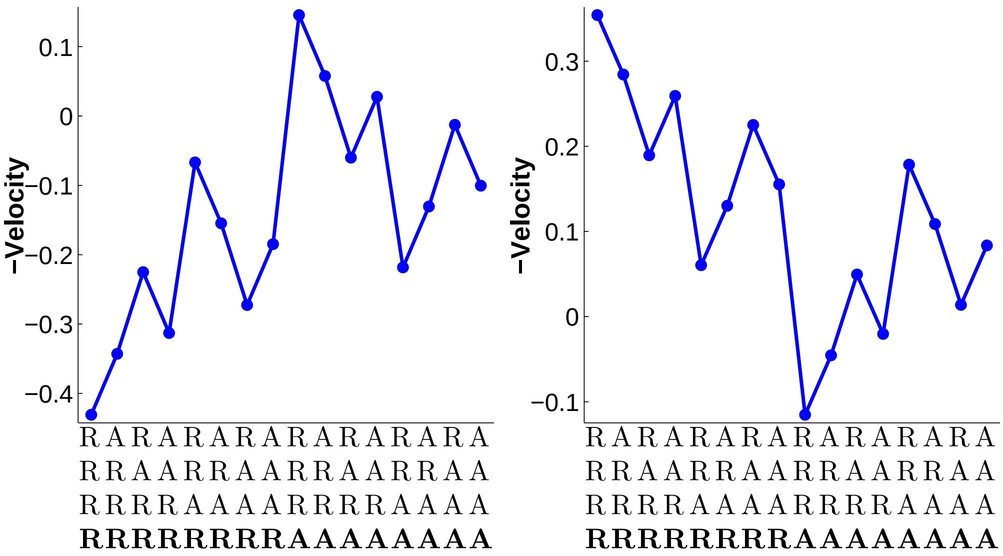

FIGURE 4.32: Spurious inversion of REP observed when the RSI is short. Left panel - the usual REP obtained by using a ratio $RSI : RT$ of 4:1; Right panel - inverted copy of REP obtained by using a ratio $RSI : RT$ of 1:4. All other parameters are equal in both cases: $\gamma = 0.6$, $f_{REP} = 1$ and $\omega_0 = 1$ (in Hertz). Note how varying the relative proportion of the ISI taken up by the stimulus results in a pattern resembling a copy of REP with a negative sign.

the harmonics; in the limit case where pulses consist of Dirac delta functions the base frequency interval repeats itself with no loss in power. The effects of these changes on the behaviour of the oscillator are complex and will not be discussed here in detail.

A more serious issue with modelling the dependence on the RSI stems from the assumption that RT is proportional to the velocity of the oscillator at the beginning of the last pulse. As discussed above, shortening the RSI increases the relative width of the pulses and this in turn affects the point at which velocity measurements are taken, defined to be the beginning of the last pulse. The qualitative effect of this shift in the point where measurements are taken depends on ω_0 and it is not the same in all cases. However, when $\omega_0 = f_{REP}$ or $\omega_0 = f_{ALT}$ - the conditions under which REP and ALT emerge - the shift in measurement point results in a perfect inversion of REP and ALT, as illustrated in Figure 4.32 for the case of REP.

The inversion of REP/ALT due to the relative width of the pulses is likely to be an artefact since it is not observed empirically as the RSI is lowered, and this exposes some limitations of the underlying assumptions built into the model. Pragmatically, it makes an analysis of the dependence of oscillator results on the RSI unrealistic for the range of parameters used in empirical research, particularly for the often used 50 ms RSI. Note that the spurious inversion of REP and ALT begins to occur when the ratio RSI/RT is above 1, whereas if we consider an average reaction time of 300 ms and a 50 ms RSI this implies a ratio of RSI/RT equal to 7.

One possible solution to the spurious inversion problem is to represent stimuli as Dirac delta functions situated at the beginning of each pulse. This would imply that the forcing of the oscillator is now proportional to positive shifts in the intensity of the stimuli relative to the background, rather than to the stimuli intensity itself. Preliminary results show that using Dirac delta functions as stimuli differs little from results obtained using square pulses, while at the same time abolishing the problem of spurious inversion of REP and ALT for short RSI. In any case, rigorously studying the dependence of oscillator results on the RSI depends on further assumptions being introduced in the model. Finally, an analysis of the dependence of the latent structure of oscillator data on the RSI would further depend on the choice of distribution for ω_0 as well as the number of degrees of freedom - i.e. number of coupled oscillators - and configuration of the system, and should be informed at a later stage with better empirical evidence.

4.4 General discussion

4.4.1 Covariance structures and underlying physical model

The possibility was discussed in Chapter 3 that sequential effects reflect two independent contributions associated with the processing of stimuli on the one hand, and of responses on the other hand. This hypothesis is partly based on the observation that the two main latent components identified in RT data - C2 and C3 - can be rotated to display patterns similar to the best evidence available about the relative contributions of stimulus and response processing: S-LRP and LRP-R. Also in Chapter 3 the possibility was discussed that different patterns of sequential effects are the product of a simple combination of a linear combination of patterns looking like C2 and C3, or correspondingly S-LRP and LRP-R. On the other hand, it was discussed above that even a single lightly damped oscillator can produce two latent components very similar to C2 and C3 (see Figure 4.30) if minor components are erroneously left out. Together these facts outline two competing possibilities: (1) sequential effects are the product of a simple linear combination of two elements resembling C2/C3 or S-LRP/LRP-R; or (2) the PCA analysis conducted in Chapter 3 left out relevant minor components. Arguments for both hypotheses will be discussed in turn.

In favour of only two fundamental components of sequential effects is the fact that, before rotation, minor latent components reflecting the effect of three-back and four-back events were not identified in RT data. By contrast, such components are clearly visible in the results of a lightly damped oscillator (see Figure 4.27, two rightmost panels). This points to a two-dimensional space for sequential effects - setting aside C4 for now since this is thought to be a by-product of the two main components - in which case sequential effects might be the product of just two components. Also in favour of only two components is the fact that a combination of C2 and C3 (together with C4) provides a good qualitative fit to individual RT data (see Appendix C). The view of sequential effects as a simple linear combination of two elements raises some additional problems if we think of the two main components of sequential effects as reflecting the two types of resonance REP and ALT: the fixed nature of relative contributions by the two elements would imply that the system is resonating irrespective of the frequency of the input, a point which will be discussed in greater detail in Chapter 5.

The second hypothesis states that the dimensionality of the space of sequential effects was in fact underestimated. In this case, the reduced dependence of C2 and C3 on events beyond the last two may be considered in and of itself as providing evidence for the fact that meaningful components were left out (see above for an explanation of why this is the case). Extra components, if they are present, are expected to explain small amounts of variance in the data, in which case confirming or refuting the second hypothesis will only be possible with a greater dataset of individual differences or alternatively with a solid theory of sequential effects.

4.4.2 The mechanism behind C4

It was shown above that even a single oscillator can produce a behaviour similar to C4. However, this was done by assuming the influence of the last stimulus to be absent. The fact that reaction times are apparently independent of the last event seems almost paradoxical, since this last event is the ones subjects are presumed to be responding to. It is hard to see how this could be possible without postulating some sort of decoupling between the mechanism triggering a decision and the mechanism behind generating expectations. In this scenario, the information responsible for expectations, and therefore reaction time, would be integrated at some point with the decision making process. Assuming the decision process is faster than the expectation generating process, any delay in the latter would induce a RT pattern depending on the second-to-last stimulus.

One possibility is that subjects are not in fact responding to the last stimulus, bur rather subjectively *generating* it, in which case the paradox may be resolved. When the RSI is very short - i.e. 50 ms - stimuli appear almost instantly after a button is pressed and this can induce in participants the illusion that the button press caused the next stimulus to appear. Anecdotally, and despite no questionnaire having been applied to measure this effect, several participants performing the experiments analysed in Chapter 3 spontaneously reported confusion as to whether they were responding to stimuli or generating them. These reports only happened with a 50 ms RSI, and never when longer RSI values were used. If the subjects were merely pressing the response button rhythmically in order to ‘generate’ the next stimulus, their responses would not require access to information about the last stimulus, while at the same time possibly reflecting subconscious expectations generated by the sequence of events. Also consistent with this view is the fact that error rates for subjects which display a pattern of results similar to C4 tend to be very high, in some cases approximating 50%.

Another possibility is that C4 in fact never occurs in isolation. The results of Jentzsch and Sommer (2002) seem to suggest it can but, on closer inspection, reveal a small difference in RT to repetition and alternations (see Figure 4.31, left panel). Another set of results closely approximating C4, that of Melis et al. (2002) (see Chapters 1 and 3), reveals a pattern that is best fit by a combination of C4 with a non-negligible contribution by C2 (not shown). It is conceivable that, while expectations are mostly driven by the second-to-last event, decisions are still being made based on the current stimulus, possibly due to greater pressure to respond quickly when the RSI is short, an effect vaguely analogous to the lowering of the decision making threshold in sequential sampling models (Ratcliff & Smith, 2004): subjects with more severe processing delays would attempt to compensate by responding with as little information as possible so as to keep reaction times reasonably low. Subjects with fast processing speed would not need to lower the threshold as much.

The apparent independence of reaction times from the last event could reflect a delay in the integration of different signals stemming from separate neurological areas. One possible way to model this effect in the context of the present framework would be to introduce time delays in the coupling between different oscillators, reflecting the finite speed of signal propagation in the brain. However, modelling such neuronal conduction delays would imply a number of additional assumptions to be built into the model, something which is discussed in more detail in Chapter 5 in the context of future modelling directions.

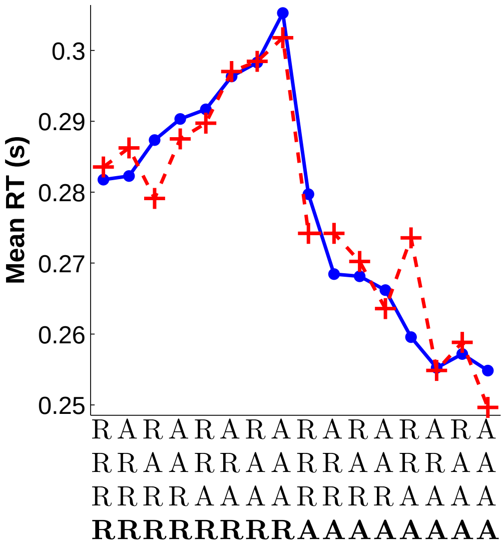

FIGURE 4.33: Results of a 2AFC with a random RSI. This experiment was a 2AFC with two horizontally displaced dots as stimuli but in which the response-stimulus interval for each trail was made random, i.e. drawn from drawn from a uniform distribution in the interval [50 1000] ms. Ten subjects took part in the experiment and the results shown are the mean across all subjects. See Appendix B for details.

4.4.3 Temporal and spatial filtering?

It should be made clear that what was proposed so far is in essence a model of *temporal filtering*, in that results are fully dependent on the arrival at constant intervals of a positive or negative pulse. Put simply, the model depends on there being temporal structure in the input and, once this structure is removed, model predictions will not exhibit any any discernible pattern. This is a problem shared by any other continuous model of sequential effects, though only two have been proposed so far (Cho et al., 2002; Gao et al., 2009). As for the remaining models, their discrete-time nature makes it unclear how, if at all, time intervals could be represented. In order to test if sequential effects in human RT also break down if temporal structure is absent, an experiment was conducted in which the RSI on each trial was itself a random variable, the results of which are shown in Figure 4.33 (the experiment is detailed in Appendix B).

The results of the random RSI experiment demonstrate that sequential effects survive the removal of temporal structure from a 2AFC. Since there is no way to predict the moment the next

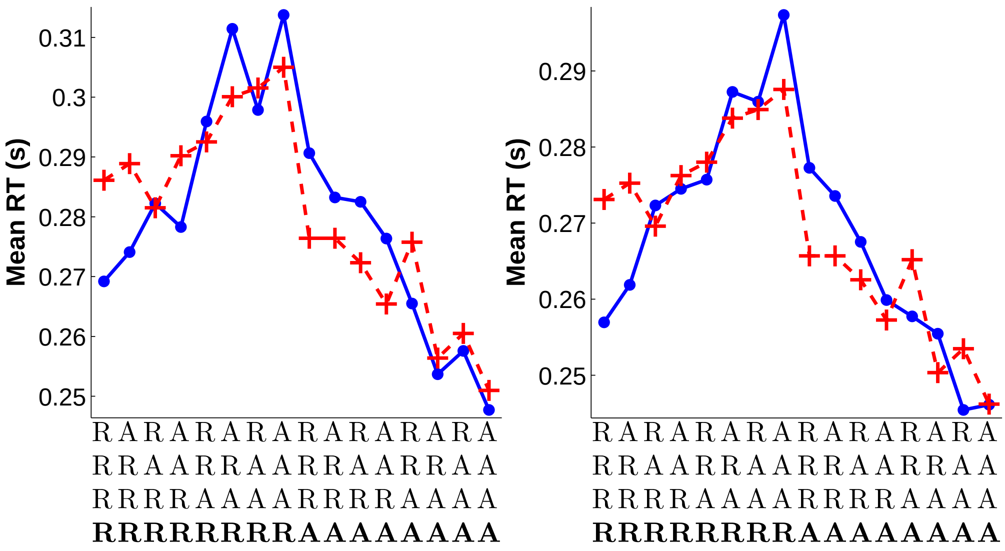

FIGURE 4.34: Results of two experiments conducted with a long - 1000 ms - RSI, both with spatially separate dots stimuli. Solid blue lines - Reaction time results averaged across those subjects performing Experiment 3 with a 1000 ms RSI (left panel) and Experiment 4 also with a 1000 ms RSI (right panel); experiments are detailed in Chapter 3. In the experiment shown on the left panel the stimuli were horizontally displaced dots, and in the experiment on the right vertically displaced dots. Dashed red lines - best fitting linear transformations of S-LRP (both panels). This data is meant to illustrate the convergence of results from experiments with spatially separate stimuli to a pattern resembling S-LRP, the pre-motor processing element of sequential effects which possibly also reflects some form of spatial filtering.

stimulus will occur, one possibility is that the pattern of results shown in Figure 4.33 reflects the spatial structure of the task, since it is possible to predict the location of the next stimulus independently of the moment in time it will appear. This makes sense if one thinks of sequential effects as the product of a pattern detection attempt: humans should be able to detect spatial as well as temporal patterns. In addition, the results of the random RSI experiment show a great deal of similarity with S-LRP, the putative pre-motor or stimulus-associated component of sequential effects (Jentzsch & Sommer, 2002). Therefore, a more complete hypothesis is then that the pre-motor component of sequential effects is in charge of detecting a spatial pattern, whereas the motor component is responsible for detecting temporal regularities. One corollary of this hypothesis is that the entire framework suggested here might really be a model of the temporal filtering occurring at the motor level.

Additional support for the spatio-temporal filtering hypothesis can be found in experiments

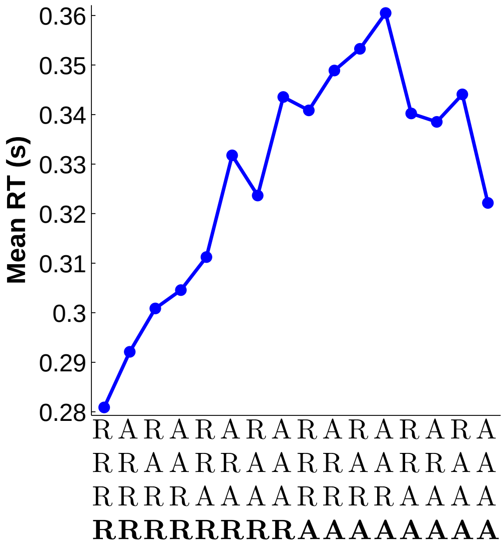

FIGURE 4.35: Results of a long RSI - 1000 ms - experiment with spatially overlapping stimuli. The stimuli used in the experiment consisted of a lower- and upper-case ‘o’. Note how in this case results do not converge to the same pattern observed for other 2AFC experiments with spatially separate stimuli shown in Figure 4.34.

conducted with a long RSI. The effects of temporal filtering, given their dependence on exponential decay, should disappear in the limit of a very long RSI, since the effects of a pulse would have dissipated before the next one arrives. Once temporal filtering subsides, what would remain would be the product of the spatial filtering component, which presumably would would take longer to decay. In agreement with this view, experiments with two spatially separated dots dots as stimuli converge to a pattern similar to S-LRP - the putative spatial filtering pattern - for long RSI values (see Figure 4.34). Conversely, an experiment in which the stimuli overlap spatially does not converge to the same pattern even with a 1000 ms RSI (see Figure 4.35), implying that spatial filtering may not be possible unless stimuli are separated in space. The hypothesis that S-LRP reflects some form of spatial filtering is nevertheless preliminary, and further empirical work is necessary in order to confirm or disprove it.

Relationship between REP/ALT and S-LRP/LRP-R

The possibility of a relationship between C2/C3 and S-LRP/LRP-R was introduced in Chapter 3. Likewise, the possibility of a relationship between ALT/REP and C2/C3 has been discussed here. Here what little evidence there is for a relationship between ALT/REP and S-LRP/LRP-R directly, i.e. irrespective of the relationship between either pair and the latent structure of sequential effects, is discussed.

A correspondence between REP and LRP-R seems at first sight fairly secure. First of all, LRP-R shows a pattern strongly resembling that of an exponential filter, which REP is equivalent to. Furthermore, LRP-R apparently disappears when the temporal structure of the input is removed, as expected if it represented some form of temporal filtering. This can be inferred from the fact that only S-LRP is present when the RSI is randomised (see Figure 4.33). Additional evidence for the absence of LRP-R when the RSI is made random stems from an inspection of individual differences in this experiment, which show greatly reduced variability when compared to a normal 2AFC: all but one of 10 subjects performing the random RSI experiment show a pattern with an alternation bias. Recall that individual differences in sequential effects are thought to be the product of varying contributions of stimulus and response processing, i.e. of S-LRP and LRP-R respectively. A reduced level of individual variation is therefore expected if one of the processing stages is nullified.

A possible relationship between S-LRP and ALT is more dubious; in order to discuss it it is useful to consider three possibilities:

1. ALT and S-LRP are the same.

2. ALT does not occur and temporal filtering occurs only through REP

3. ALT and S-LRP co-exist

The first hypothesis relies on S-LRP disappearing when temporal structure is disrupted, since ALT is also a form of temporal filtering, just like REP. We must therefore exclude this hypothesis if we assume the results of the random RSI experiment to be equivalent to S-LRP, a likely scenario given the great deal of similarity between the two (see Figure 4.33).

In the second hypothesis, temporal filtering would presumably be represented only by REP, with no exponential filtering of alternations - or ALT - occurring. This possibility depends on a correspondence between LRP-R and REP one the one hand, and S-LRP and some form of spatial filtering on the other hand. The first premise that LRP-R and REP are equivalent has been discussed above as likely. Investigating the possible correspondence between S-LRP and some form of spatial filtering is a more complex matter. One immediate possibility would be to remove spatial structure from the task and see if results reduce to the temporal filtering component LRP-R. Using abstract symbols appearing in the same location - such as ‘A’ and ‘B’ - might not be enough since any pair of characters will have non-overlapping parts. Perhaps a better way to completely remove spatial structure is to use stimuli based on colour. An experiment with different coloured dots was conducted by Jentzsch and Sommer (2002), the results of which show a strong repetition bias but are otherwise sufficiently different from LRP-R to render the experiment inconclusive with respect to the hypothesis that only a temporal filtering component remains.

The final possibility is that ALT and S-LRP are different elements contributing towards sequential effects. ALT and S-LRP display similar patterns so it might be hard to tease apart their influence with a latent variable approach. Preliminary results from a PCA performed on an artificially generated dataset in which ALT and S-LRP were mixed reveals that, provided enough noise is added, the two components might become indistinguishable (not shown). Experimental evidence for the fact that motor processing sometimes displays a pattern similar to ALT, instead of REP as is the case with LRP-R, would go a long way towards validating this possibility.

The view of sequential effects as reflecting some form of spatio-temporal filtering is one of the main propositions of this thesis and is discussed in more detail in Chapter 5.

5

Discussion

In order to conclude this dissertation, it is important to first review what was discussed in its different sections. Chapter 1 included an in-depth review of the field of sequential effects, which was followed by three research chapters. Chapter 2 discussed the computational nature of sequential effects in terms of different transition probabilities that could be used. It was found that humans switch from using first order transition probabilities in a task with two alternatives to using just the relative frequency of stimuli in a task with three elements. In Chapter 3 a more bottom-up strategy towards understanding the structure of sequential effects was taken by performing a latent variable analysis of individual differences. Three latent variables related to sequential effects were identified - two main and one minor - and a relation between the two main components and two separate processing stages involved in sequential effects was proposed, with the minor component argued to be a consequence of processing constraints. Still in Chapter 3, individual differences, as well as the dependence of sequential effects on the RSI, were explained in terms of the latent structure of sequential effects. In Chapter 4 an entirely novel framework for modelling sequential effects was proposed, motivated by several limitations of most models suggested so far as well as the need to explain all facets of sequential effects. The framework suggested, based on the physics of oscillatory systems, was found to be successful in reproducing key aspects of sequential effects such as two central components with symmetrical biases for repetitions and alternations, as well as being able to produce two latent components similar to those encountered in empirical data with minimal assumptions about the nature of individual differences.

There are several objectives to this section. Firstly, given the wide-reaching implications of the content of this dissertation for the field of research into sequential effects, it is important to discuss how results fit within the context of previous research into the subject. This goal will be achieved by contrasting two different perspectives on sequential effects, a dichotomy which, while never before made explicit, is useful in that it adds some perspective to an often fragmented field of research where considerable redundancies between models exist. The discussion of the two views will also be useful in achieving another important objective of this section: to integrate the different sections of this thesis into one coherent whole. Towards the end a new overarching perspective on sequential effects will be discussed by proposing that these effects can be understood as some form of spatio-temporal filtering. With this new perspective in mind, some possible future research directions are suggested, with particular focus on the modelling front. Finally an attempt will be made at putting the results of this dissertation, and more generally sequential effects, into a wider context by discussing a broad philosophical view of the way the human mind works.

5.1 Two different views of sequential effects

At this point in the history of research into sequential effects it is useful to draw a distinction between two different perspectives on sequential effects. The first view, which will be referred to as the *classical or statistical view*, sees sequential effects as a consequence of the tracking of different types of statistics about the sequence. The second view, which will be referred to as the *two component view*, sees sequential effects as consisting of two discrete and independent components related to the detection of two types of pattern: repetitions and alternations. The first view has been dominant since the work of Laming (1969) and underlies most models of sequential effects suggested so far. The second view has only been clearly articulated once by Maloney et al. (2005), although several other authors seem to imply a similar idea (e.g. D. Hale, 1969). The point here is not to make a decision about which view is correct; in fact, the two perspectives might be considered to be complementary if we consider them to lie at different levels of Marr's hierarchy (Marr & Vision, 1982). Still the the second perspective will be argued to be of better use in understanding the full range of sequential effects, as well as having more solid empirical grounding.

5.1.1 The classical or statistical view

All attempts to model sequential effects so far have revolved around two types of information: the relative frequency of the stimuli and the relative proportion of repetitions and alternations in the sequence. These two sources of information are usually represented in models by two types of exponential filter: one applied to the raw sequence of stimuli and the other to the sequence of repetitions and alternations. For simplicity sake, the first filter will be referred to as a 'simple' exponential filter and the second as an A/R filter. The usefulness of a combination of the two types of filter was first noted by Laming (1969) and subsequently variations of the theme were presented

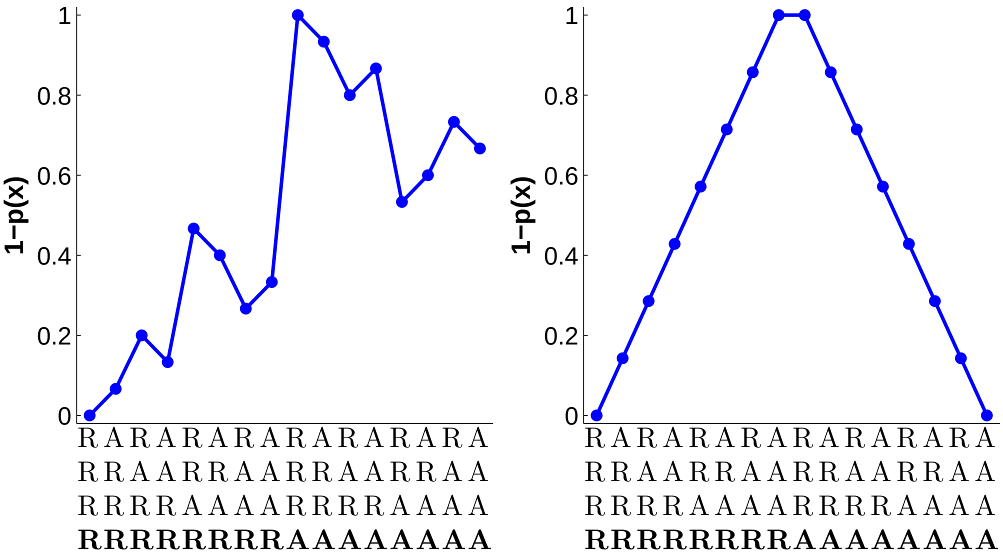

FIGURE 5.1: Predictions generated by both types of exponential filter. Left panel - simple exponential filter applied to the sequence of stimuli; Right panel - exponential filter applied to a sequence of repetitions and alternations of stimuli, i.e an A/R filter. Both plots were produced with $p(x_t) = \sum_{i=0}^{n-1} \alpha^i S_{t-i}$ and $\alpha = 0.5$; the number n of stimuli used in calculating $p(x_t)$ was 5 in the case of the simple exponential filter and 4 in the case of the A/R filter, reflecting the fact that a sequence of five stimuli corresponds to a four-long sequence of repetitions and alternations. In both cases $p(x_t)$ was normalised by $\sum_{i=0}^{n-1} \alpha^i$ in order to make probability values vary between 0 and 1.

by including either one type of filter or both (K. C. Squires et al., 1976; Jentzsch & Sommer, 2002; Cho et al., 2002; Yu & Cohen, 2008; M. Wilder et al., 2009; Gokaydin et al., 2011; M. Jones et al., 2013). The predictions generated by both types of filter are shown in Figure 5.1 in terms of $1 - p$ so as to facilitate comparison with reaction time results.[1]

The success of the combination of two filters is largely anchored on the good quality of fit to results from experiments conducted with a long response-stimulus interval (RSI). Figure 5.2 shows the remarkably good fit of a simple - i.e. non-weighted - sum of the two filters to the data of Cho et al. (2002) This pattern of results - obtained with an 800 ms RSI - is nevertheless somewhat uncharacteristic of long RSI experiments in that it displays a repetition bias, whereas more commonly results from such experiments show an alternation bias (e.g. Soetens et al., 1985). The reason for this repetition bias probably lies with the choice of overlapping figures as stimuli

[1] Recall that reaction times are generally assumed to be proportional to $1 - p$, where p is the probability of the next event.

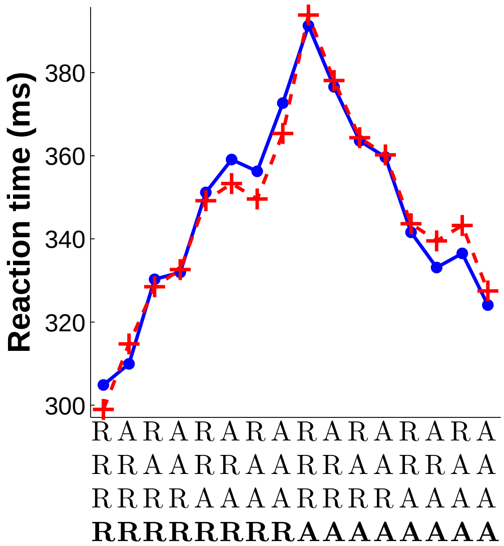

FIGURE 5.2: Illustration of the good fit of a sum of exponential filters to some datasets. Solid blue line - Reaction time data adapted from Cho et al. (2002). Dashed red line - sum of two types of exponential filter, one applied to the sequence of stimuli and the other to the sequence in terms of repetitions and alternations, both shown in Figure 5.1. The experiment was a regular 2AFC with an upper and lower case ‘o’ as stimuli and an 800 ms RSI. Best fitting α, the parameter regulating the decay of the filters, was 0.66. The sum of both filters was linearly transformed in order to fit empirical data.

by Cho et al. (2002), which is known to extend the repetition bias observed when the RSI is short to higher values thereof.[2] Nevertheless, the results shown in Figure 5.2 are clearly not spurious: a replication of the experiment by Cho et al. (2002) analysed in Chapter 3 produced a very similar pattern.

Despite its success in fitting some empirical results, the combination of two different types of filter has several shortcomings. First of all it sidesteps the issue of how the mind turns a sequence of stimuli into a sequence of repetitions and alternations, which is left in need of an explanation. Another perhaps more serious issue lies with the fact that there is no combination of the two filters shown in Figure 5.1 that explains the pattern of results commonly observed with a short RSI,[3] rendering the theory applicable only to sequential effects observed when the RSI is long. But even if we restrict ourselves to the case of long RSI experiments, the combination of filters is unable to reproduce one recurring feature of data: an alternation bias, or faster reaction times overall to

[2] Chapter 1 contains more details concerning the effects of using overlapping figures as stimuli.

alternations. Results of experiments conducted with a long RSI more often than not display such an alternation bias, and the same feature is also often observed in individual participant data, even when the average of the group in which such individuals are included does not display the said bias. So in short the combination of two exponential filters is successful in a relatively restricted set of experimental conditions.

The incompatibility between short RSI results and the two-filter approach has received little attention in the literature. This may be partly due to the traditionally held view that short RSI results are the product of an entirely different mechanism (Soetens et al., 1985), which has led some authors to dismiss them altogether as not being part of the computational theory of sequential effects (M. Wilder et al., 2009; M. Jones et al., 2013).[4] However, the evidence presented in Chapter 3 suggests otherwise, in that differences between short and long RSI results were found to be of a largely quantitative, rather than qualitative, nature. In fact, the entire range of different types of sequential effects was found to be well described by only three latent variables, of which two explain most variance irrespective of RSI. If confirmed, these results suggest a common structure underpinning all types of sequential effects, raising questions about the need to postulate separate mechanisms.

In the next section an alternative view of sequential effects will be discussed, based on empirical evidence for the existence of two separate processing stages: one associated with stimulus and the other with response processing. As this 'two-component' is presented it will concomitantly be discussed why it may hold an advantage over the combination of two exponential filters discussed above.

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3 See the literature review section for a description of this pattern of results, sometimes referred to as 'benefit-only' by contrast with the 'cost-benefit' - i.e. inverted 'v' - pattern of sequential effects typical of long RSI results.

4 There is one exception to this trend in the work of Gao et al. (2009) which attempts to unify short and long RSI results, albeit through the use of an arguably overly complex model.

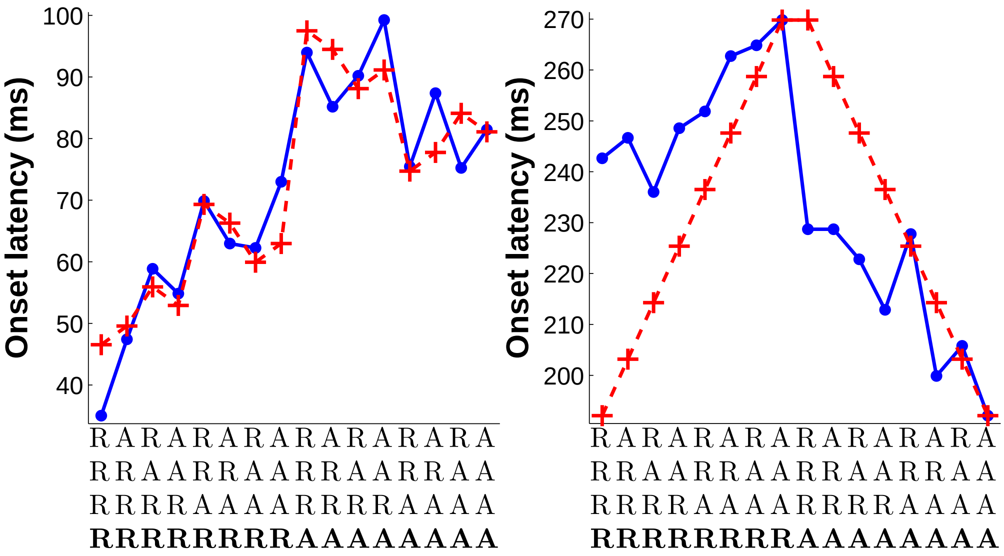

FIGURE 5.3: Fit of both types of exponential filter to evidence about the two processing stages of sequential effects. Solid blue lines - LRP-R (left panel) and S-LRP (right panel). Dashed red lines - simple exponential filter fit to LRP-R with $\alpha = 0.47$ (left panel); A/R exponential filter shown together with S-LRP (right panel); in this case the filter was not fit to the data. Recall that S-LRP and LRP-R reflect the relative contribution of pre-motor and motor processing respectively towards sequential effects (Jentzsch & Sommer, 2002). Data adapted with permission from the author.

5.1.2 The emerging two component view

There are three main sources of evidence supporting the existence of two separate processing stages involved in sequential effects: EEG studies (Jentzsch & Sommer, 2002), behavioural experiments (Maloney et al., 2005; M. H. Wilder et al., 2013) and finally the latent variable analysis performed in Chapter 3. Collectively, the evidence points to a stage which is pre-motoric in origin and associated with the processing of stimuli, and a second stage associated with motor control and the processing of responses.[5] Furthermore, all three lines of enquiry point to similar patterns of contribution by each stage towards sequential effects, best illustrated by the measurements of pre-motor - S-LRP - and motor - LRP-R - processing time performed by Jentzsch and Sommer (2002) and shown in Figure 5.3. Note how S-LRP and LRP-R display approximately symmetrical biases with respect to whether repetitions or alternations are preferred.

[5] This evidence is also discussed in Chapters 1, 3 and 4.

Some authors have proposed a relationship between the two processing stages of sequential effects and the two types of exponential filter discussed above (M. Jones et al., 2013). In order to discuss this possibility the two exponential filters are shown together with the putative respective counterparts in Figure 5.3. Based merely on quality of fit, it would seem that a correspondence between LRP-R and a simple exponential filter is at first sight a sensible proposal, whereas the same cannot be said of S-LRP and an A/R filter. The suggestion that S-LRP corresponds to an A/R filter is motivated at least in part by two factors: firstly, in the context of the classical theory, once an exponential filter is accounted for there is only one other alternative mechanism available, and that is the A/R filter; secondly, it is hard to reproduce the alternation bias displayed by S-LRP with any form of discrete-time exponential filtering, upon which almost all models of sequential effects are based.

In order for a correspondence between S-LRP and an A/R filter to hold it becomes necessary to account for the differences observed between the two (Figure 5.3, right panel), in particular with respect to the alternation bias displayed by S-LRP. M. Jones et al. (2013) suggest that a ‘cue competition’ mechanism between the two types of exponential filter is responsible for the alternation bias of S-LRP. There are a number of issues with this theory, not least in that it relies on interpreting different types of statistics as ‘cues’, but perhaps its most fundamental problem is that the mechanism proposed is only necessary in order to explain S-LRP, not LRP-R. In other words, while LRP-R is fit by the simple exponential filter in isolation, the other component - S-LRP - is fit with a combination of both types of filter interacting with each other via the cue competition mechanism.

Instead of forcing an equivalence between S-LRP and an A/R filter a possible alternative is to accept S-LRP and LRP-R at face value as reflecting two *separate and independent* contributions towards sequential effects. If we take this view, the relative contribution of stimulus processing towards sequential effects, as measured by S-LRP, displays an alternation bias independently of response processing, i.e. of LRP-R. Some support for the independence of both stages comes

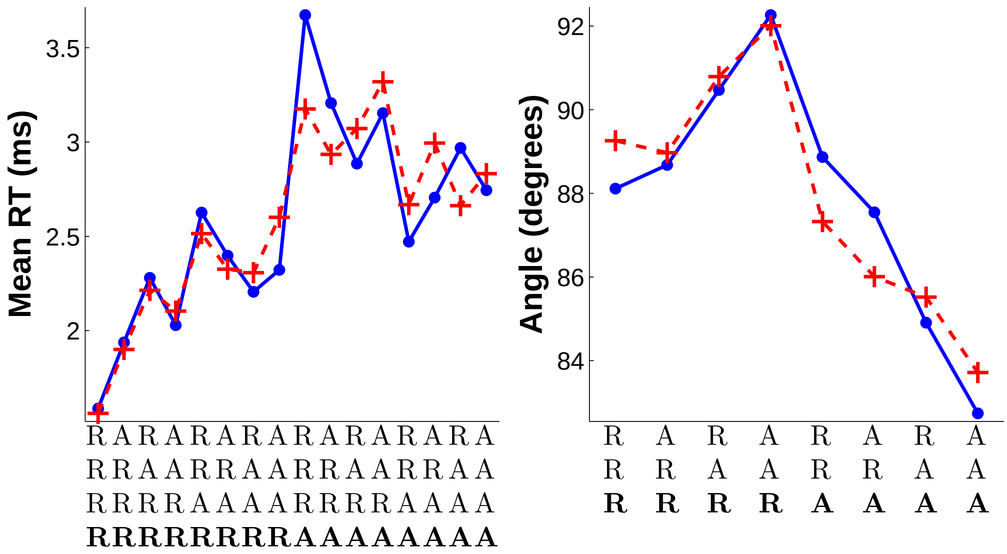

FIGURE 5.4: Evidence from behavioural experiments for stimulus and response processing. Solid blue lines - reaction time data from an experiment meant to remove the influence of stimuli, adapted from M. H. Wilder et al. (2013) (left panel); subjective point of equality data from an experiment meant to remove the influence of responses, adapted from Maloney et al. (2005). Dashed red lines - LRP-R (right panel) and S-LRP (right panel). Note that the right panel shows results as a function of the last four stimuli because this is the way empirical data was presented in the original article. The way S-LRP would look like as a function of the last four stimuli was estimated by averaging the corresponding five-back sequences two-by-two.

from experiments which aimed at selectively removing the influence of stimuli on the one hand, and of responses on the other hand, from sequential effects (Maloney et al., 2005; M. H. Wilder et al., 2013). When the influence of stimuli is removed, a reaction time pattern similar to LRP-R is obtained (see Figure 5.4, left panel); when the influence of responses is removed instead, a pattern resembling S-LRP is recovered (Figure 5.4, right panel). The results obtained when response processing is disrupted are particularly hard to fit with the theory that an interaction between stimulus and response processing is responsible for the alternation bias displayed by S-LRP.

Additional evidence for the existence of two independent processing stages can be found in the results of the latent variable analysis conducted in Chapter 3. Two main latent variables were identified which display opposite and equally strong first order biases (see Figure 5.5) much like S-LRP and LRP-R. Some care should be taken when interpreting the latent structure of sequential

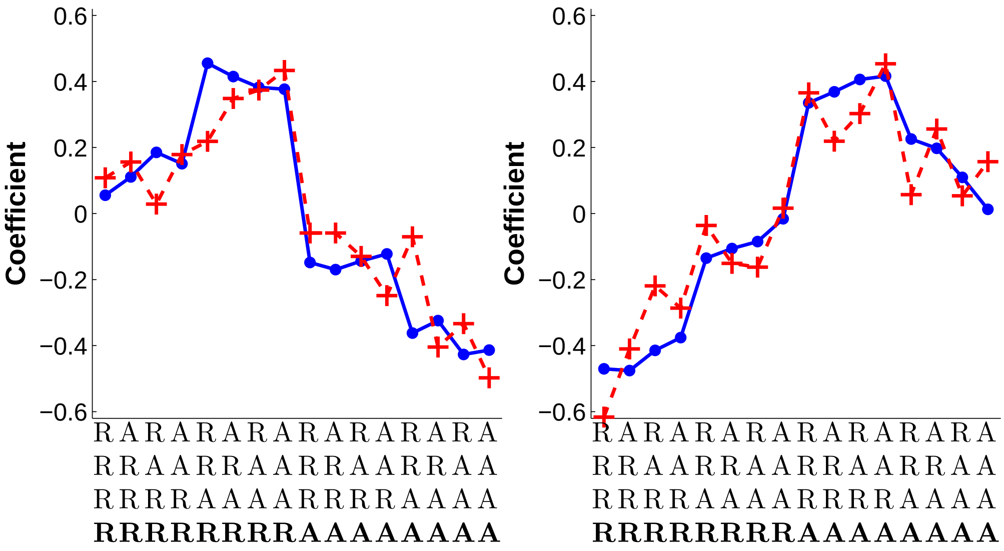

FIGURE 5.5: A comparison of the two main latent components of sequential effects with S-LRP and LRP-R. Solid blue lines - coefficient patterns of the two main components of sequential effects - C2 (left panel) and C3 (right panel) after rotation using S-LRP and LRP-R as targets. Dashed red lines - S-LRP (left panel) and LRP-R (right panel) shown for comparison.

effects since, as discussed in Chapter 4, the relationship between covariance structure and underlying physical truth can be subtle and nuanced. Nevertheless, when considered together with other sources of evidence, the latent structure of sequential effects adds some weight to the idea that there are two main independent components of sequential effects.

To the empirical evidence discussed so far one could add an argument based on the explanatory potential of the two competing views, i.e. classical and two-component. When considering only the typical pattern of results obtained with a long RSI - the cost-benefit or inverted ‘v’ pattern - it would seem that there is little difference between both views: a combination of S-LPR and LRP-R is at least equally as good at capturing a cost-benefit pattern as is a combination of the two exponential filters (see Figures 5.2 and 5.6). However, when taking into consideration the full spectrum of different patterns of sequential effects, a combination of elements resembling S-LRP and LRP-R is capable of describing a far wider range results than a combination of two exponential filters, including sequential effects observed with a short RSI - i.e. the so-called benefit-only pattern - as well as a host of individual differences (see Chapter 3). On the other hand, even if

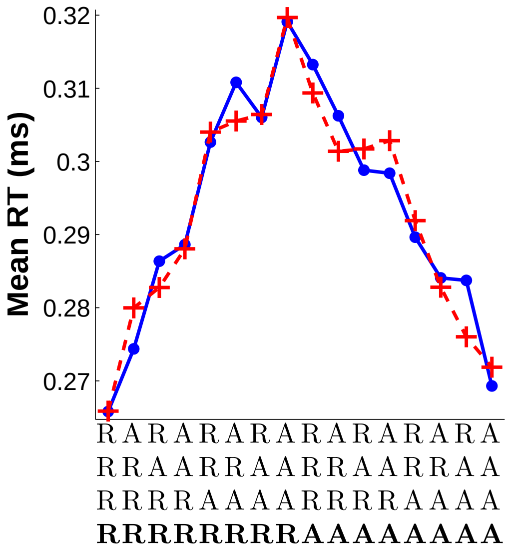

FIGURE 5.6: Fit of a combination of S-LRP and LRP-R to a typical cost-benefit pattern of sequential effects. Solid blue lines - average across all subjects performing any version of a 2AFC with an 800 ms RSI included in Chapter 2. Dashed red line - linear combination of S-LRP and LRP-R of the form $b + w_1 SLRP + w_2 LRPR$.

we incorporate the cue competition mechanism, a combination of two filters will not be able to capture results typically observed with a short RSI (M. Jones et al., 2013).

If we accept the two-component view of sequential effects, then we are left with the task of finding a mechanism which could produce a pattern such as S-LRP with its alternation bias. In Chapter 4 such a mechanism was proposed, based on the physics of oscillatory motion. It turns out that the two most fundamental types of behaviour of a single oscillator display a great deal of similarity to the two patterns - S-LRP and LRP-R - thought to represent the relative contributions of stimulus and response processing towards sequential effects.

5.1.3 Resonance and the exponential filtering of alternations

A new framework for modelling sequential effects was proposed in Chapter 4 based on the physics of oscillatory harmonic motion. The model makes use of the fact that, when represented as square

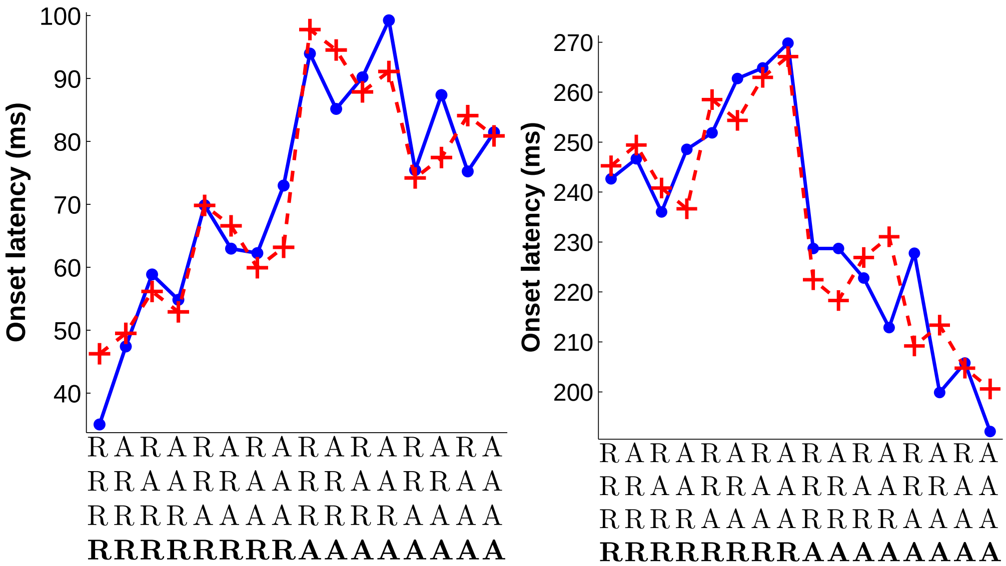

FIGURE 5.7: A comparison of the two types of oscillator resonance ALT and REP with S-LRP and LRP-R respectively. Solid blue lines - LRP-R (left panel) and S-LRP (right panel). Dashed red lines - REP (left panel) and ALT (right panel) linearly transformed to fit the EEG components. Both REP and ALT were generated for this illustration with $\gamma = 1.5$.

impulses, repeating and alternating patterns can be distinguished on the basis of their frequency, referred to as respectively as f_{REP} and f_{ALT}. By setting the natural frequency of the oscillator ω_0 to match either f_{REP} and f_{ALT}, the system can be made to resonate with - or detect - repetitions or alternations respectively. When the oscillator is tuned to repetitions it effectively behaves like the simple exponential filter at the sequence level described above, producing a pattern or results - referred to as REP - which, much like its discrete-time counterpart, shows a great deal of similarity with LRP-R (Figure 5.7, left panel). In addition, by tuning the oscillator to resonate with alternations - i.e. by setting ω_0 equal to f_{ALT} - it is possible to obtain a pattern of results - referred to as ALT - resembling S-LRP with its alternation bias (see Figure 5.7, right panel). Crucially, both REP and ALT are produced by *exactly the same mechanism* and are equally fundamental types of oscillator behaviour. Moreover a pattern of results with an alternation bias was produced without introducing any additional assumptions.

One possible interpretation of the two types of resonance - REP and ALT - shown in Figure 5.7 is that these act as detectors of repetitions and alternations respectively. Note that REP is

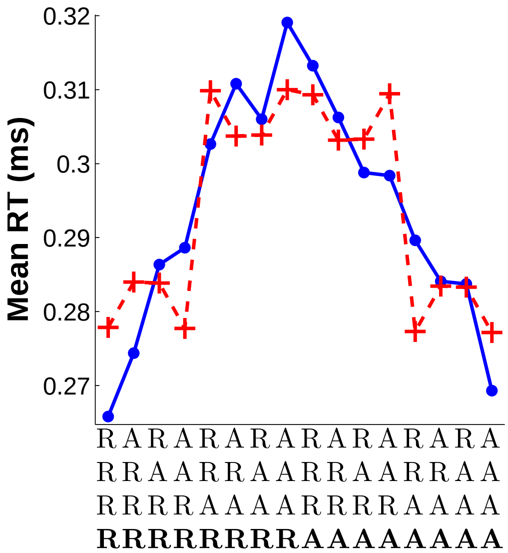

FIGURE 5.8: Fit of a combination of REP and ALT to the typical cost-benefit pattern of sequential effects. Solid blue line - average reaction time across all subjects performing any variation of a 2AFC with an 800 ms RSI (see Chapter 3). Dashed red line - best fitting combination of REP and ALT of the form $b + w_1 REP + w_2 ALT$. Note the difficulty in reproducing the smooth inverted 'v' shape of the cost-benefit pattern with a combination of REP and ALT. $\gamma = 1.5$ for both REP and ALT.

only sensitive to repetitions of stimuli, and ALT to alternations thereof. Some authors have in fact argued in the past that two separate mechanisms must exist in order to detect repeating and alternating patterns at the same time. D. Hale (1969) observed that a decrease in reaction times occurred with increasing length of both repeating and alternating runs of stimuli, which could not be explained by a single passively facilitating trace. More recently, Maloney et al. (2005) made a similar argument from a different perspective, and concluded that sequential effects were the product of an attempt at completing two types of pattern: repetitions and alternations. Note that in a system with multiple coupled oscillators it is possible for more than one frequency to be filtered, and therefore for resonance with repetitions and alternations at the same time to occur. The possibility is therefore raised here that the proposed separation of mechanisms in charge of detecting repetitions and alternations takes root in the independence of the normal modes of some form of dynamical system (see below).

The view of the two fundamental components of sequential effects as mapping onto two types of resonance of an oscillatory system is not without problems of its own. First there is the issue of the differences in detail between the patterns of ALT and S-LRP on the one hand, and REP and LRP-R on the other hand which (see Figure 5.7). These differences are probably meaningful, as can be seen by contrasting Figures 5.6 and 5.8: while a combination of LRP-R and S-LRP provides an excellent fit to the typical cost-benefit pattern of sequential effects, a combination of REP and ALT shows a notable degree of distortion relative to the smooth inverted 'v' shape typical of a cost-benefit patterns obtained when the RSI is long. That REP and ALT provide a less than perfect fit is perhaps unsurprising given that these are linear filters, whereas oscillatory activity in the brain is more likely to be nonlinear in nature (see below for a more detailed discussion of this point).

A second problem with the resonance view is that, in its current form, the model depends on the temporal regularity of the input, making it in effect a model of temporal filtering. If one were to present the stimuli to the model at random time intervals, the results would be equally random. However, empirically randomising the interval between the presentation of the stimuli *does not* abolish sequential effects. In fact, it seems to make results converge to the component of sequential effects with an alternation bias, i.e. S-LRP (see Figure 5.9). If one accepts that the results of the random RSI experiment reflect the same underlying construct as S-LRP, it follows that the pre-motor or stimulus-associated component of sequential effects is *temporally insensitive*. The same is not true of the motor component of sequential effects - LRP-R - which does disappear when a random RSI is used. Possible solutions to this problem are discussed below in the context of the view of sequential effects as spatio-temporal filtering. As a side note, issues with the temporal regularity of the input are common to all continuous time models, although only two other have been suggested previously (Cho et al., 2002; Gao et al., 2009). As for the remaining models, their discrete time nature makes it difficult to study any questions related to the time interval between stimuli (e.g. Laming, 1969; M. Jones et al., 2013).

A third issue with the oscillator model is that REP and ALT only occur when the natural frequency of the oscillator ω_0 is equal to either f_{REP} or f_{ALT}. If ω_0 does not match either of these

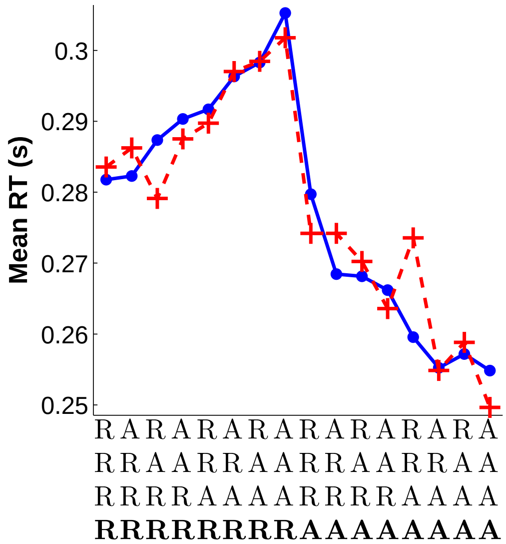

FIGURE 5.9: Results of a 2AFC with a random RSI. The experiment was a 2AFC with two horizontally displaced dots as stimuli, but in this case the RSI was not constant but rather different for each trial and drawn randomly from a uniform distribution in the interval [50 1000] ms. Ten subjects took part in the experiment and the results shown are the mean across all subjects. Appendix B contains all the details of the experiment.

two frequency values it will filter other minor intermediate frequencies and no longer conform to either REP or ALT (see Chapter 4). However, sequential effects are apparently the product of two *fixed* patterns looking like S-LRP and LRP-R, one manifestation of this being the fact that the typical cost-benefit pattern can be observed for a wide range of RSI values, a parameter which determines the average frequency of the stimuli. If resonance were to explain all these results the implication would be that the underlying system is resonating *irrespective of input frequency*. One possible solution to this conundrum might be to consider that in a system with many degrees of freedom f_{REP} and f_{ALT} will always fall within the frequency band of one of the normal modes of the system. Alternatively, as discussed below, some form of synchronization might be occurring between neural oscillators and the frequency of the input.

Before discussing possible ways to improve the oscillator model, as well as outlining a more general view of sequential effects, we must first turn to a discussion of how the statistical nature of sequential effects in different tasks, investigated in Chapter 2, fits within the context of the two component view of sequential effects.

5.2 The statistics humans use in different tasks

It was discussed in Chapter 2 that when humans change from a task with two possible stimuli - 2AFC - to one with three - 3AFC - they stop using first order transition probabilities and begin to use only information about the relative frequency of the stimuli, or 0-th order statistics. It is unclear how these results relate to the two-component view that has since been discussed in Chapters 3, 4 and above. A more general but closely related question concerns the relationship between the computational or statistical level of sequential effects and the underlying process-level mechanisms. While a final answer to either of these questions will not be given here, there are some hints of how the different statistics studied in Chapter 2 might relate to the two processing stages of sequential effects. In order to discuss this relationship the cases of first and 0-th statistics will be dealt with separately.

It seems reasonable to assume that, in order to track first order statistics - e.g. $P(A|B)$ - information about the relative abundance of repetitions and alternations in the sequence must be available. In fact, the first order model discussed in Chapter 2 produces a cost-benefit pattern of results very similar to a sum of two types of exponential filter discussed above (see Figure 5.2), one of which represents the A/R ratio. At the process level, this implies that both components of sequential effects must be active, if we rely on an interpretation of these as being sensitive to repetitions and alternations respectively and exclusively.[6] So the use of first order statistics in a 2AFC reported in Chapter 2 is likely to rely on contributions from both pre-motor and motor processing stages.

[6] Note that this is implied by the patterns of S-LRP and LRP-R, but it is not known yet if the relative contributions of both processing stages will always display a separate sensitivities to repetitions and alternations.

The relationship between the tracking of 0-th order statistics - e.g. $P(A)$ and $P(B)$ - and the underlying processes is more direct. The tracking of - exponentially discounted - stimulus frequencies is effectively equivalent to a simple exponential filter, which in turn shows a great deal of similarity to the relative contribution of the motor processing stage in isolation (see Figure 5.3, left panel). However, recall that it was found in Chapter 2 that humans use 0-th order statistics in a 3AFC, and no empirical evidence for how LRP-R looks like in this case exists. Nevertheless, it seems reasonable to assume that motor processing represents the exponential filtering of the sequence, irrespective of the number of alternative stimuli. If we accept this premise, then only the motor processing stage of sequential effects is active when humans are performing a 3AFC.

A teleological argument can be invoked in order to justify the changes observed when switching from a 2AFC to a 3AFC, based on the fact that alternations become vanishingly rare in a sequence of three equiprobable stimuli.[7] The only regular pattern observed with a non-negligible frequency in a random sequence with three alternatives is repetitions of the same element, and then only fleetingly. If we think of the two process-level components of sequential effects as detecting repetitions and alternations separately, it follows that the alternation detector should not manifest itself in a 3AFC, since there are no alternations to be found. Conversely, repetitions and alternations are equally abundant in 2AFC, and both detectors would be triggered. More work is necessary in order to confirm or disprove that the changes observed when increasing the number of sequence elements from two to three are indeed the result of the pre-motor stage becoming inactive. However, if this is found to be true, it could be interpreted as providing evidence for a view of sequential effects as as the trace of an attempt at completing two different types of patterns - repeating and alternating.

The interpretation of sequential effects as a combination of two detectors, one of repetitions and one of alternations, while potentially true at some level, is nevertheless simplistic for several

[7] Some authors (e.g. Audley, 1973) consider an ‘alternation’ to be a sequence of two different stimuli, no matter what they are, and in this sense the sequence XYZYXZ is alternating. However, this is clearly a different situation to the alternation of two stimuli, which forms a clear *pattern*.

reasons made clear below. In the next section an attempt is made at delineating a more nuanced view of sequential effects as some form of spatio-temporal filtering. In general, from this point on the content of this discussion should be taken to be of a largely speculative nature, although possible empirical support will be shown whenever possible.

5.3 Sequential effects as spatio-temporal filtering

In order to predict the future, any intelligent organism must answer three fundamental questions: 'what?', 'where?' and 'when?'. Moreover, as pointed out several times throughout this dissertation, predictions are only possible if there are patterns in the input. Combining these two premises leaves us with three possibilities: either there is a pattern in time, in space, or in the nature of the objects recognized. These three types of pattern can exist relatively independently of each other or combine to form complex spatio-temporal patterns. Furthermore, the human perceptual system imposes complex constraints on what is perceived depending on the objects identified and their trajectories in time and space. For instance, in the familiar language of the 2AFC experiments discussed extensively in this work, two dots on the screen can be interpreted as the same object or as different ones depending on the distance in time and space between them (Maloney et al., 2005). Similarly, an object can be perceived as transforming into another if sufficiently close intermediate steps are included, as anyone who has watched a film can attest to. So while the questions of 'what?', 'where?' and 'when?' can be considered separately, in practice they are deeply intertwined.

Throughout the history of research into the subject it seems to be often implicitly assumed that sequential effects are the product of a pattern detection attempt. However, it was never made explicit what type of pattern it is that humans are attempting to discover: temporal, spatial or in other properties of the stimuli. Sequential effects are observed in experiments with a wide range of different stimuli such as two separate dots (e.g. Soetens et al., 1985; Jentzsch & Sommer, 2002), spatially overlapping figures (e.g. Laming, 1968; Cho et al., 2002), stimuli defined by other properties such as colour (e.g. Jentzsch & Sommer, 2002), an even in modalities other than vision (e.g. K. C. Squires et al., 1976). Differences between such experiments are often framed in the context of response-stimulus compatibility: experiments where stimuli overlap are often taken to present a reduced degree of mapping between the spatial configuration of the stimuli and of the responses. Little or no attention has been given to the fact that distinguishing stimuli based on shape, colour or form is fundamentally different from a distinction based on spatial location. This may reflect an implicit assumption from the authors of such studies that the question humans are trying to answer is 'what?', irrespective of 'where?' and 'when?'.

The mathematical structure of most sequential effects models in the literature may have contributed to the lack of specification regarding the type of pattern humans are attempting to detect when displaying sequential effects. Almost all models proposed so far are of a discrete time nature and therefore not easily - if at all - amenable to conceptualising continuous time. Moreover, no model so far has incorporated space in any way, the model proposed in Chapter 4 being the first to do so in the simplest manner possible. However, there is substantial evidence that both space and time are important in sequential effects. First of all, issues of compatibility between the spatial configuration of stimuli and responses point to a role of space in sequential effects. With respect to time, sequential effects have been shown to depend on the temporal interval between the stimuli, highlighting the need to include this as a parameter in any complete model. Finally, an experiment where the time interval between stimuli was randomised (see Figure 5.9) not only further highlights that time must be taken into consideration, but also indirectly points to the need to conceptualize space: if temporal structure is removed from the input then whatever is left - spatial structure in this case - must be responsible for the sequential effects observed. So while there is no doubting the historical importance of the Markov approach, it is likely to be insufficient - or at least cumbersome - if one is to gain a more complete understanding of sequential effects and human pattern detection in general.

The possibility that sequential effects are due to some form of spatio-temporal filtering was first introduced in Chapter 4. The hypothesis, in a nutshell, is that the two components of sequential effects, in addition to being responsible for detecting two different patterns - repetitions and alternations - also represent an attempt at detecting patterns in two separate domains: space and time. The pre-motor component - S-LRP - would be associated with the detection of spatial patterns whereas the motor component - LRP-R - would instead be responsible for detecting temporal patterns. A combination of these two elements would lead to an optimal spatio-temporal prediction about the next event, represented by the typical cost-benefit - or inverted 'v' - pattern of sequential effects. An interesting side observation is that this pattern of sequential effects, if it does represent an optimal average prediction as has been suggested before (Yu & Cohen, 2008), is only seldom observed at the individual level, despite being consistently observed when multiple subjects are averaged. It seems therefore that there might be a 'wisdom of crowds' aspect to sequential effects, in that the average of a group of non-optimal predictors represents an optimal prediction. That this is the case would make sense from an evolutionary standpoint, since it guarantees that at least a few individuals will be precisely correct.

The interpretation of the two components of sequential effects as detecting repetitions and alternations separately is compatible with the spatio-temporal filtering hypothesis: repetitions and alternations are different in that the latter have separable temporal and spatial dimensions, whereas in the case of repetitions time and space are inextricably linked. Put simply, repetitions occur in the same location and so have no relevant spatial structure. If, by a crude analogy with the temporal filtering model proposed in Chapter 4, one were to propose a putative spatial filtering mechanism, this could never be tuned to repetitions, since these do not have a characteristic spatial frequency. It is not so clear why the temporal filtering element of sequential effects should always display the same pattern with a repetition bias, since it was shown in Chapter 4 that it is equally easy to temporally filter alternations as it is repetitions. Also in Chapter 4 the possibility was discussed that the motor component of sequential effects might sometimes be tuned to detect alternations, in which case it would results in a pattern - ALT - so similar to the spatial filtering element - S-LRP - as to be indistinguishable from it.

It is tempting to think that, in the same way as the temporal structure of the stimuli was removed by making the time interval between stimuli random, resulting in the isolation of a putative spatial filtering component (see Figure 5.9), the inverse could be done: removing spatial structure and recovering the temporal filtering element. One possible way to design a task without spatial structure would be through the use of stimuli equal in every respect except for colour. However, we are not assured that this would abolish a temporally insensitive component to sequential effects: in much the same way as it is possible to derive a prediction about the next spatial location independently of time, the same is in principle possible regarding the colour of the next stimulus. So far S-LRP has been discussed as reflecting spatial filtering partly because it was identified in an experiment with spatially separate stimuli (Jentzsch & Sommer, 2002). In addition, the experiment where the RSI was made random, which resulted in a pattern similar to S-LRP (see Figure 5.9), also made use of two separate dots as stimuli. When stimuli are spatially separate the 'what?' and 'where?' questions are equivalent, but this is not the case in general. So while there is no evidence available at the moment that would support such a view, it may be the case that a temporally independent component of sequential effects will occur even when stimuli are overlapping, a form of 'object filtering' so to speak.

5.4 Future directions

5.4.1 Empirical

The renewed perspective on sequential effects presented in this work raises several questions calling for further empirical work, the most important being perhaps the fixed nature of the two components of sequential effects. It was hypothesized in Chapter 3 that variation in sequential effects might be the product of different combinations of two components similar to S-LRP and LRP-R, in which case the patterns of these two elements would remain fixed, with only the magnitude of their contribution changing. So far there is strong evidence for the relevance of two components looking like S-LRP and LRP-R (see above and Chapters 3 and 4) in a specific set of experimental circumstances - i.e. long RSI - and on average for groups of subjects. On the other hand there is as yet no evidence for the way in which the relative contribution of stimulus and response processing - as measured by S-LRP and LRP-R or another method - would change both when the RSI is short as well as across different individuals performing the same experiment. Measuring the two processing stages in an experiment with a short RSI by using the LRP method was found to be difficult due to baseline problems (Jentzsch & Sommer, 2002), so another method may have to be used. With respect to individual differences however there seems to be no impediment to the use of the LRP method so long as the RSI is long enough, and the two-stage theory would greatly benefit from such as study.

The flipside to the question of whether the two components of sequential effects are fixed is whether these do in fact vary and in what way. As discussed above, there is some reason to believe that if S-LRP represents some form of spatial filtering it might always display a pattern with an alternation bias, since repetitions have no characteristic spatial frequency. However, the question remains of whether or not a temporal filtering element with an alternation bias can occur, and whether this is confounded in reaction time data with S-LRP. An analysis of individual differences in S-LRP and LRP-R would again be useful in this respect: if temporal filtering of alternations does in fact occur, then this might be observable in LRP-R, which would sometimes display a pattern with an alternation bias.

Further empirical research is also necessary in order to investigate the view of sequential effects as spatio-temporal filtering. It is in principle possible to design experiments where the spatial and temporal structure of sequential effects is selectively varied and to draw conclusions from the effects of these manipulations in both reaction time as well as well as event-related potentials. For instance, the possibility was discussed above that using stimuli distinguished by colour would prevent any form of spatial filtering, thereby reducing sequential effects to their temporal filtering - motor - component. If this premise holds true, further randomizing the interval between successive stimuli in a manner similar to the experiment shown in Figure 5.9 could abolish sequential effects altogether. Notwithstanding the possibility that sequential effects will persist due to an attempt at detecting a pattern in the colour of the stimuli, proof of concept of the effects of using stimuli distinguished by colour, as well as other experimental manipulations, should be obtained. Finally, some attention should be given to individual differences in any future experiment given the greater awareness of their importance brought about by this dissertation.

5.4.2 Theoretical

A novel approach to modelling sequential effects was suggested here which, despite showing some promise, is nevertheless still incomplete in several important respects. In order to suggest ways in which the model can be improved, as well as discuss some further caveats of the general approach, it is important first to clarify what a spatially extended version of the model proposed in Chapter 4 would look like. While the discrete oscillatory units considered in Chapter 4 were meant to represent different spatial locations, this is of little practical consequence if distance between oscillators is not represented, either by introducing coupling delays or representing space explicitly, i.e. adding one or more spatial dimensions. In general, it should be kept in mind that the model proposed in Chapter 4 was always intended as a simplification of a spatially extended model. During the discussion of how the model can be augmented to include space one crucial aspect will remain unchanged: every version of the model will be linear. Following this a brief discussion about non-linear systems will ensue in which it is briefly speculated how the properties of such systems may be of use in explaining the more complex aspects of sequential effects.

Time delays

Perhaps the simplest possible extension of the oscillator model is to introduce time delays in the coupling between the different oscillatory units. Unlike the case of two masses connected by a physical object such as a spring, two oscillatory units in the brain are not expected to influence each other instantaneously, but rather some delay is likely to occur due to the finite speed of neuronal signal propagation. Mathematically speaking a pair of delay-coupled oscillatory units can be described by the following pair of delay-differential equations

$$\begin{aligned} \ddot{y}_1 + \gamma\dot{y}_1 + \omega_0^2 y_1 + k(y_2(t-\tau)) &= F_1(t) \\ \ddot{y}_2 + \gamma\dot{y}_2 + \omega_0^2 y_2 + k(y_1(t-\tau)) &= F_2(t) \end{aligned} \tag{5.1}$$

analogous in every respect to the system presented in Chapter 4 except for the fact that now the different oscillators influence each other after a constant delay given by τ. Note that although space is not parameterised explicitly, the delays in coupling reflect the time that it takes for signals to travel in space, which is therefore represented implicitly.

In Chapter 3 two main latent components of sequential effects were identified - C2 and C3 - and a third minor one - C4 - thought to be related to processing delays. This interpretation of C4 relies on the fact that a similar pattern is observed in reaction time data from a group of elderly participants performing a 2AFC, but not in young subjects performing the exact same experiment (Melis et al., 2002). One possibility is that the differences observed between both groups are due to age related loss of myelination, in which case the vague 'processing delays' discussed in Chapter 3 might be made more concrete as conduction delays. One possible way to model this phenomenon would be to use a system with coupling delays such as the one in (5.1). However, careful consideration of what such a model for C4 would look like exposes some of the limitations of the oscillator-based models in general. Firstly, it is unclear how many oscillatory units are involved in sequential effects, how these map onto different regions of the brain and finally what the topology of the network is. It would seem from information about the two processing stages of sequential effects that there are two main signals contributing towards sequential effects: one with origin in the visual cortex and one in the motor cortex. However, it is unclear how these sources of information integrate, and whether there is any modulation from more central areas like the pre-frontal cortex. An understanding of some or all of these aspects might be necessary when investigating the possible role of conduction delays in sequential effects and in particular in the emergence of C4. Notwithstanding this, a demonstration that an oscillatory system with coupling delays is capable of generating a pattern such as C4 would be useful in providing a proof of concept that the effect can be replicated within the oscillator-based framework.

Spatially extended models

A more obvious way in which the oscillator model can be extended is to introduce one or more spatial dimensions. This can be done easily by using the well-known wave equation, which is a partial differential equation given by

$$\frac{\partial^2 u}{\partial t^2} - c^2 \nabla^2 u + \gamma \frac{\partial u}{\partial t} = F(x, t) \tag{5.2}$$

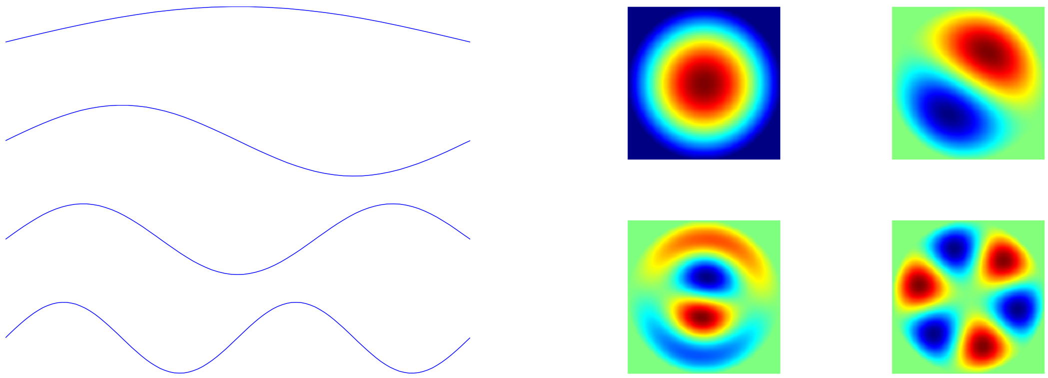

FIGURE 5.10: Example normal modes of a string and of a surface shaped like a circle. Left panel - First four normal modes of an idealized massless string tethered at both ends. Right panel - Sample normal modes of a circularly shaped surface, chosen as illustrative of the diversity of different possible patterns. Less regular shaped surfaces exhibit more intricate and complex eigenfunctions.

where x is a vector of any number of spatial coordinates, c is the speed of wave propagation, γ is the damping coefficient, ∇^2 is the Laplacian operator[8] and $F(x, t)$ represents the forcing term. Note that the model in Chapter 4 can be considered a particular case of the wave equation where each discrete oscillator is taken to represent a particular location in space and the velocity of propagation is infinite. Conversely, in what is a classical pedagogical example, the one-dimensional wave equation can be derived by taking the infinite limit of a chain of discrete oscillators (French, 2003). Much like discrete systems have normal modes, represented by vectors with a finite number of entries, the wave equation has normal mode *functions*, examples of which are shown in Figure 5.10 for one and two dimensions. In a discrete system the number of normal modes is equal to the number of oscillators by definition, but this number is theoretically infinite in the case of a continuous system. In practice however, most physical systems are composed of discrete units and this limits the number of normal modes. This may not be an important consideration in the case of a liquid surface with atoms as unit elements, but it might be a relevant in the case of a system composed of neurons.

In order to make the connection with a discrete system clearer, it is useful to perform a thought

[8] Which is equal to $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ for the case of three dimensions.

experiment on how one could reproduce a resonance behaviour similar to that discussed in Chapter 4 but in the case of a spatially continuous system.[9] The example used will be a unidimensional ‘string’ tethered at both ends so that these are fixed points. Now consider the second normal mode function of the string, analogous to the ‘breathing’ mode of two discrete oscillators: in this mode the left and right halves of the string are moving with the same frequency and in anti-phase (see Figure 5.10; left panel, second mode from top), and the middle point is stationary. Now take the middle points of each half - 1/4 and 3/4 of the distance of the rope - to correspond to the two discrete oscillators of the discrete system. Next, apply pulses representing the stimuli to the two halves of the string and measure velocity at any point in the string except the ends or the middle, making sure the point chosen for measurements is always the same. Finally, make sure to set the propagation velocity c high enough so as to minimise transient behaviour in favour of standing wave behaviour.[10] A perfectly analogous scenario could be described for two dimensions, except that the pulses would have to be applied to the centre of any two different troughs and/or peaks - shown in blue and red in Figure 5.10 - of the normal mode surfaces.

In addition to clarifying the relationship between discrete and continuous systems the above example serves as an introduction to several of the difficulties we expect to face in dealing with spatially extended systems. Firstly, *boundary conditions* had to be set, i.e. the behaviour - free or fixed or more complicated - at the border of the system had to be specified. Secondly, the pulses representing the stimuli were applied to *particular locations* in space in order for the system to exhibit a resonating behaviour; if the pulses had been applied at nodes rather than peaks, no resonance would occur. This can be taken as an illustration of a more general principle of spatially extended systems: *temporal filtering implies spatial filtering* (Nunez, 1995). Put simply, the system resonated because the input had a particular spatial as well as temporal frequency, the spatial element being determined by the relative position of the points at which the pulses were applied.

9 This is meant to be an informal visual example, but if doubts should persist about its feasibility the effect has been observed by solving the wave equation numerically in one dimension.

10 For a discussion of travelling and standing waves please refer to a physics textbook such as Main (1993).

The possibility was discussed above that sequential effects are due to some form of spatio-temporal filtering, when in fact we see that any physically realistic model of oscillatory activity forces a consideration of the spatial as well as temporal structure of the input.

Further complications resulting from introducing space in the model could be added to those already mentioned. For instance, it was merely stated that the pulses should be applied to specific points on the string, but one must specify the input to other parts of the string as well, since the stimuli are now represented by functions of space as well as time - $f(x,t)$ - which must be defined for the entire spatial domain. In addition, the speed of wave propagation had to be set high enough so that delays in wave propagation would be negligible. Under conditions where this speed would limiting, such as high temporal frequency of the input, complex travelling wave phenomena such as interference might occur and potentially have an impact on sequential effects. Finally most real media which support wave-like behaviour have different speeds of propagation for different frequencies, in which case a relation between temporal and spatial frequency - termed a dispersion relation - must be specified. In an example of potential relevance, a dispersion relation has been estimated for the human cortex when considered as a continuous medium for propagating brain waves (Nunez, 1995).

Non-linearity, synchronization and pattern formation

Whether in a discrete or spatially extended form, all versions of the model proposed so far suffer from the same limitation: their linearity. Real physical systems however are more often than not non-linear (Strogatz, 2000) and, despite being far less mathematically tractable, display a far richer range of behaviours, such as complex spatio-temporal pattern formation (Golubitsky & Stewart, 2002) and synchronization (Pikovsky, Rosenblum, & Kurths, 2001). Synchronization of oscillatory activity in different areas of the brain has in fact been observed empirically and suggested as a possible solution to the so-called 'binding problem', i.e. the fusion of different perceptual and cognitive aspects into a single entity (Singer & Gray, 1995). Most models of neuronal activity are in fact non-linear in nature (Izhikevich, 2010) and so it is likely that, moving forward, a more realistic model of sequential effects will be non-linear. In this it is speculated briefly as to the possible usefulness of some properties of non-linear systems in explaining different aspects of sequential effects.

Throughout this work, it has been purposefully left unclear what type or scale of neuronal oscillation the abstract model in Chapter 4 is meant to be representing. Nevertheless, we are almost assured of one of the properties of these oscillations: their non-linearity. The reason for this is that oscillatory activity in the brain seems to be *self-sustained*, i.e. it persists in the absence of any external input (Regan, 1989). Mathematically speaking, this behaviour is termed a limit cycle, and it can only be reproduced with non-linear models. This does not all at once invalidate the model presented in Chapter 4, since non-linearity does not preclude a behaviour similar to that of a linear system under some regimes, particularly if the forcing is relatively weak. Indeed, a preliminary analysis of a forced Van der Pol oscillator[11] hints that such a system is capable of displaying a behaviour similar to the resonance patterns of a linear system described in Chapter 4 (not shown).

Linear resonance and non-linear synchronization are similar phenomena in that an interaction between two systems leads to both oscillating with the same frequency (Pikovsky et al., 2001). However, the two phenomena are different in that oscillatory activity involved in synchronization is self-sustained, i.e. it persists in the absence of external forcing. On the contrary, the oscillations of a linear system will eventually die out due to energy dissipation, except in the trivial case of no damping. Furthermore, some models of synchronization allow for two interacting systems to oscillate together with a frequency different from the natural frequency of either system in isolation. By contrast, resonance only happens when the frequency of the driving system is equal of very close

[11] The Van der Pol oscillator is the simplest formulation of a type of non-linearity know to underpin many models of neuronal oscillations such as for instance the Fitzhugh-Naguno model (Izhikevich, 2010).

to the natural frequency of the system being forced. Properties such as these might make synchronization helpful in addressing one of the difficulties with the linear system mentioned above: that a phenomenon similar to resonance is apparently observed irrespective of input frequency. Irrespective of such considerations, and given the non-linear and self-sustained nature of neuronal oscillations, the author would like to venture a guess that the true underlying mechanism of sequential effects is some form of non-linear synchronization, of which linear resonance is only an approximation.

The complex spatio-temporal pattern formation capabilities displayed by some non-linear dynamical systems may also come in use when attempting to model another aspect of sequential effects: the proposed spatial filtering element. The behaviour of the pre-motor component of sequential effects - S-LRP - suggests a not so straightforward filtering mechanism, one that is capable of generating a prediction based on spatial location independently of time. That some form of complex behaviour should emerge is perhaps unsurprising if we consider the putative locus of the pre-motor element of sequential effects, the visual cortex, known to generate complex geometric patterns spontaneously in conditions of sensory deprivation (Sacks, 1999), a phenomenon which has been successfully captured with a non-linear pattern formation system (Bressloff, Cowan, Golubitsky, Thomas, & Wiener, 2002). Such a system is expected to exhibit considerably more complex types of behaviour than resonance, and among these may potentially lie the explanation to for the proposed time-independent spatial filtering element of sequential effects.

5.5 Wider implications and conclusion

Guiding this work was an overarching philosophical vision which deserves an attempt at being made explicit. The core idea of this vision is that the mind does not set out as a clean slate to learn all the possible regularities which might exist in the universe, but rather that it contains an implicit 'catalogue' of such regularities which it attempts to match with sensory data, with the best fitting pattern being used in order to learn and to guide behaviour. Moreover, while such a catalogue is necessarily influenced by experience, it is not assumed to be built from all the patterns seen before, but rather to be determined largely by the very physics of the brain, and in particular by the patterns that it is able to form *spontaneously*. The inspiration for this view on how the mind works comes from multiple sources, but centred around one main theme: if there is no sensory input, the brain tends to form patterns of its own (Bressloff et al., 2002); if the information available happens to be random humans often find a pattern despite all evidence to the contrary (Nickerson, 2002). In general, it would seem that the mind fills the gaps in knowledge with the patterns that it either forms spontaneously, or with those closest to what little information is available. These patterns impart structure on what is perceived and learned, which becomes a mixture of reality with what is found *a priori* in the mind.

Sequential effects are in this context taken to be the simplest manifestation of the above stated principles: repetitions and alternations both have corresponding forms of dynamical behaviour in the brain, which are latent in the behaviour of even a single oscillator. In this sense a repeating or alternating pattern is not so much learned as *detected*, by resonance or a more complex pattern matching mechanism. In other words, the question implicitly being asked in sequential effects is not 'what pattern is present in the sequence?' but rather two separate questions are being posed at the same time: 'is the pattern repeating?' and 'is the pattern alternating?'. The last two questions are far simpler than the first, which heralds one possible advantage of pattern matching over some universal learning mechanism: economy of resources. Searching for just any type of pattern implies that such a pattern can be of any length, which would require storing large amounts of information. However, the exponential decay observed in sequential effects and more generally in human memory points to a very short storage capacity, on the order of four or five stimuli. How are we to reconcile this rapid decay with the obvious fact that humans learn on all time scales? This answer to this question may be all too simple: the pattern was already latent in the mind, it was merely activated.

There are several corollaries to the pattern matching view. For instance, learning would be seen as a process of selecting those patterns which are most useful in any given situation, while at the same time trimming those which are not useful, a process more akin to tuning a musical instrument than a processing computer. That some form of trimming is indeed taking place is suggested by the much larger number of neuronal connections present in the infant brain when compared to that of an adult. Another corollary is that the classical problem of inductive inference might not be a problem at all: we do not abstract away from a particular to a general domain, we simply match the same pre-existing abstract structure to similar problems, a view which finds echo in the writings of some philosophers (Popper, 1959). Thinking in terms of pattern matching may also provide a natural answer to the problem of essentialism: ever since Plato and his cavern scholars have noted a human tendency to find an essence to the structure of similar problems which seems to transcend physical reality. The reason for this apparently perfect 'world of ideas' might be that when the same pattern is found in two different domains, it is matched to exactly the same pre-existing dynamical state in the brain. Furthermore, by virtue of the very nature of spontaneously generated patterns these tend to be regular and symmetrical, and this may explain the apparent perfection of ideas as opposed to the crudity of reality. Coming back again to sequential effects, one can make an oscillator resonate with a slightly irregular input, but once this is gone the oscillator will eventually display a perfectly regular oscillatory motion. The input may have consisted of noisy square impulses, but the remaining essence is a perfect sinusoid.

Finally, the pattern matching view suggests a course for research into the mind itself: if attempting to understand how humans learn in a particular situation, proceed by finding the system which can reproduce *spontaneously* the pattern which is being learned. It was in fact a similar reasoning which gave rise to the idea of using oscillators to model sequential effects, so this very thesis can be taken as an example of this principle at work.

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A

Appendix A: Supplementary information to Chapter 3

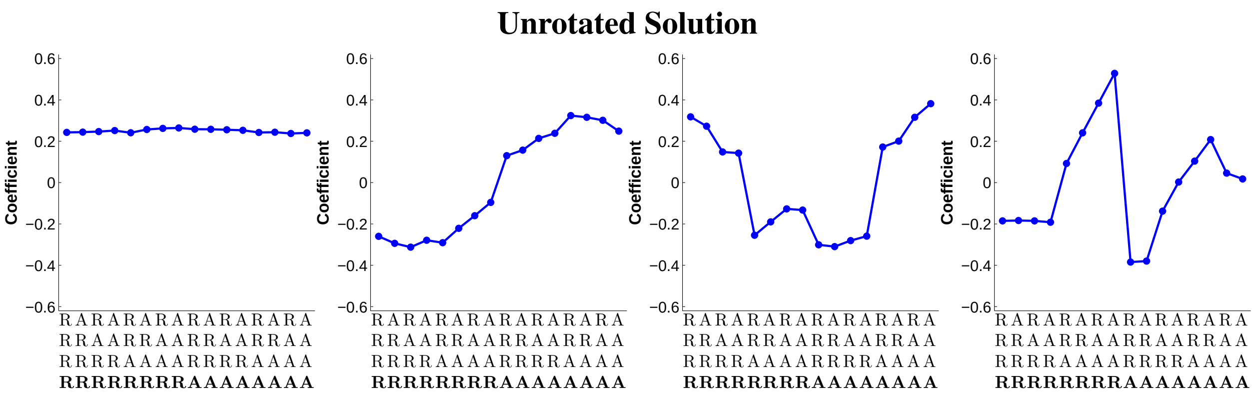
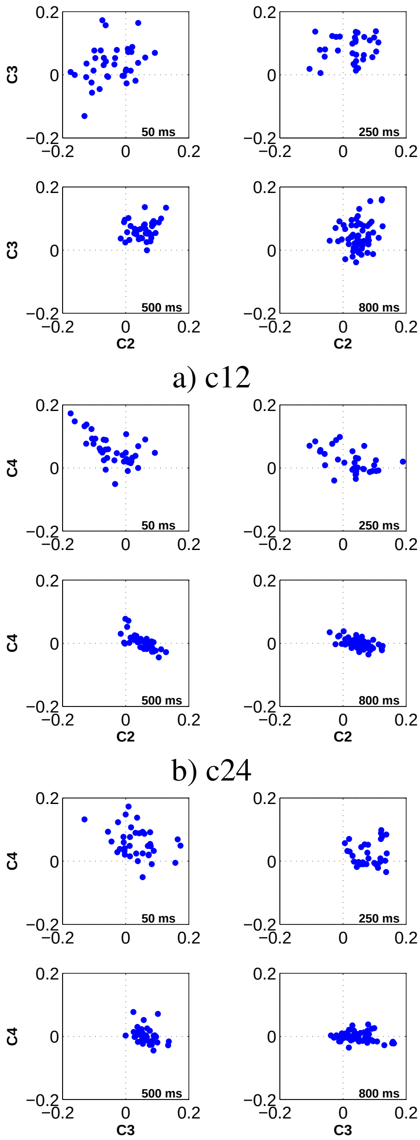


FIGURE A.1: Unrotated latent structure. Coefficient patterns of the four first components extracted with PCA, before rotation. From left to right, components are ordered by amount of variance explained: 78%, 12%, 4.9% and 1.25%; variance explained by the remaining components goes 0.9%, 0.54%, 0.39%,...,0.089%. The first component is clearly interpretable the effect of overall individual mean RT; the second and third components - C2 and C3 - can be interpreted as the effects of the last and second-to-last events respectively; the fourth component exhibits an approximate dependence on the second-to-last independently of the last event, visible as an overall similarity between the left and right halves of the plot.



a) c12

b) c24

c) c34

FIGURE A.2: Individual scores for all 158 participants on the three latent components related to sequential effects.(a)-(b) panels with scores on one particular component plotted against those on another component. Within each panel, individual RSI subgroups are plotted separately. Details of how the scores were calculated are detailed in the text. Note that the scores were those obtained from the global PCA analysis including all participants. Note that, for a 500 and 800 ms RSI, most subjects have a score on C4 close to zero, reflecting the absence of this component for long RSI values (panels (b) and (c)). In addition, note the correlation between C2 and C4 score for low RSI (middle panel, 50 and 250 ms subgroups) discussed in the main text. Finally, observe the single subject which exhibits a significantly negative score on both C2 and C3 (top panel, 50 ms subgroup); note that the good qualitative nature of the fit to this subject (see Appendix C) is indicative that these negative scores may not be spurious. In other words, it might be possible - yet rare - to have a negative score on both C2 and C3.

Recalculation of component scores

Under normal circumstances the PCA model's prediction for the j-th individual is obtained through $\mathbf{x_j} = \boldsymbol{\mu} + \sum_{i=1}^{N} s_i^j \mathbf{C_i}$, where μ is the grand mean array, N is the number of components retained, $\mathbf{C_i}$ is the coefficient pattern for each component and s_j^i is the the score of subject j on component i. If we replace the grand mean with a simple constant, our model becomes $\mathbf{x_j} = b_j + \sum_{i=1}^{N} s_i^j \mathbf{C_i}$, with b equal to individual overall mean RT. If we further discount the mean RT by subtracting it from each individual, we can set the baseline RT at zero for all subjects, in which case our model further reduces to $\mathbf{x_j} = \sum_{i=1}^{N} c_i^j \mathbf{V_i}$, where the notation has been changed to highlight the fact that the scores are now linear coefficients and the coefficient patterns simply vectors equal to the coefficient patterns identified with PCA. Individual scores will be estimated by fitting a linear combination of coefficient patterns to each individual's data with the overall mean subtracted. As expected, the linear coefficients thus obtained are almost perfectly correlated to the scores obtained with PCA ($r = 0.92$, $r = 0.97$ and $r = 0.89$ respectively for C2, C3 and C4, $p << 1e-3$ in all cases). It is to these linear coefficients that we refer throughout as individual 'scores'.

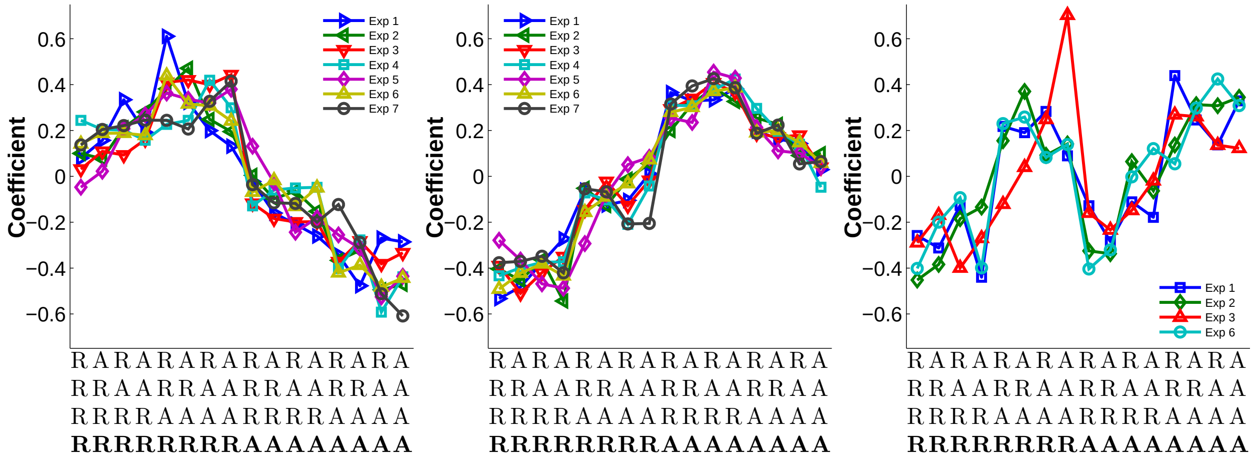


FIGURE A.3: Coefficient patterns obtained by performing a PCA on different subgroups of participants performing different experiments. Plots show, from left to right, C2, C3 and C4. Experiments 8 and 9 were not analysed as they do not include enough subjects allowing for PCA to be conducted. All experiments considered (1 through 7) yielded a C2 and C3 significantly similar to those obtained in the global analysis including all subjects. Only experiments 1, 2, 3 and 6 yielded a C4 significantly similar to the global components. The reason for this is probably the small number of participants in each subgroup together with the fact that C4 explains a relatively small amount of variance. Together, these results clearly indicate that the latent structure obtained with the global analysis is not an artifact of grouping different experiments.

Invariance of latent structure with RSI and Experiment

The non-standard approach of analysing data from multiple experiments together might raise concerns regarding whether the latent structure is constant across conditions. For instance, it would be possible in principle for a component to be present exclusively in one experiment in which case our results would be an artefact of mixing qualitatively different results. In order to dispel these doubts extra care was taken to demonstrate that the latent structure of sequential effects is invariant with respect to both RSI as well as experimental design. This is particularly relevant in the case of different RSI values, given the prevalent view that short and long RSI sequential effects are qualitatively different. In order to evaluate how the latent structure varies, the same analysis which was conducted for all subjects together will be performed in different subgroups separated according to RSI, irrespective of experiment, and according to experiment performed, collapsing across RSI. Different latent structures were obtained, one for each subgroup, and four components were retained each case. It was then necessary to evaluate whether these components were the same as the ones in the global pool of subjects, and this was done with recourse to Tucker index of similarity (Gorsuch, 1983) according to the following procedure: the index was calculated between all putative components of the same type (say C1), one at a time, and the global corresponding component (C1 in this case), and similarly for the remaining three components. The significance of the calculated coefficient values was assessed by holding one vector fixed and randomly permuting the other, allowing a p value to be estimated (Abdi, 2007).

B

Appendix B: Random RSI experiment

Participants

Ten subjects took part in this experiment. Participants were undergraduate students at the University of Adelaide and were given course credit for their participation; all gave their informed consent to taking part in the experiment; all had normal or corrected-to-normal eyesight.

Stimuli

Stimuli consisted of two horizontally displaced dots, approximately 5 cm apart and 1 cm in diameter.

Procedure

Subjects sat approximately 60cm away from the computer screen, inside a darkened room. The stimuli were white (approximately 3 cm tall) and the background was gray. Stimuli were displayed using Psychophysics Toolbox 3 and Matlab r2008a on a 15" Macintosh MacBook Pro running MacOSX 10.6. Responses were made using a Cedrus RT-530 response time box, which has one central round button surrounded by four rectangular buttons. The RT box was placed to the right of the computer if the subject was right-handed, and to the left if left-handed.

Responses were carried out using the two index fingers, one placed on the left button and one on the right button of the response box. Subjects were instructed to respond as quickly and accurately as possible to the stimulus by pressing the button on the same side as the stimulus shown. After a random response-stimulus interval, different for every trial, the next stimulus appeared. Each value of RSI was drawn randomly from a uniform distribution in the interval [50 1000] ms. The only feedback was a beep whenever a button was pressed.

The experiment consisted of 13 blocks of 120 trials each, with a small break in between each block and a longer break (approximately 10 min) after the seventh block. Each subject was given one block of training before beginning. The data from the training blocks was not used in the analysis. Sequences were generated randomly for each subject, with each element sampled from a uniform distribution over the elements. The relative frequencies of both the stimuli were equal within each block and so for the whole experiment.

C

Appendix C: All individual data

This appendix shows all 158 individual participants in the seven experiments described in Chapter 3, organised by experiment and response-stimulus interval (RSI). Each individual panel in each of the plots shown in the next pages corresponds to data from a single individual performing a particular experiment with a particular RSI. Solid blue lines show mean reaction times. Red dashed lines show a linear combination of the coefficient patterns of the first four latent components identified with PCA (see Chapter 3). Inset plots show the scores on the three components responsible for sequential effects - C2, C3 and C4.

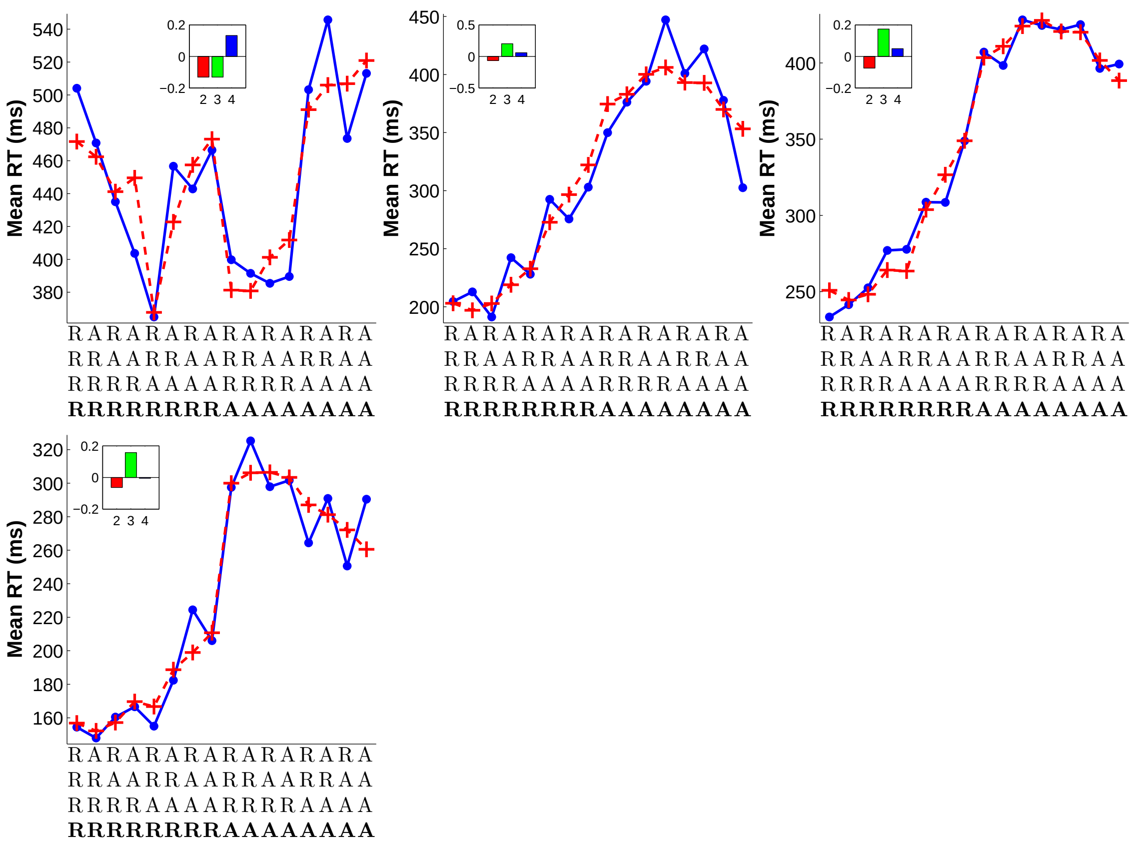


FIGURE C.1: Individual data for Experiment 1 with a 50 ms RSI.

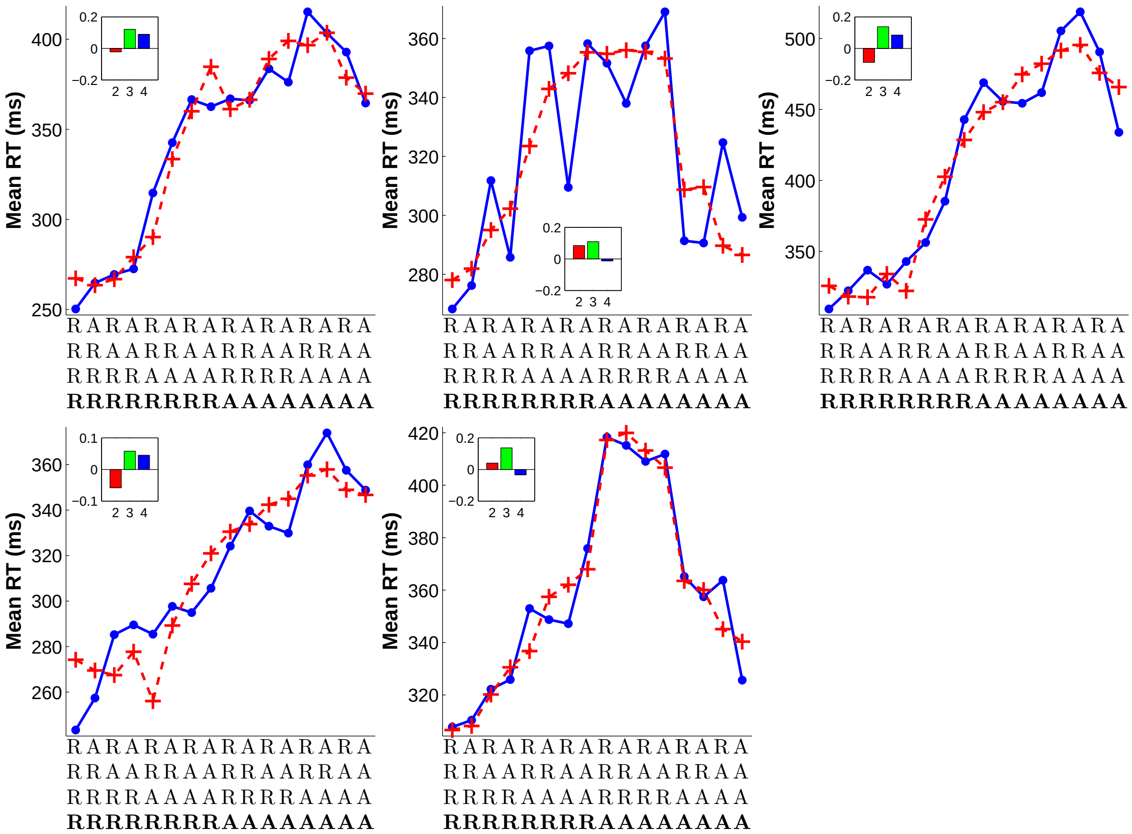


FIGURE C.2: Individual data for Experiment 1 with a 250 ms RSI.

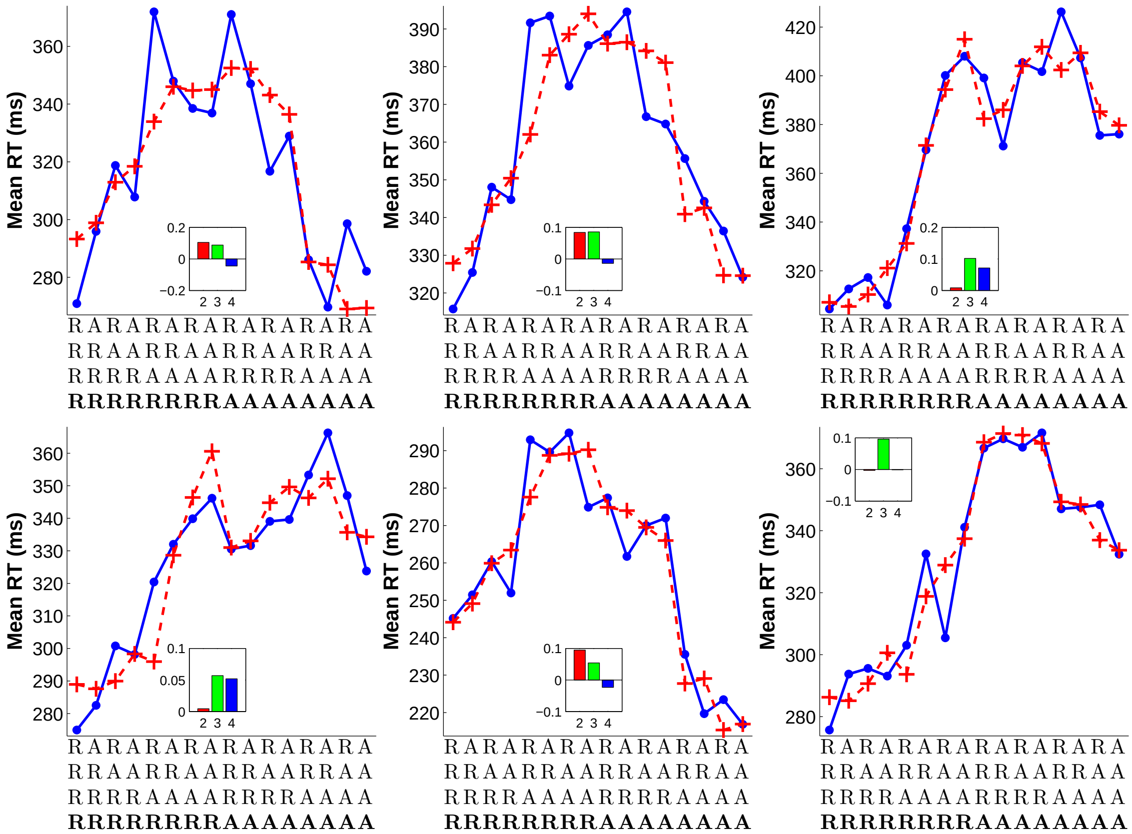


FIGURE C.3: Individual data for Experiment 1 with a 500 ms RSI.

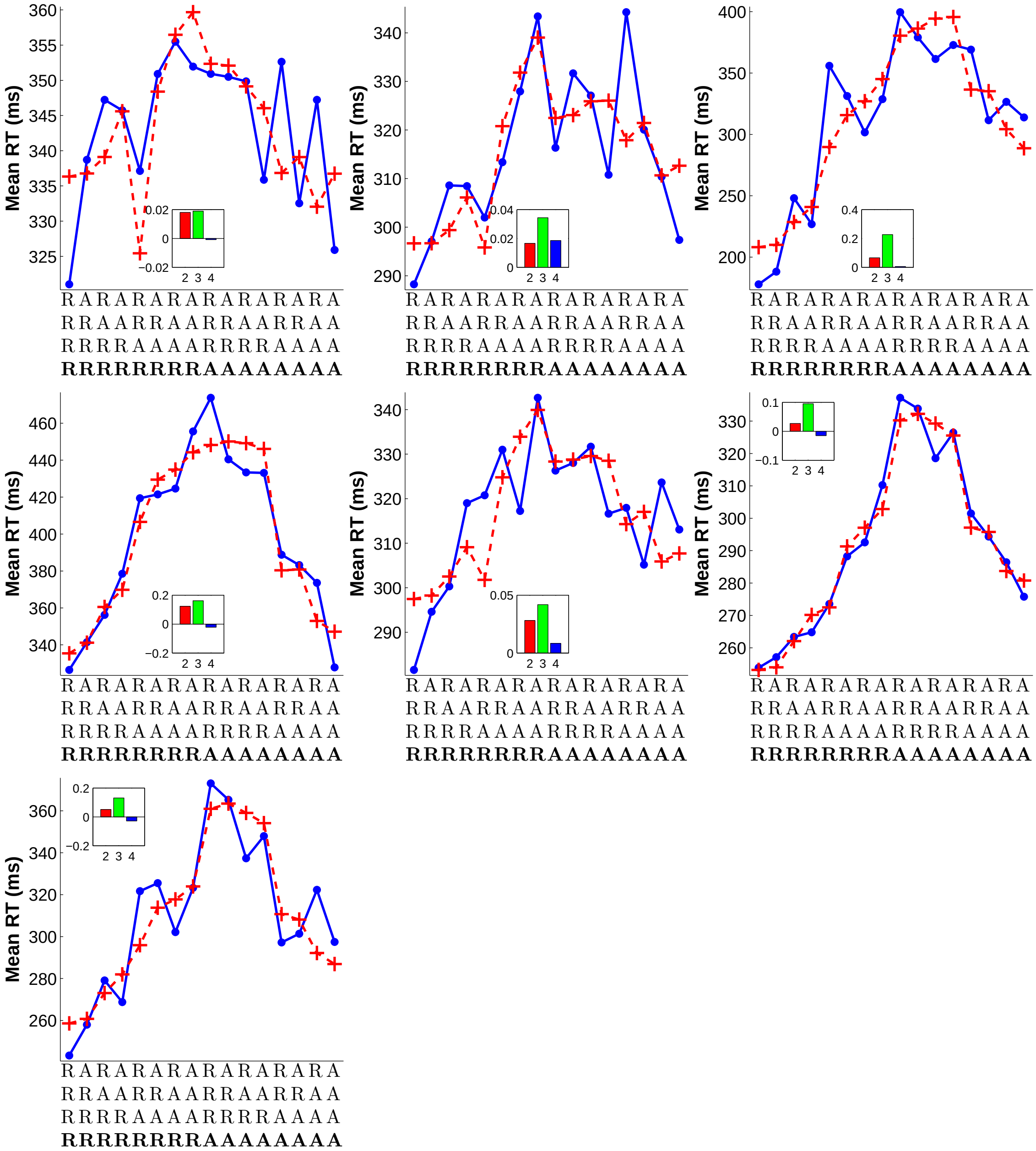


FIGURE C.4: Individual data for Experiment 1 with a 800 ms RSI.

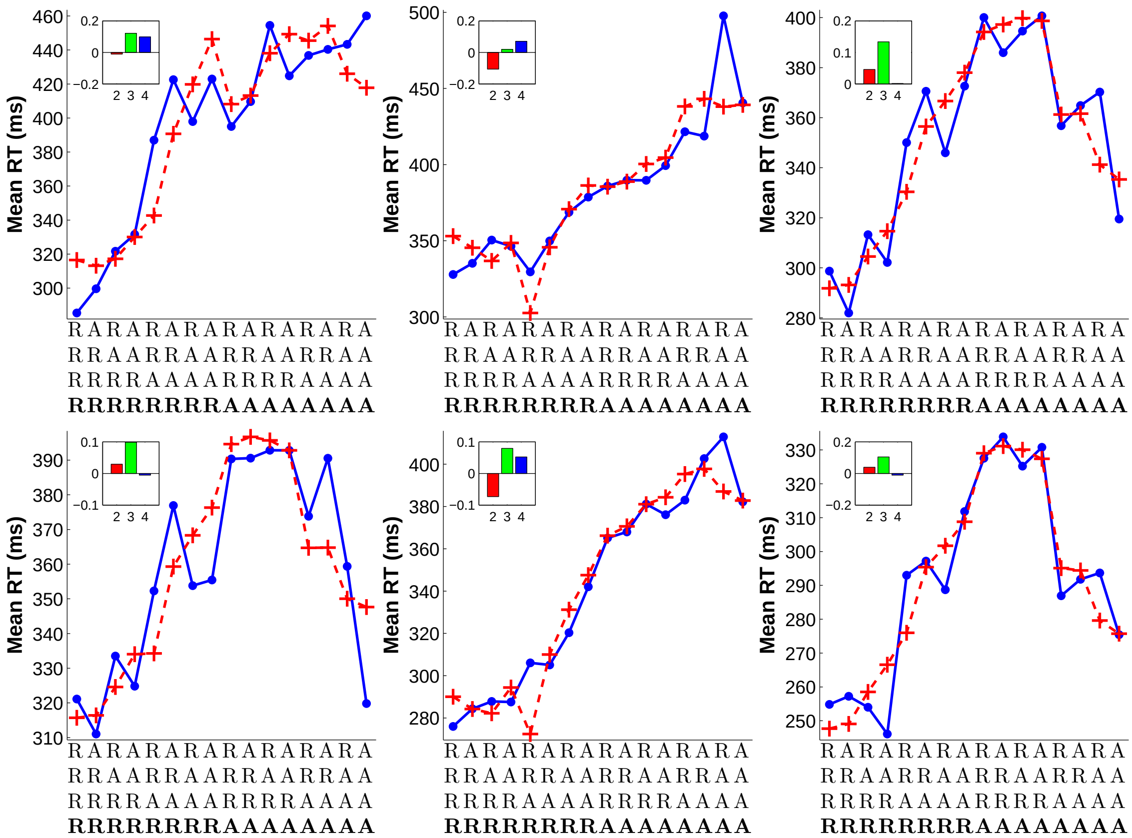


FIGURE C.5: Individual data for Experiment 2 with a 250 ms RSI.

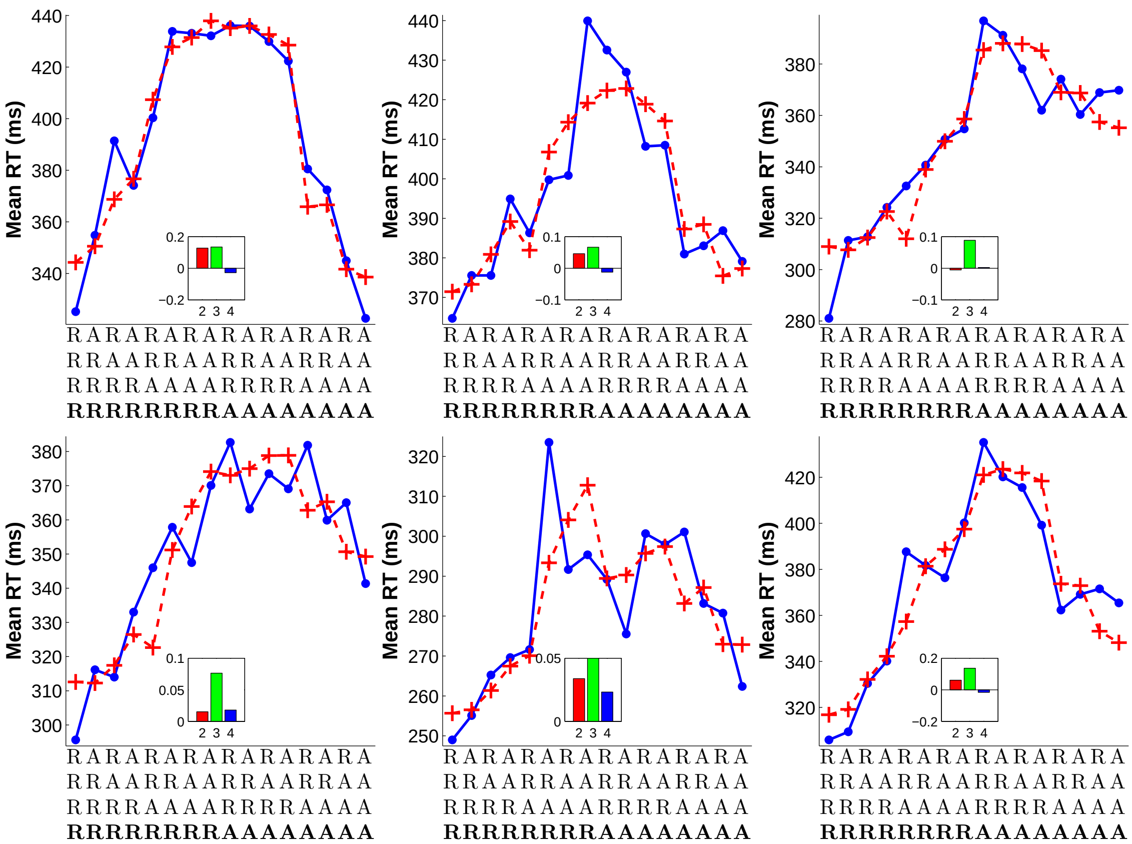


FIGURE C.6: Individual data for Experiment 2 with a 500 ms RSI.

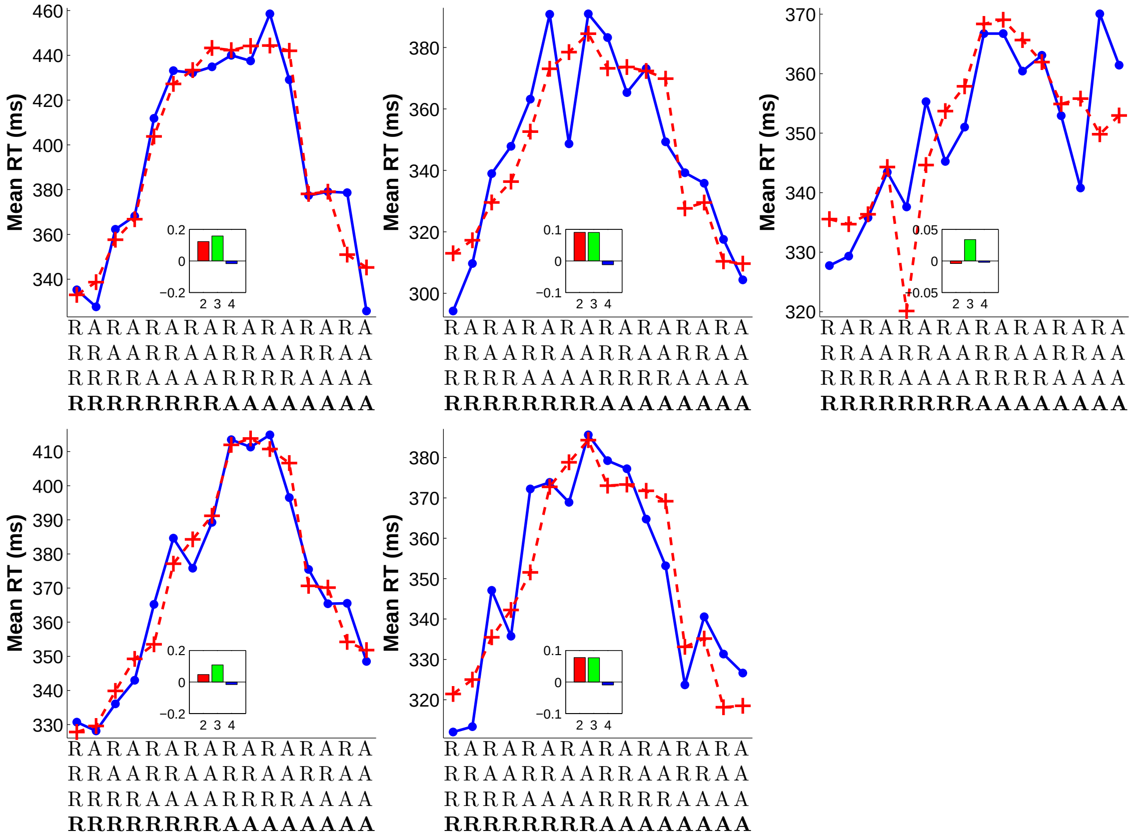


FIGURE C.7: Individual data for Experiment 2 with a 800 ms RSI.

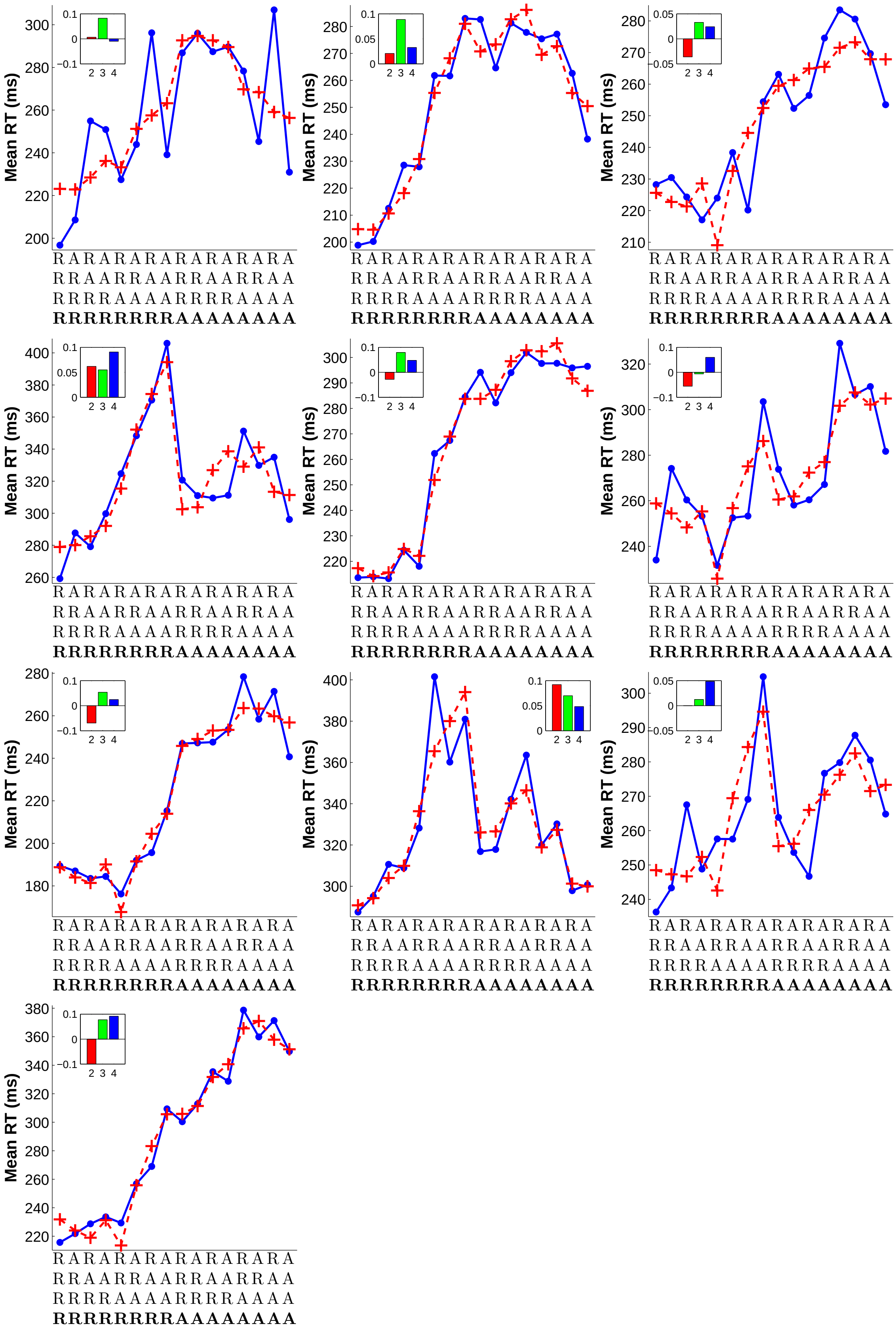


FIGURE C.8: Individual data for Experiment 3 with a 50 ms RSI.

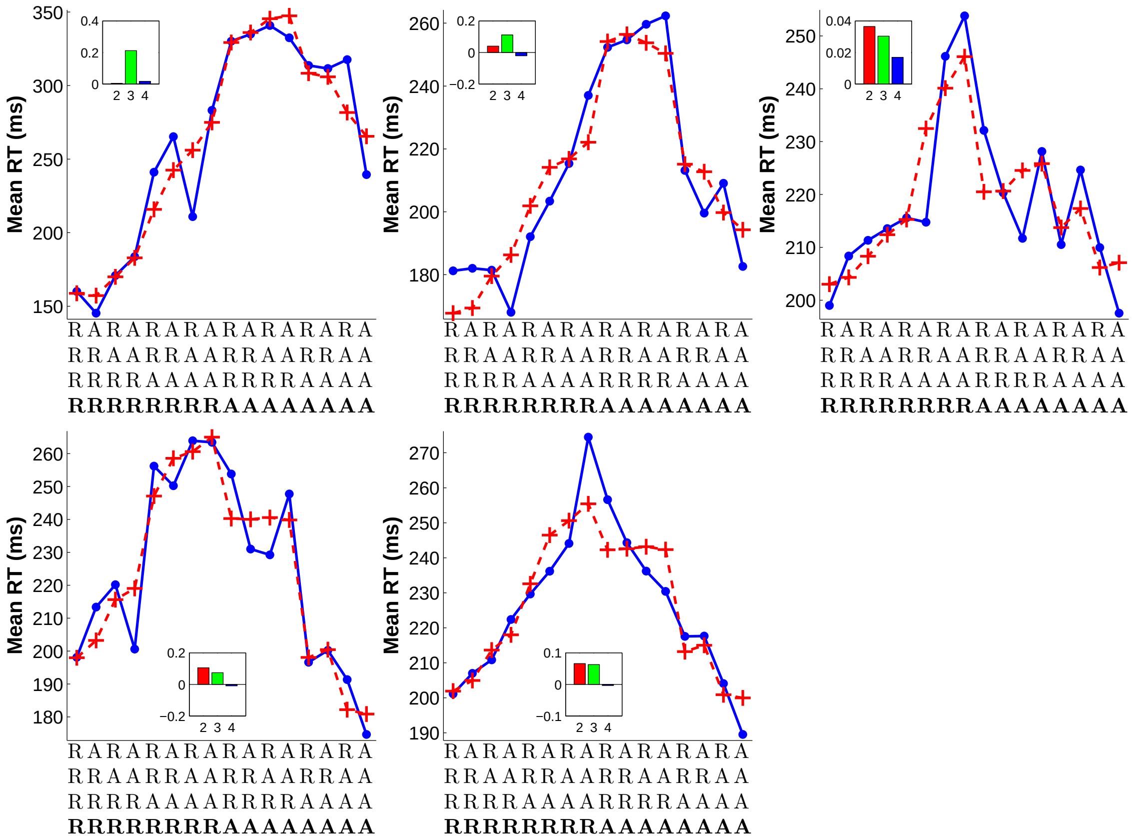


FIGURE C.9: Individual data for Experiment 3 with a 250 ms RSI.

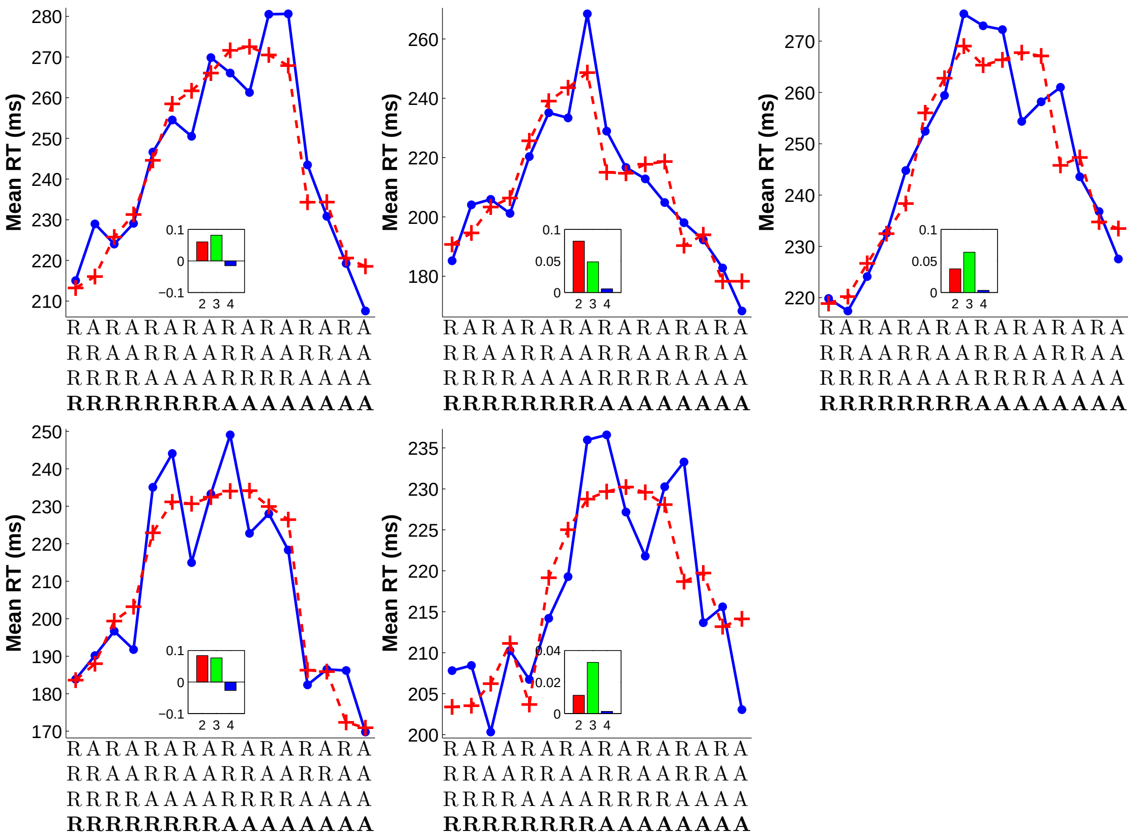


FIGURE C.10: Individual data for Experiment 3 with a 500 ms RSI.

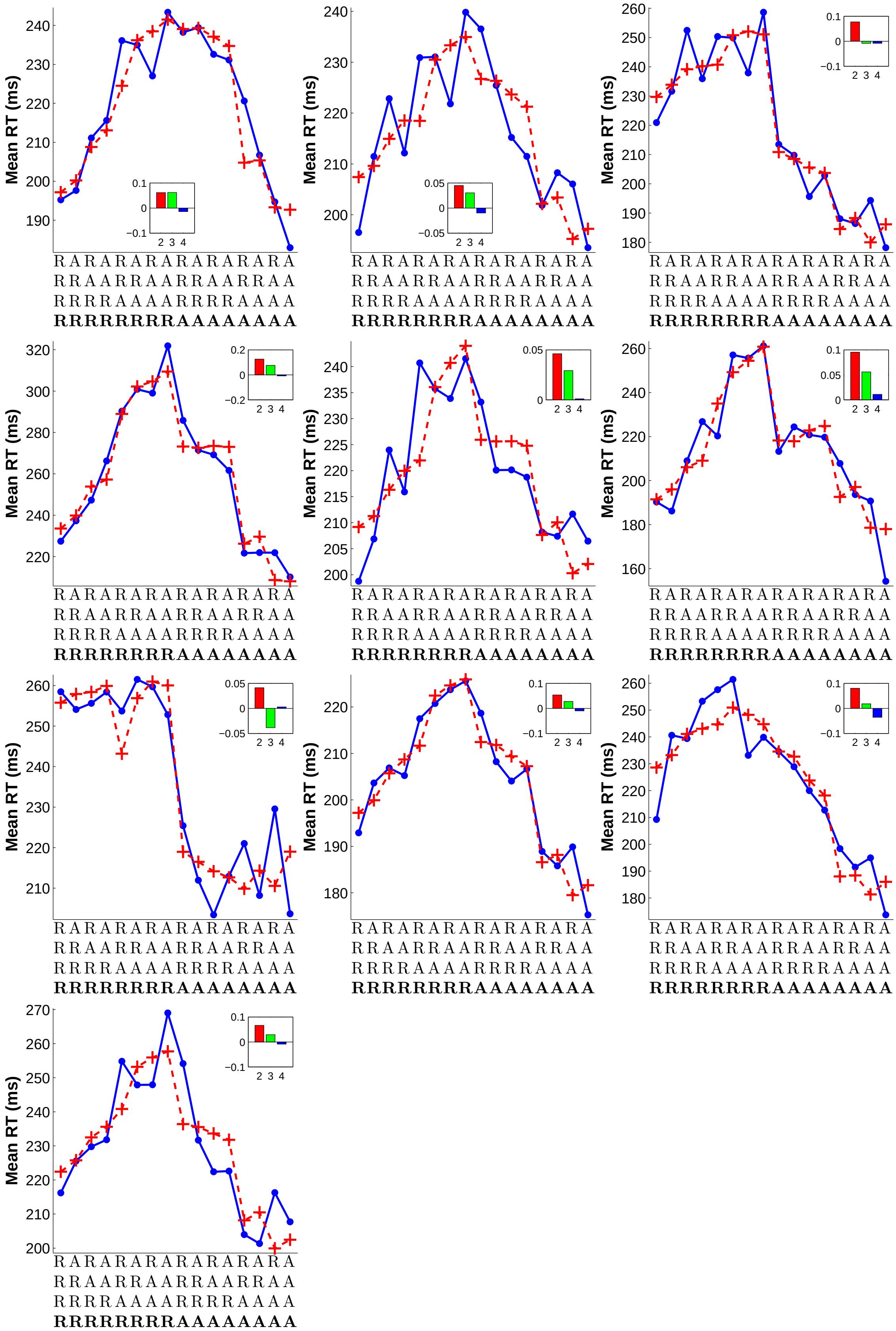


FIGURE C.11: Individual data for Experiment 3 with a 800 ms RSI.

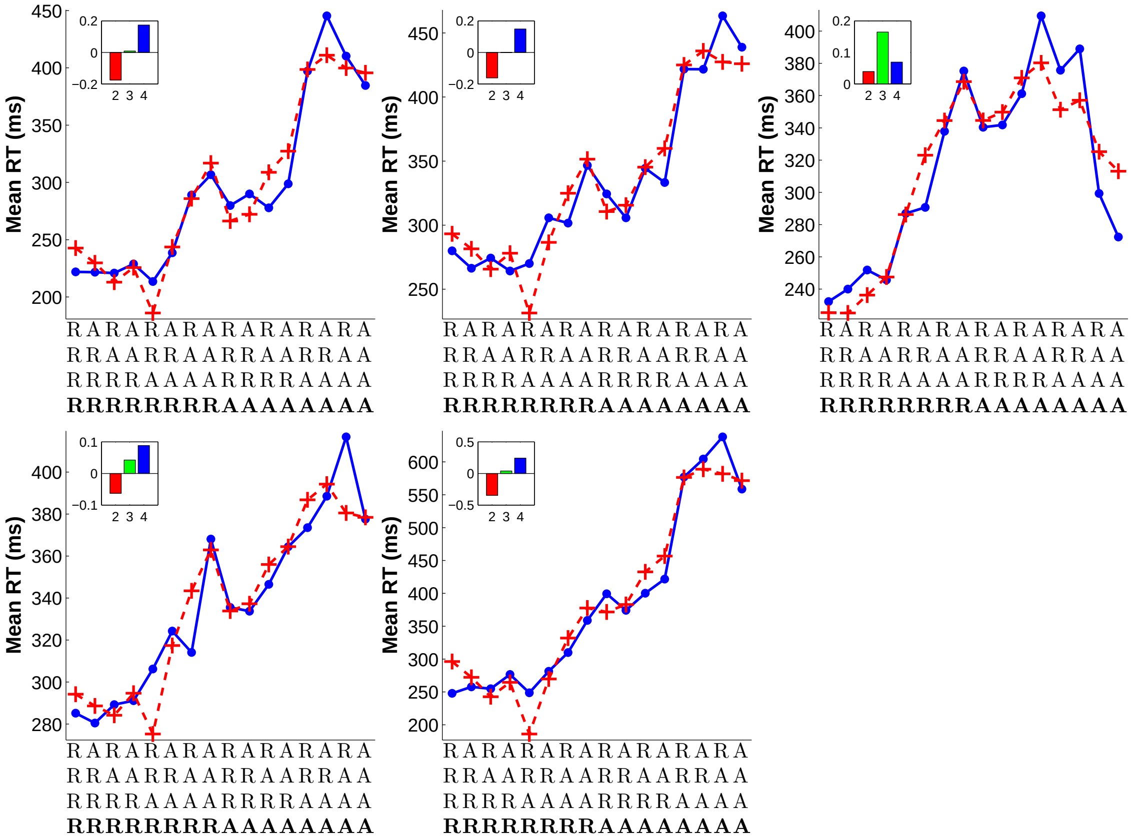


FIGURE C.12: Individual data for Experiment 4 with a 50 ms RSI.

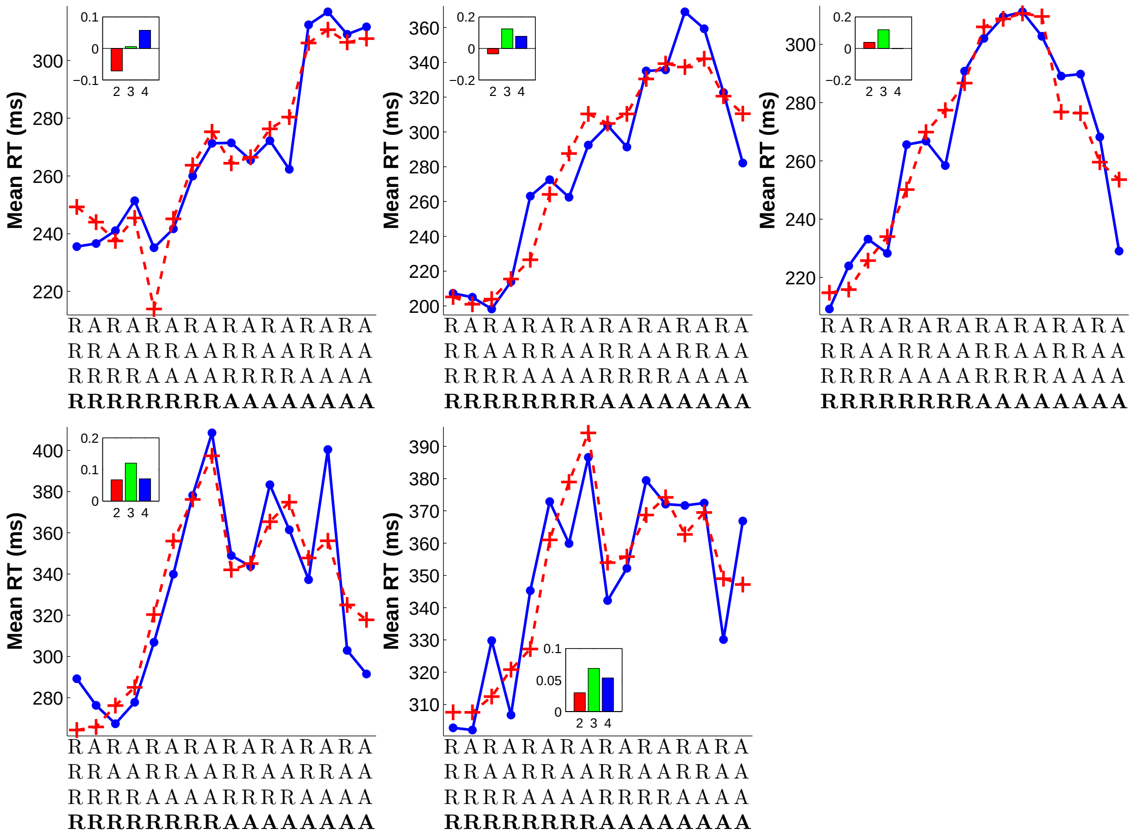


FIGURE C.13: Individual data for Experiment 4 with a 250 ms RSI.

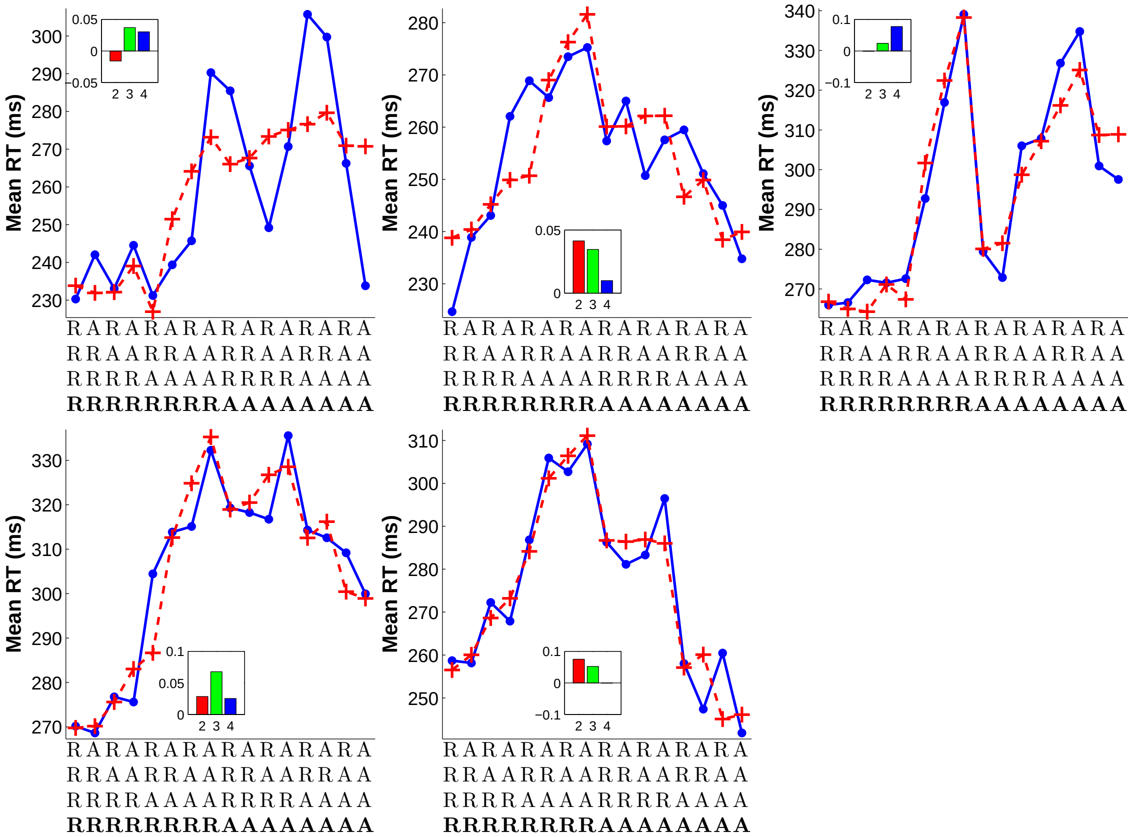


FIGURE C.14: Individual data for Experiment 4 with a 500 ms RSI.

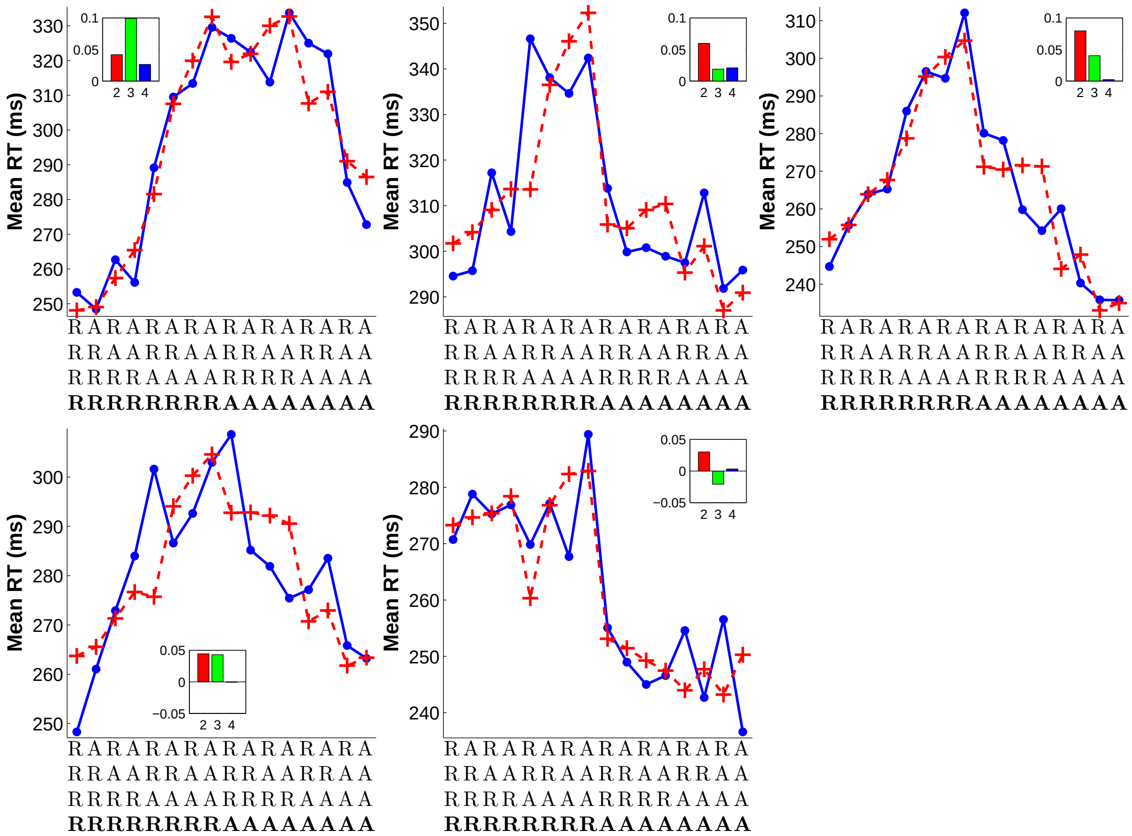


FIGURE C.15: Individual data for Experiment 4 with a 800 ms RSI.

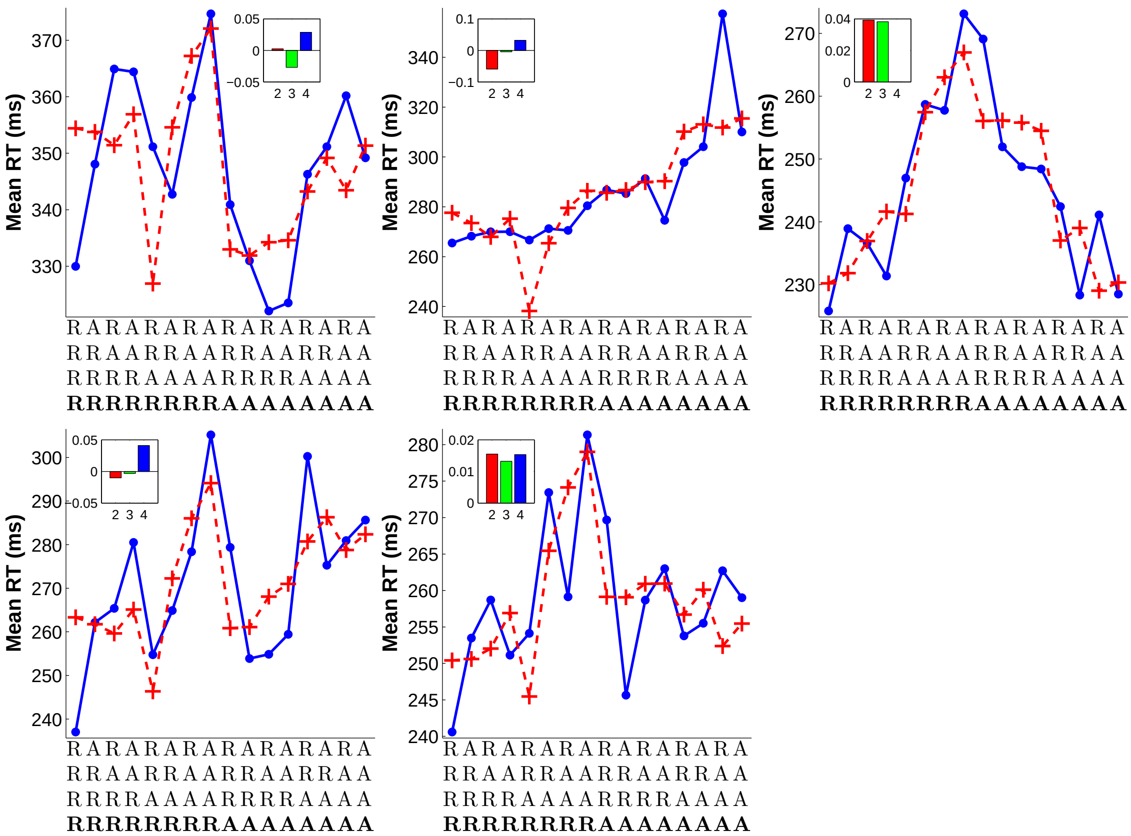


FIGURE C.16: Individual data for Experiment 5 with a 50 ms RSI.

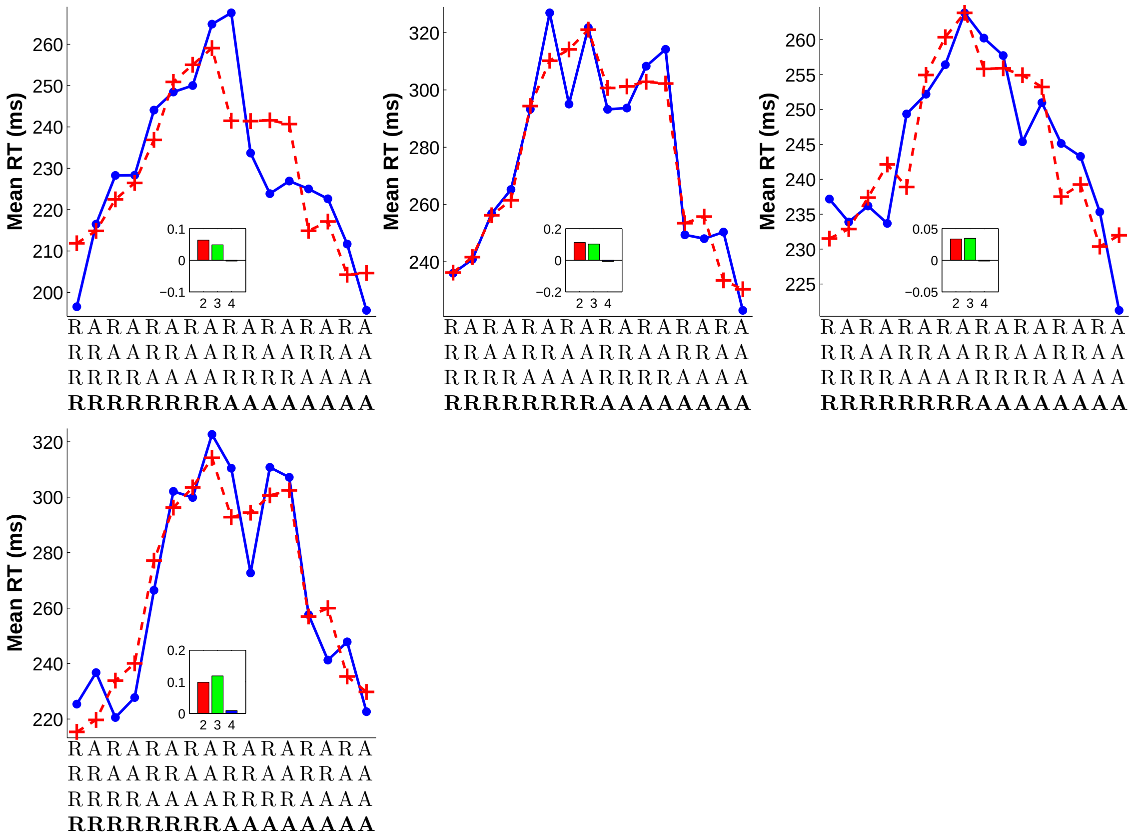


FIGURE C.17: Individual data for Experiment 5 with a 250 ms RSI.

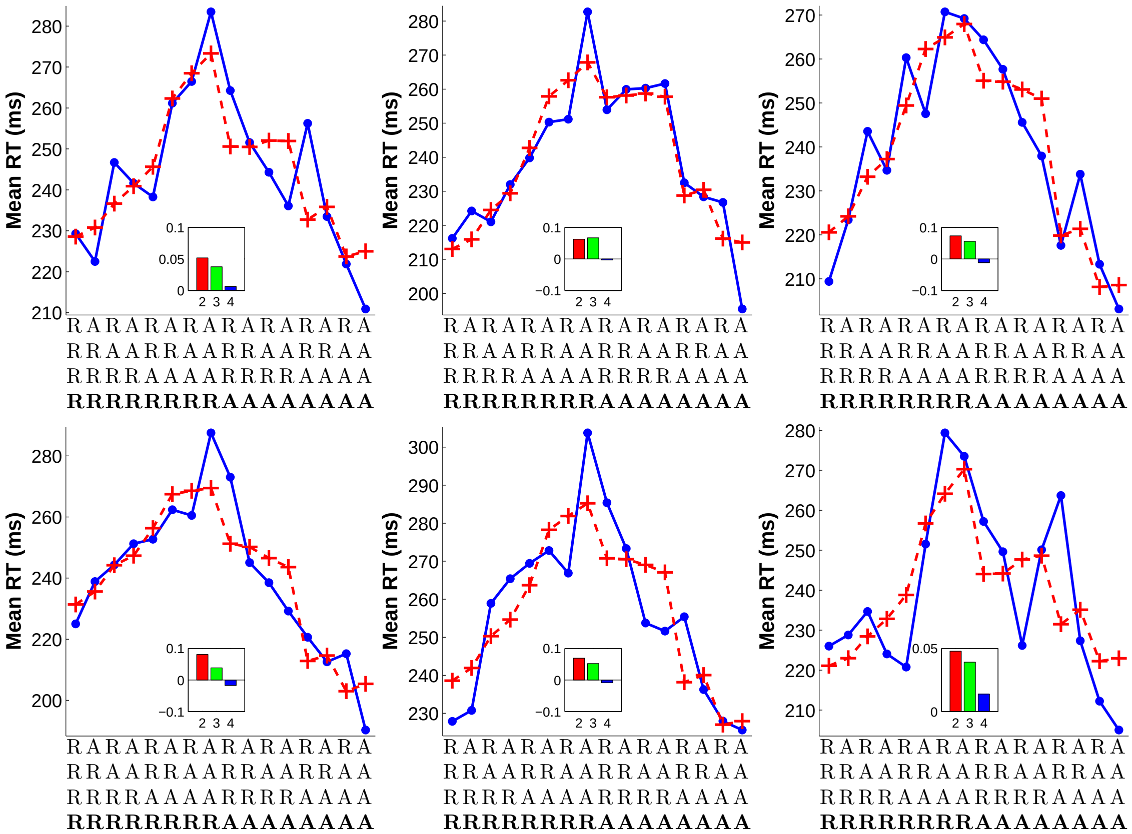


FIGURE C.18: Individual data for Experiment 5 with a 500 ms RSI.

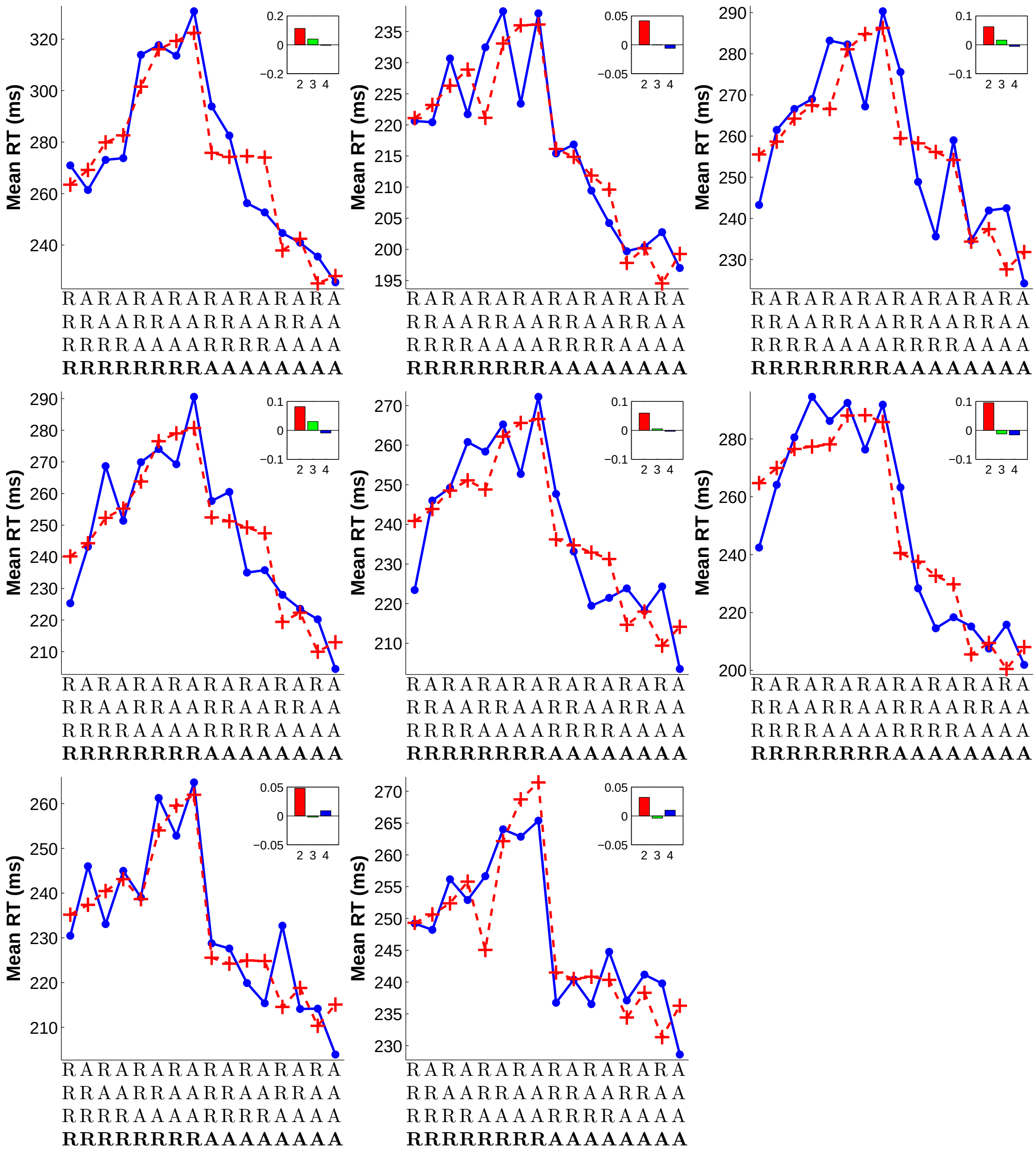


FIGURE C.19: Individual data for Experiment 5 with a 800 ms RSI.

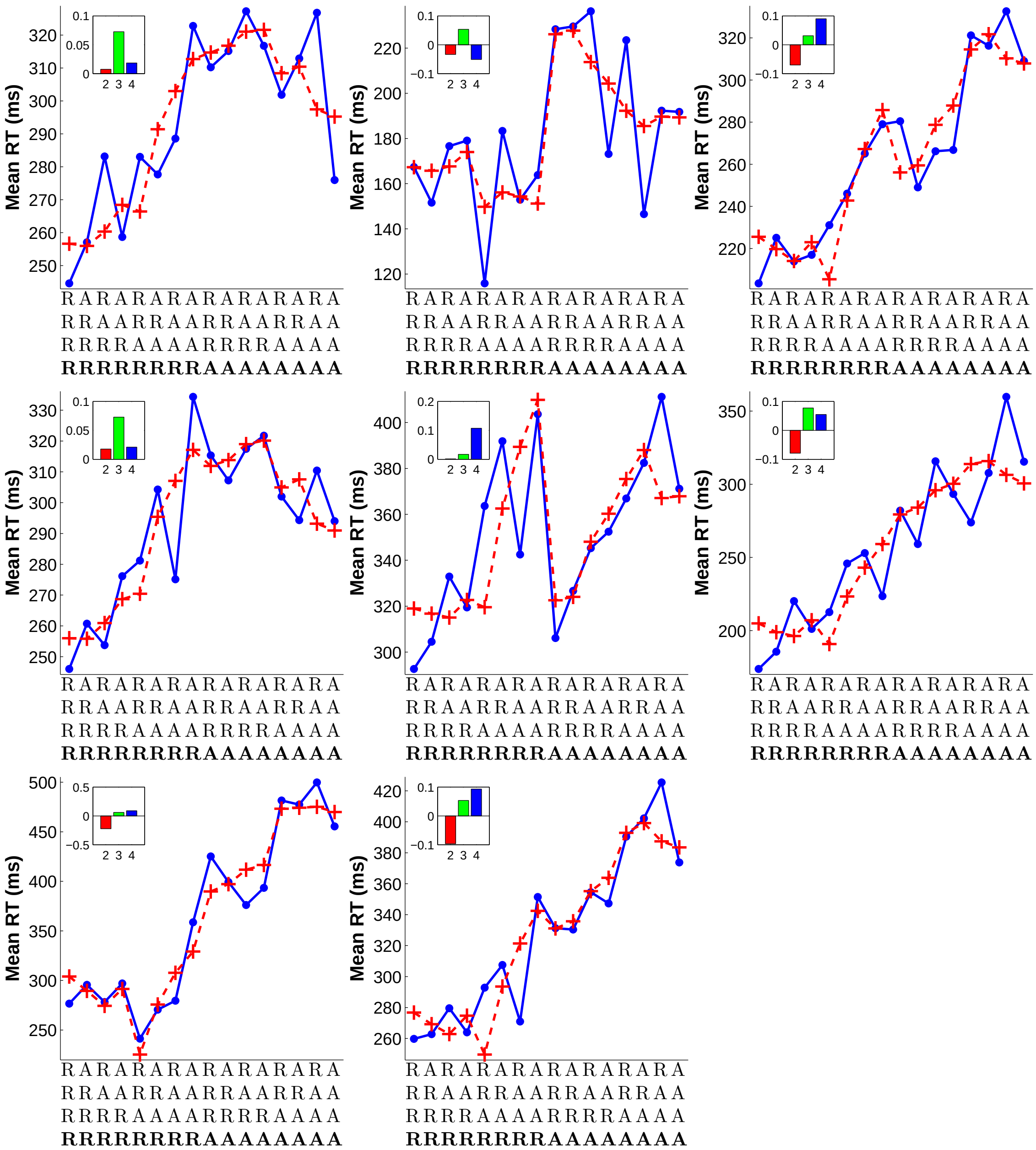


FIGURE C.20: Individual data for Experiment 6 with a 50 ms RSI.

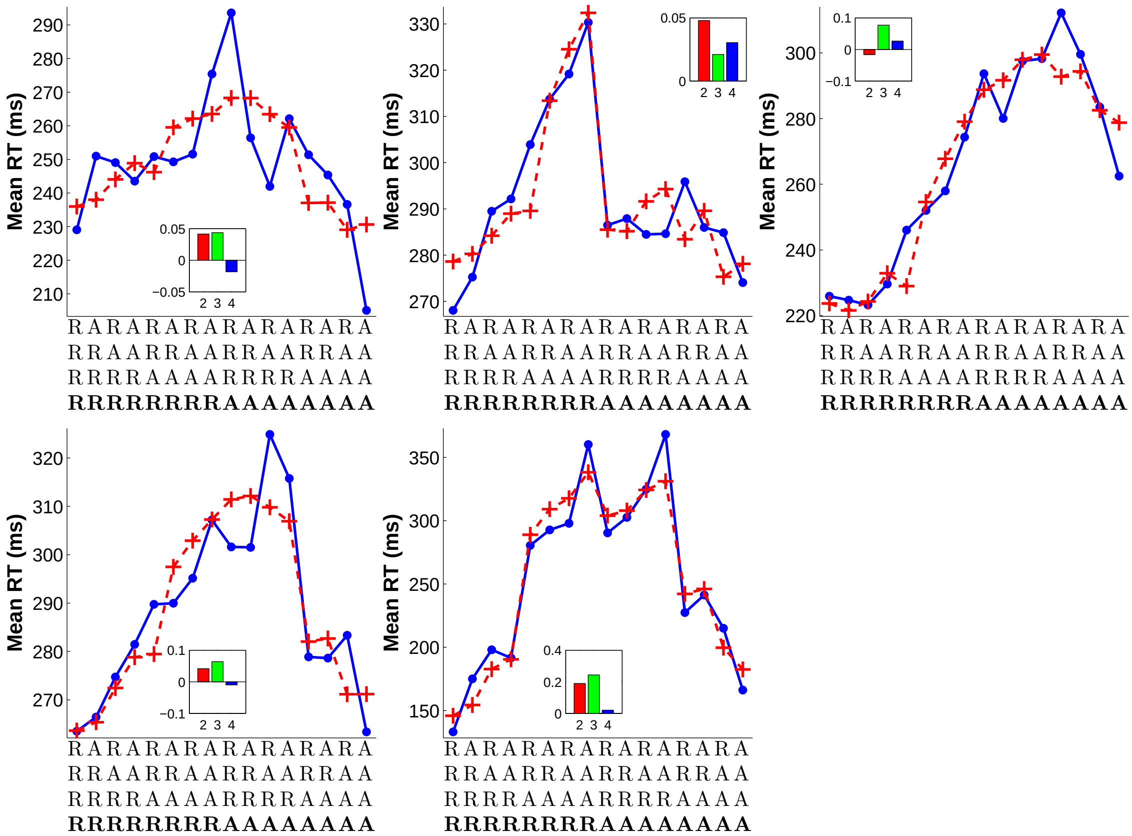


FIGURE C.21: Individual data for Experiment 6 with a 250 ms RSI.

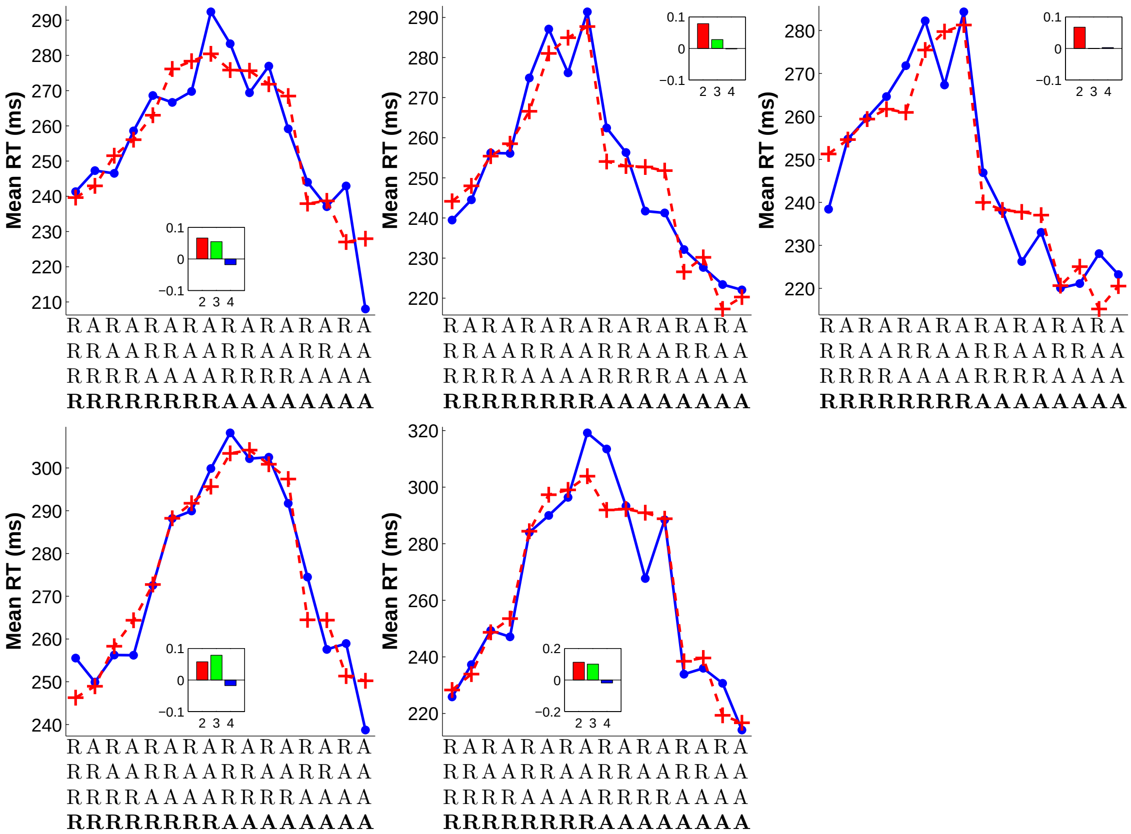


FIGURE C.22: Individual data for Experiment 6 with a 500 ms RSI.

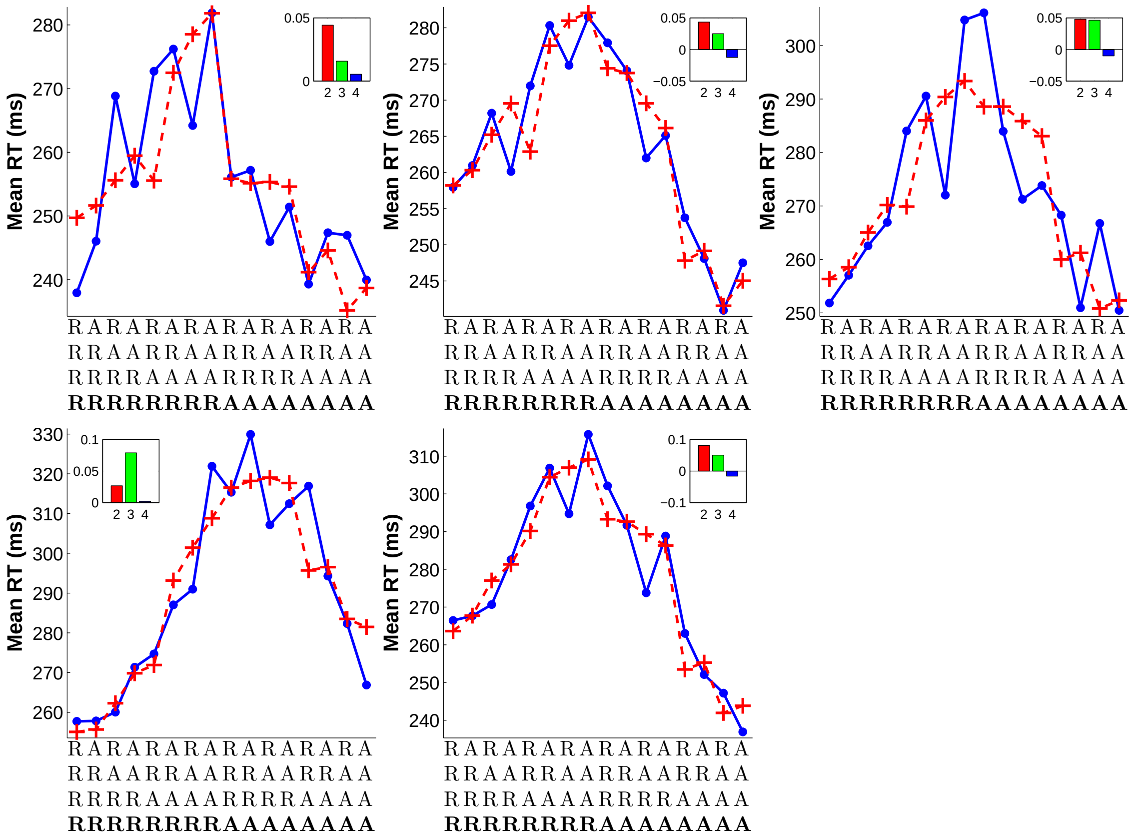


FIGURE C.23: Individual data for Experiment 6 with a 800 ms RSI.

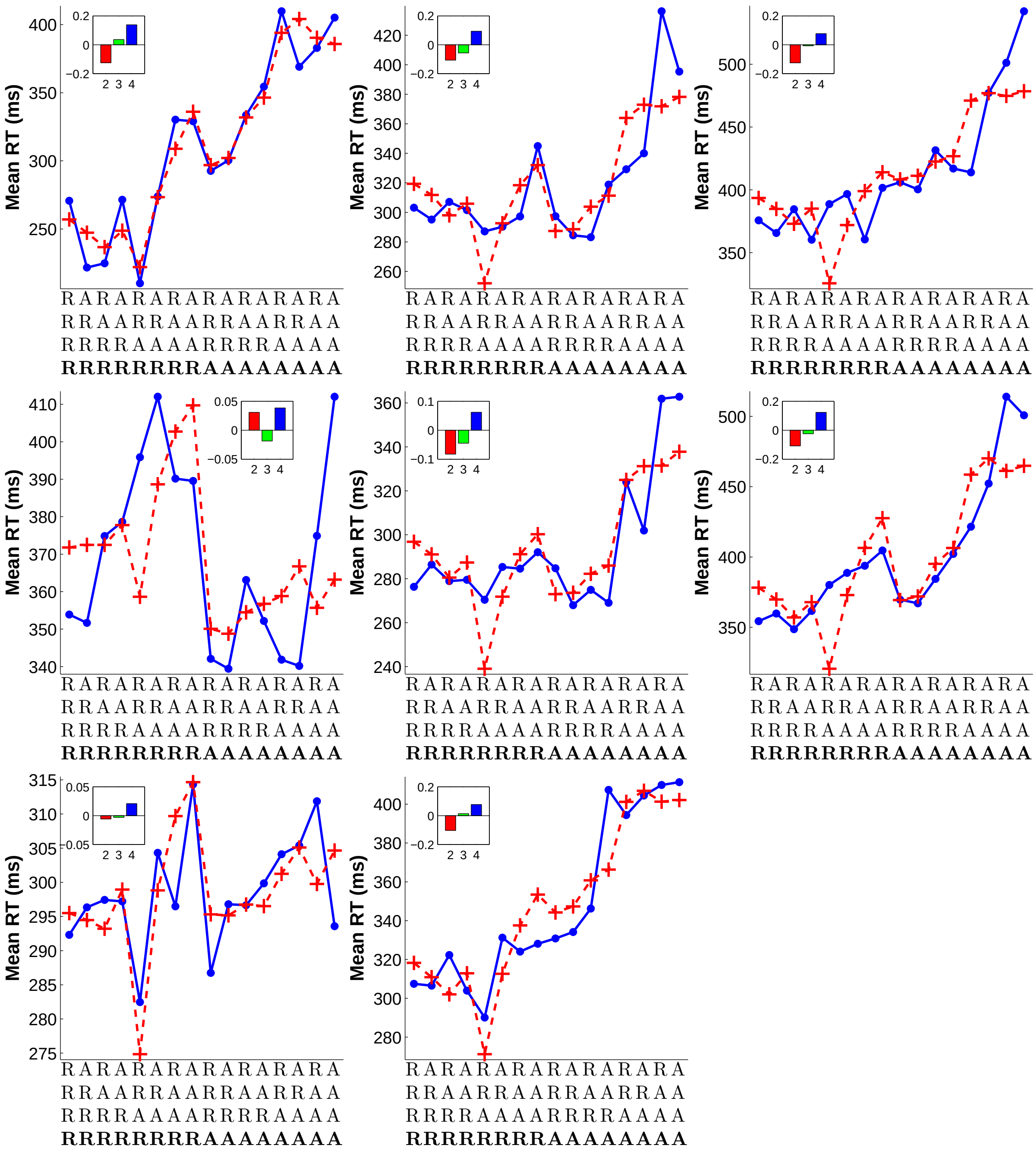


FIGURE C.24: Individual data for Experiment 7 with a 50 ms RSI.

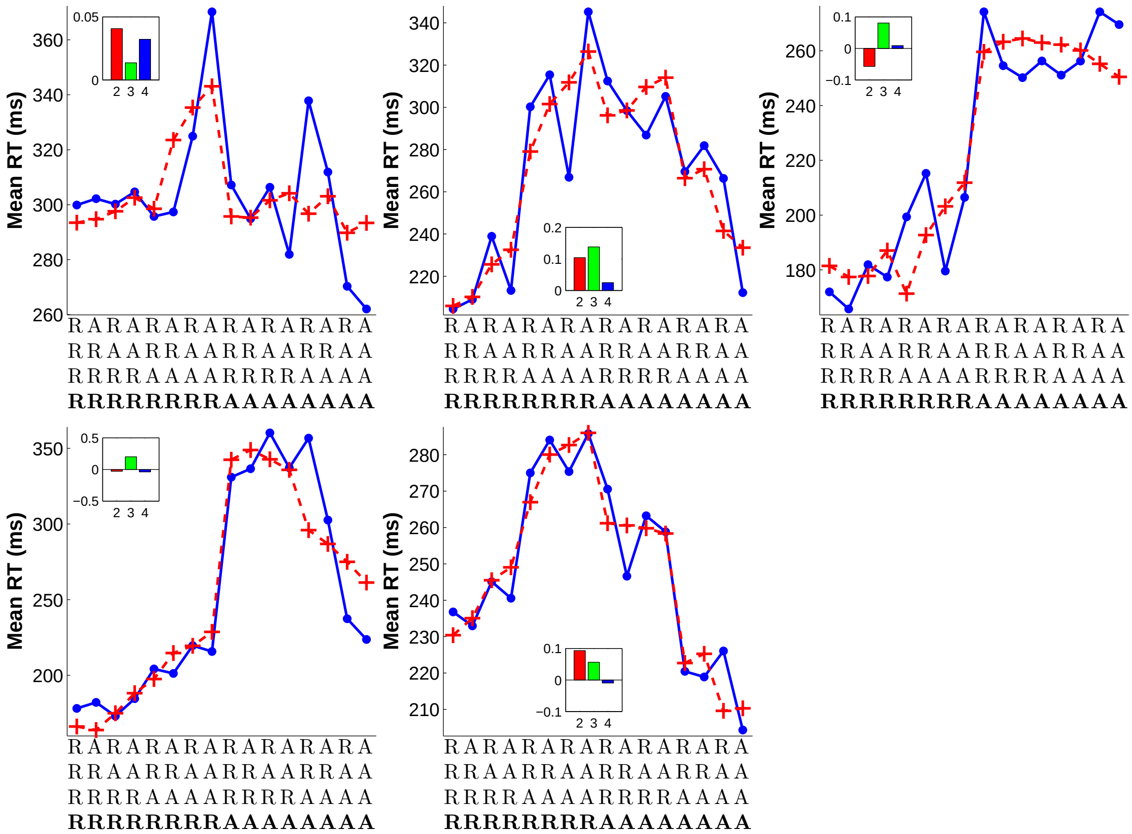


FIGURE C.25: Individual data for Experiment 7 with a 250 ms RSI.

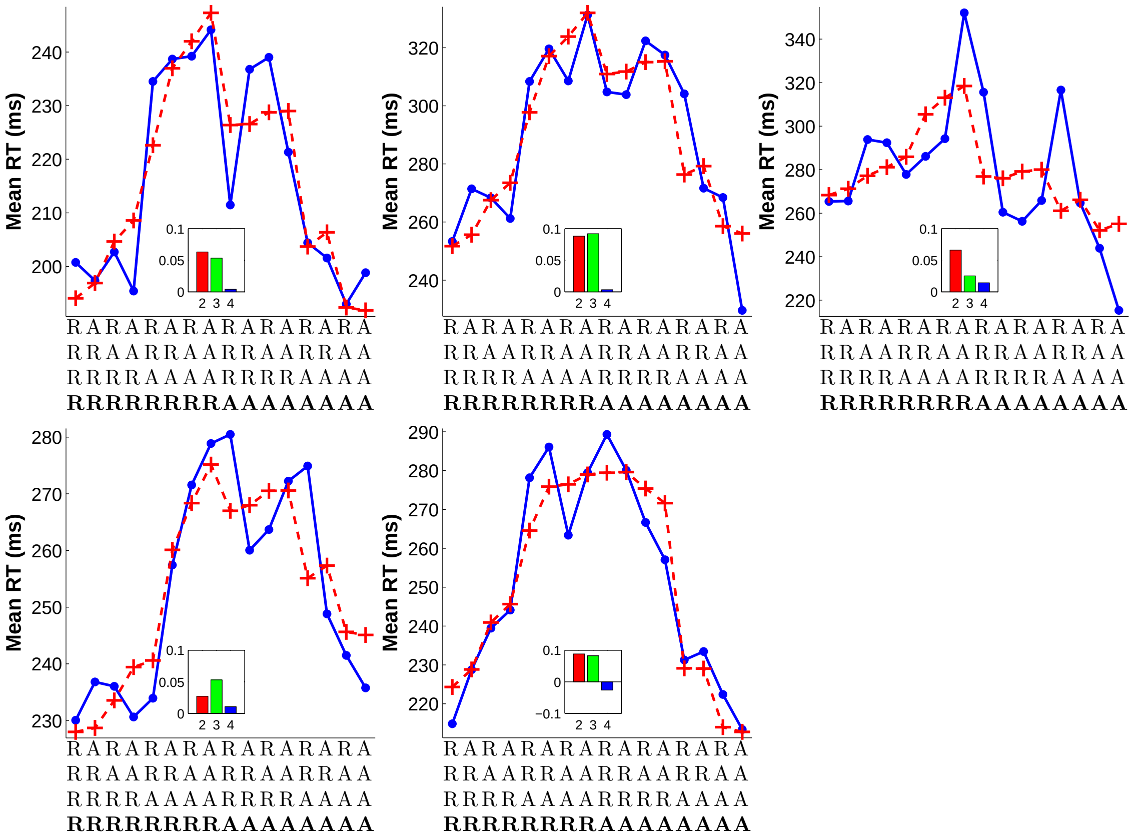


FIGURE C.26: Individual data for Experiment 7 with a 500 ms RSI.

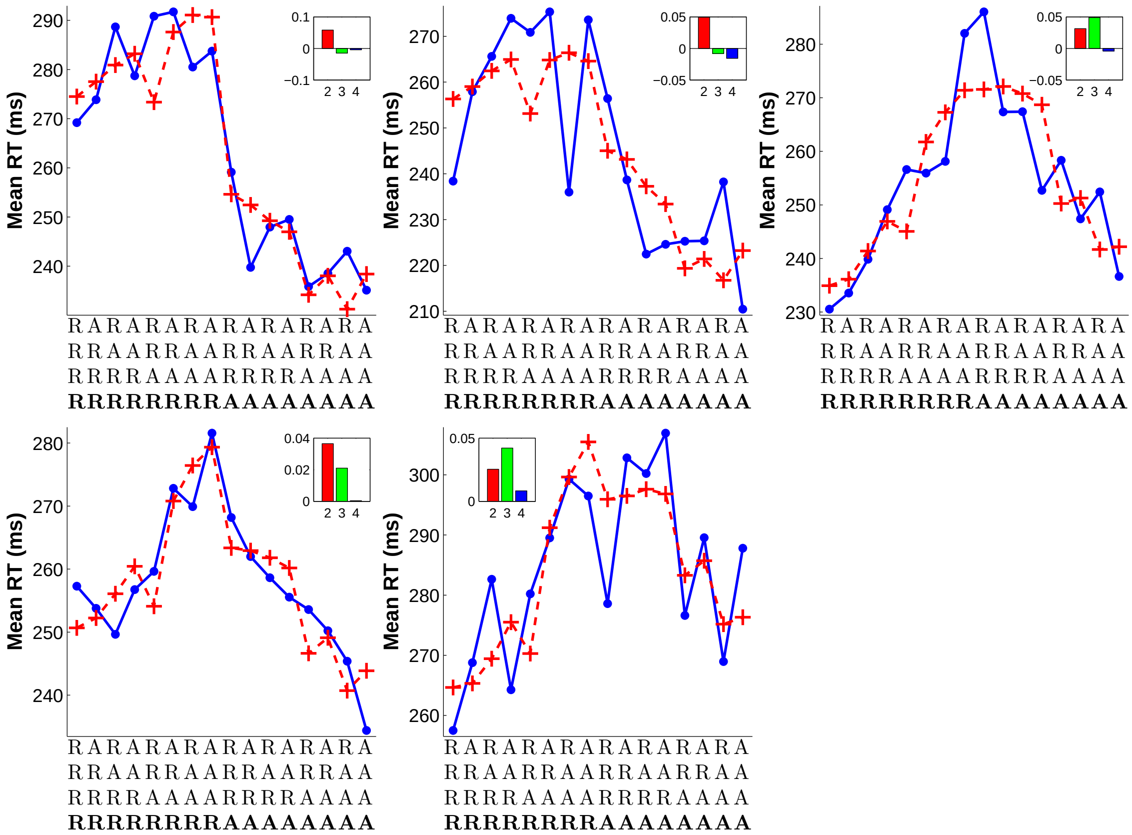


FIGURE C.27: Individual data for Experiment 7 with a 800 ms RSI.

D

Appendix D: Solution of differential equations

All differential equations were solved by using the Laplace transform method. The Laplace transform is defined as

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t) dt \tag{D.1}$$

where s is a complex number. The transform of a derivative function is

$$\mathcal{L}(f'(t)) = sF(s) - f(0) \tag{D.2}$$

where $F(s)$ is the Laplace transform of $f(t)$ and $f(0)$ the value of the original function at $t = 0$. This result can be achieve by substituting $f'(t)$ for $f(t)$ in (D.1) and integrating by parts. Iterating (D.2) twice we arrive at the formula for the Laplace transform of the second derivative

$$\mathcal{L}(f''(t)) = s^2 F(s) - sf(0) - f'(0) \tag{D.3}$$

Finally, the Laplace transform of a Heaviside step function is equal to

$$\mathcal{L}(c\mathcal{H}(t)) = \frac{c}{s} \tag{D.4}$$

where c is a constant equal to the amplitude of the step.

We are now ready for a short description of the method for the case of a second order differential equation. Consider the case of the equation for a single harmonic oscillator, given by

$$\ddot{y} + \gamma\dot{y} + \omega_0^2 y = F(t) \tag{D.5}$$

Where $F(t) = c\mathcal{H}(t)$ is a Heaviside step function. Taking the Laplace transform of both sides of (D.5), and making use of the linearity of of the operator $\mathcal{L}(.)$, gives

$$s^2Y(s) + \gamma sY(s) + \omega_0^2 Y(s) - sy(0) - \gamma y(0) - y'(0) = \frac{c}{s} \tag{D.6}$$

which, solving for $Y(s)$ yields

$$Y(s) = \frac{\frac{c}{s} + sy(0) + \gamma y(0) + y'(0)}{s^2 + \gamma s + \omega_0^2} \tag{D.7}$$

which is the Laplace transform of $y(t)$. The key to the method then lies in taking the inverse transform of $Y(s)$ to recover $y(t)$, according to

$$y(t) = \mathcal{L}^{-1}(Y(s)) \tag{D.8}$$

which usually involves taking partial fractions to reduce the polynomial on the right side of (D.7) to a a sum of functions for which the transform is known. This is practical only if the initial conditions $y(0)$ and $y'(0)$, i.e. the initial position and velocity, are both 0, but it becomes intractable if both are given any other value. Since arbitrary initial conditions are necessary for our purposes, the inverse transform was calculated using the analytical software package Mathematica; the calculations and resulting expressions for $y(t)$ and $y'(t)$ are shown in Appendix E.

Recall that in the model proposed in Chapter 4 $F(t)$ is usually a set of five consecutive pulses with different signs. Deriving a single expression for $y(t)$ for such a complex input using the Laplace transform method would quickly make calculations intractable due to nesting of the solutions for each pulse. Therefore the solution for an arbitrary number of pulses will be calculated *iteratively*, by alternating two types of period: one in which $F(t) = c\mathcal{H}(t)$, corresponding to each pulse or equivalently the period during which a stimulus is shown; and another period where $F(t) = 0$ corresponding to the inter-stimulus interval. The position and velocity at the end of each period are used as initial conditions for the next one, and so on until five pulses have been applied. Note that in order to calculate model results only the sign of the last pulse is necessary (see Chapter 4), so in order to save computational time the position and velocity at the end of the last pulse are not calculated. This process is extremely efficient computationally since, for each sequence of five pulses, only eight time points must be calculated for both the position - $y(t)$ - and velocity - $y'(t)$ - of the oscillator.

In order to calculate results for more than one oscillator the system is fist diagonalised, i.e. transformed into canonical or normal model coordinates, the solution is calculated as above for each normal mode, and the original coordinates are then recovered by applying the inverse transformation of that which first diagonalised the system.

E

Appendix E: Mathematica code

The following page shows Mathematica code used to compute the analytic solution of a single damped oscillator. The code calculates the inverse transform of $Y(s)$, the Laplace transform of $y(t)$, i.e. the solution of the damped oscillator, assuming a Heaviside step forcing function. In addition, the code calculates the first derivative of $y(t)$, i.e. the velocity of the oscillator. Finally, the expressions for both position and velocity as simplified.

Heaviside step function

```
In[7]:= Ft = c HeavisideTheta[t]
Out[7]= c HeavisideTheta[t]
```

Laplace transform of Heaviside step function

```
In[8]:= Fs = LaplaceTransform[Ft, t, s]
```

$$\text{Out[8]= } \frac{c}{s}$$

Inverse Laplace transform to obtain y(t)

```
In[9]:= ft = FullSimplify[InverseLaplaceTransform[
           (Fs + s x0 + v0 + gamma x0) / (s^2 + gamma s + omega0^2 ) , s, t]]
```

$$\text{Out[9]= } \frac{1}{2\,\text{omega0}^2\sqrt{\text{gamma}^2-4\,\text{omega0}^2}}\, e^{-\frac{1}{2}\left(\text{gamma}+\sqrt{\text{gamma}^2-4\,\text{omega0}^2}\right)t} \left(-c\left(\left(-1+e^{\sqrt{\text{gamma}^2-4\,\text{omega0}^2}\,t}\right)\text{gamma}+\left(1+e^{\sqrt{\text{gamma}^2-4\,\text{omega0}^2}\,t}-2\,e^{\frac{1}{2}\left(\text{gamma}+\sqrt{\text{gamma}^2-4\,\text{omega0}^2}\right)t}\right)\sqrt{\text{gamma}^2-4\,\text{omega0}^2}\right)-2\,\text{omega0}^2\,\text{v0}+\text{omega0}^2\left(\left(-\text{gamma}+\sqrt{\text{gamma}^2-4\,\text{omega0}^2}\right)\text{x0}+e^{\sqrt{\text{gamma}^2-4\,\text{omega0}^2}\,t}\left(2\,\text{v0}+\left(\text{gamma}+\sqrt{\text{gamma}^2-4\,\text{omega0}^2}\right)\text{x0}\right)\right)\right)$$

Calculate derivative of y(t)

```
In[10]:= dft = FullSimplify[D[ft, t]]
```

$$\text{Out[10]= } \frac{1}{2\sqrt{\text{gamma}^2-4\,\text{omega0}^2}}\, e^{-\frac{1}{2}\left(\text{gamma}+\sqrt{\text{gamma}^2-4\,\text{omega0}^2}\right)t}\left(2\,c\left(-1+e^{\sqrt{\text{gamma}^2-4\,\text{omega0}^2}\,t}\right)+\text{gamma v0}+\sqrt{\text{gamma}^2-4\,\text{omega0}^2}\,\text{v0}+2\,\text{omega0}^2\,\text{x0}+e^{\sqrt{\text{gamma}^2-4\,\text{omega0}^2}\,t}\left(-\text{gamma v0}+\sqrt{\text{gamma}^2-4\,\text{omega0}^2}\,\text{v0}-2\,\text{omega0}^2\,\text{x0}\right)\right)$$

F

Appendix F: Frequency spectra

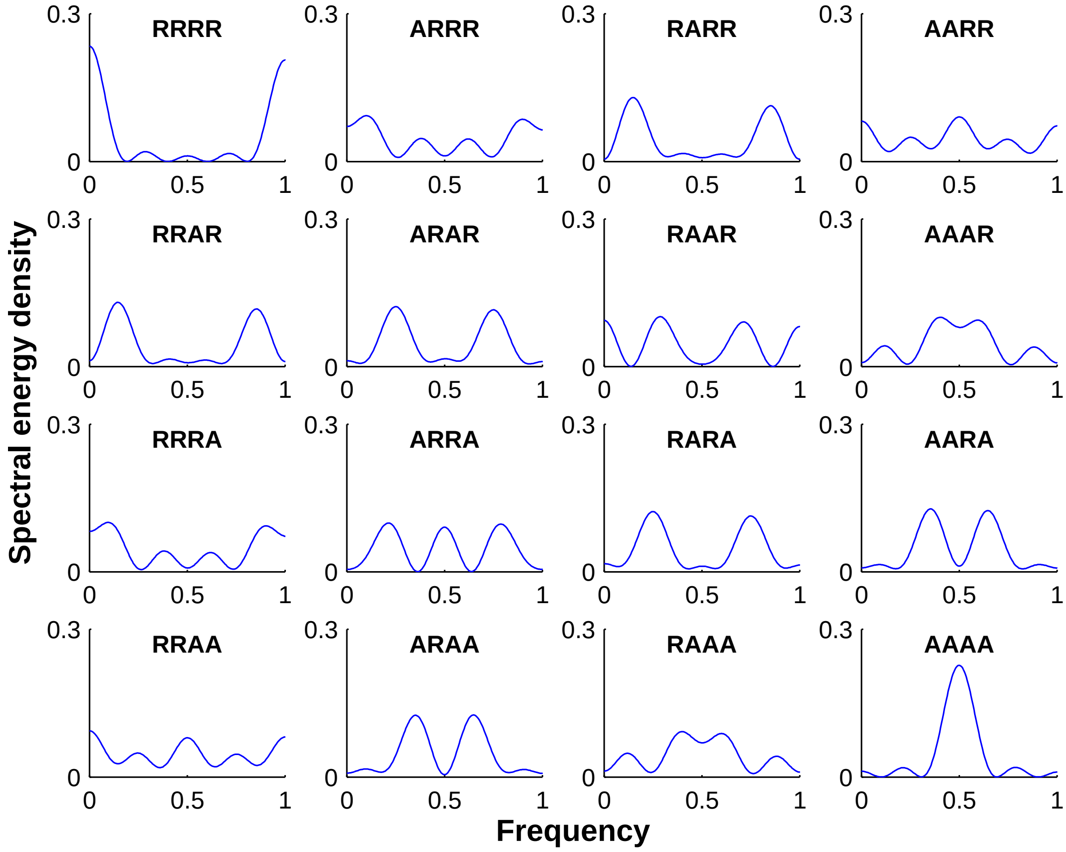


FIGURE F.1: Frequency spectra of all sixteen sequences. Note that all sequences which are left-right symmetric have the same spectrum.