Prediction of load-carrying capacity of piles using a support vector machine and improved data collection

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ABSTRACT

Model development for the prediction of the axial load carrying capacity of piles, at least at the model verification stage, relies on the measured data at full scale. Artificial intelligence and machine learning approaches use data in the whole process of model development and verification, making it necessary to incorporate reliable and diverse data. This study aims to develop a more accurate model for predicting pile capacity based on cone penetration test and full scale static pile load test data, employing a support vector machine (SVM) technique. Furthermore, it draws on the concept of support vectors to make suggestions for compiling additional data that are more representative of the problem, leading to enhancing the accuracy of the future models. In fact, in models developed using the SVM technique, those samples within a dataset for which a model shows the greatest uncertainties are detected as support vectors and are the only data that contribute to model development. In previous studies an examination of the distribution of input parameter values and the concentration of support vectors in input intervals were considered as guidelines for collecting further data. Geotechnical problems, however, are more complicated in that the importance of each input in model development and also the interaction of those parameters should be taken into account. Therefore, this study employs a sensitivity analysis of the model input parameters and other statistical analyses to improve the existing support-vector based approaches to data collection.

Keywords: pile, load-carrying capacity, support vector machine, improving data collection

1 INTRODUCTION

The model development process for the purpose of quantifying the behaviour of geotechnical problems always involves model verification and inspecting the generalisation ability of the proposed model. In fact, each model should be assessed to determine whether its outcomes are consistent with the intended behaviour and how accurate the responses are. Therefore, a set of relevant data should be available for the verification stage. The data set should be of high quality in terms of measurement accuracy and diversity to ensure that it is a representative of the general phenomenon. Moreover, the number of input parameters (input variables) should match the greatest number of factors that contributes to the problem. The utilised dataset is of greater importance when the data are used to develop models with artificial intelligence and machine learning techniques, like artificial neural networks (ANNs) and support vector machine (SVM) algorithms as these methods draw on the data to discover the underlying relationship between the contributory variables to the problem and the output. The application of artificial intelligence and machine learning approaches has largely been received favourably by geotechnical engineering researchers as they have been successfully employed in modelling geotechnical problems and phenomena with a considerable generalisation ability. SVM is a relatively new machine learning method that has demonstrated very encouraging results. The prediction of settlement and capacity of foundations (Samui 2008a; Pal and Deswal 2010; Samui 2011; Kordjazi et al. 2014), liquefaction assessment (Oommen et al. 2010), and slope stability analysis (Samui 2008b) are but a few examples of studies regarding the application of SVM in geotechnical engineering. Interestingly enough, in addition to being a powerful modeling technique, SVM can be used to provide the user with advice regarding the values of the variables in the dataset that the training set (i.e. the dataset that is used for developing a model) lacks and which, by adding more these samples to the training set, might improve model performance (Oommen and Baise, 2010). Oommen and Baise's approach relies on the concept of the support vectors, which are a portion of the training data that contributes to the final model. After a statistical analysis of the support vectors, the intervals that further data would be required can be determined. This study aims to take advantage of the SVM algorithms to develop a more reliable model for the prediction of the load-carrying capacity of axially loaded piles. To minimise the prediction error in the model, this study incorporates as many contributory variables to the pile capacity problem as possible, including pile geometry, pile tip condition, pile material, and the method of installation, along with soil properties in the form of CPT results. This paper also examines the application of the Oommen and Baise (2010) approach to determine those areas of uncertainty in the dataset that, by feeding the training set with more samples, would enhance model performance. Since there are a considerable number of input variables in the model and also these input variables are intricately related to each other to define pile behaviour under loading, applying the data collection approach might lead to a wide range of data that are required to improve the model, so this approach might need adjusting to fit this problem. Therefore, a sensitivity analysis has been conducted to refine this approach and give priority to each variable in the data selection process.

2 SUPPORT VECTOR MACHINE

In order to provide background to the data collection process, this section briefly introduces SVM algorithms and the nature of support vectors. For more detailed information readers are referred to the existing literature, such as Dibike et al. (2001).
The solution to a regression problem within the SVM, applied to a dataset (x_1, y_1), …, (x_l, y_l), $x \in R^m$, $y \in R$ is a linear function $f(x)$ as given below:

$$f(x)=w.x+b \tag{1}$$

where, l is the number of samples; x is the input vector; y is the output value; w is the weight vector; b is the bias and $w.x$ shows the inner product of the two vectors, w and x. It has been shown in the SVM literature that the following optimisation problem can be developed by incorporating aloss function with an ε-insensitive zone (Figure 1) (Dibike et al. 2001):

$$L\left(\alpha^*,\alpha\right)=-\varepsilon\sum_{i=1}^{l}\left(\alpha_i^*+\alpha_i\right)+\sum_{i=1}^{l}y_i\left(\alpha_i^*-\alpha_i\right)-\frac{1}{2}\sum_{i=1}^{l}\sum_{j=1}^{l}\left(\alpha_i^*-\alpha_i\right)\left(\alpha_j^*+\alpha_j\right)\left(x_i.x_j\right) \tag{2}$$

where, α and α^* are Lagrange multipliers and $L(\alpha, \alpha^*)$ indicates the Lagrange function. The function above must be maximised subject to the following constraints:

$$\begin{cases}\sum\alpha_i^* = \sum\alpha_i \\ 0 \le \alpha_i^* \le C \\ 0 \le \alpha_i \le C\end{cases} \tag{3}$$

to determine the coefficients α and α^*. The regression function within the SVM is then calculated using:

$$w_0 = \sum_{\substack{Support\\vectors}} (\alpha_i^* - \alpha_i)x_i \tag{4}$$

$$f(x) = \sum_{\substack{Support\\vectors}} (\alpha_i^* - \alpha_i)(x_i.x) - \frac{1}{2}w_0.[x_r + x_s] \tag{5}$$

where, $w_0$is the optimum weight vector; x_r and x_s are the support vectors. Samples which have a non-zero Lagrange multiplier are known as support vectors (Dibike et al. 2001) and these samples contribute to the final formulation of the model. Therefore, only a portion of the training samples are used in the model development. These samples have prediction errors larger than $\pm\varepsilon$ and fall outside the 2ε-width band, as shown in Figure 1. This concept will later be used to offer recommendations on data collection for future studies. The SVM algorithm can also be expanded to non-linear regression by the use of Kernel functions (Smola and Schölkopf 1998; Dibike et al. 2001).

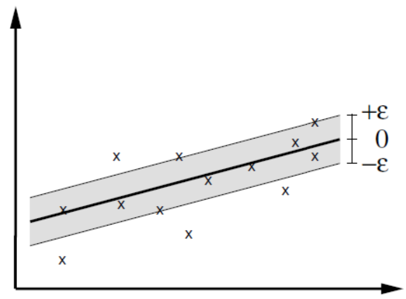

Figure 1.ε-insensitive loss function (those samples outside the grey band are support vectors) (Smola and Schölkopf 1998)

3 MODEL DEVELOPMENT

This study employs a set of 108 samples of actual, full-scale field measurements, containing pile properties, results of cone penetration tests(CPTs) along the length of each pile (and below their tip), and finally the ultimate bearing capacity of each pile measured from full-scale static pile load tests for each case. All piles were subjected to compression static load tests and the ultimate capacity was calculated based on the results of these load tests. The pile fails when rapid pile movement occurs with respect to a constant value of applied load or small increase in it, and the load corresponding to the failure load is considered the ultimate capacity (Fellenius 2001). In cases where this failure point was not easily recognizable on the load-settlement curve, the 80% criterion has been used (Hansen 1963). The database includes a wide range of pile and soil types: the pile materials – steel, concrete and composite; pile shape – square, round, octagonal, triangle, pipe and H section; pile tip condition – closed and open; and the type of installation – driven and bored. The soil types include sands, clays and silts, in individual and multiple layers, with the soil properties of each varying spatially and naturally as measured by the CPTs. This dataset has mainly been compiled from the published literature and for further information on the data, readers are referred to Kordjazi et al. (2014), in which a complete list of the references for all the samples is available.

3.1 Data preparation

Data preparation refers firstly to defining the input variables and the corresponding output for the model. In this work, the results of CPTs are considered as quantifying the soil properties. The CPT results consist of cone tip resistance (q_c) and sleeve friction (f_s) along the pile's embedded length. In order to account more accurately for the variability of soil properties along the pile length, the embedded length of the pile is sub-divided into three segments of equal thickness and the average of $q_c$and f_s are calculated along each segment. Therefore, the input variables used in the development of the SVM models are the: (1) type of static pile load test (maintained or constant rate of penetration); (2) pile material (steel, concrete and composite); (3) method of pile installation (driven or bored); (4) pile tip (closed or open); (5) embedded length of pile (L_{embed}); (6) perimeter of the pile in contact with the soil (O); (7) cross-sectional area of the pile tip (A_{tip}); (8) average cone tip resistance along the embedded length of the pile (q_{c1}, q_{c2} and q_{c3}); (9) average sleeve friction along the embedded length of the pile (f_{s1}, f_{s2} and f_{s3}); and (10) average cone tip resistance beneath the pile tip (q_{ctip}) The ultimate load-carrying capacity of the pile (P_u) is the single output variable.

The second step in preparing the data is preprocessing and selecting the data for training the model and subsequently testing its performance. In order to assess the performance of the trained models, a portion of the data is set aside for this purpose. As a result, training and testing sets are usually employed in the model development process, used respectively for training the model and evaluating its performance. In this paper, 83cases are used for training and the remainder (25 cases), are used for testing. Statistical characteristics for these subsets are summarised in Table 1. As another part of the data preprocessing, all variables (input and output) are scaled between 0.0 and 1.0 by normalising each variable against their maximum values (Goh & Goh 2007; Pooya Nejad et al. 2009).

3.2 Training the model

To increase the generalisation ability of a SVM model, non-linear models are usually developed using kernel functions. In this study, in order to develop non-linear models, a radial basis kernel function (RBF) is employed (Dibike et al. 2001):

$$K(x, x_i) = exp\left(-|x - x_i|^2 / 2\sigma^2\right) \tag{6}$$

Table 1: SVM input and output characteristics

Model variables	Data set	Statistical parameters				
		Mean	St. Dev.[a]	Minimum	Maximum	Range
A_{tip}(m^2)	Training Set	0.1827	0.1783	0.0080	0.7854	0.7774
	Testing Set	0.1519	0.1176	0.0080	0.5030	0.4950
O(mm)	Training Set	1625.12	866.92	585.0	7341.3	6756.3
	Testing Set	1425.68	416.67	858.0	2510.0	1652.0
L_{embed}(m)	Training Set	15.70	10.36	5.50	67.00	61.50
	Testing Set	16.17	8.47	6.50	36.50	30.00
q_{c1}(MPa)	Training Set	4.13	3.17	0.02	15.07	15.05
	Testing Set	3.76	2.85	0.24	11.72	10.74
f_{s1}(kPa)	Training Set	74.17	59.03	0.73	283.94	283.21
	Testing Set	69.49	43.67	10.76	176.75	159.7
q_{c2}(MPa)	Training Set	6.21	5.86	0.32	30.71	30.39
	Testing Set	5.73	4.09	1.00	18.42	17.43
f_{s2}(kPa)	Training Set	103.52	96.91	2.12	618.66	616.54
	Testing Set	100.65	63.69	15.00	303.08	288.00
q_{c3}(MPa)	Training Set	7.36	6.07	0.27	32.59	32.32
	Testing Set	6.63	4.76	0.70	23.05	22.36
f_{s3}(kPa)	Training Set	126.57	95.58	7.99	396.57	388.58
	Testing Set	139.18	100.87	25.00	388.00	363.00
q_{ctip}(MPa)	Training Set	8.75	6.28	0.25	27.11	26.86
	Testing Set	9.03	6.03	1.15	22.30	21.15
P_u(kN)	Training Set	1988.20	1811.60	60.00	10910.00	10850.00
	Testing Set	1891.68	1302.78	520.00	5850.00	5330.00

[a]Standard deviation.

where, the constant σ is the width of the RBF and the kernel parameter must be specified by user. Other parameters which need to be defined by the user are ε and C, the loss function and the penalty parameter, respectively. Positive values should be assigned to these parameters to train a model. The optimal values of these parameters are usually obtained through a process of trial-and-error and there is no hard-and-fast rule for determining the best values; however, there are some useful hints assisting the user in assigning values that lead to better performance, especially for the initial guesses (Samui 2008a; Kordjazi et al. 2014). In this work, the SVM toolbox (Gunn 2001) in MATLAB has been used to develop the SVM model.

3.3 Model performance

Several models have been trained by assigning various values to the design parameters, namely the constants C, ε, and the kernel parameter in a trial-and-error process.The coefficient of correlation (R) and the root mean square error (RMSE) were used to compare the performance of each model against the training samples and testing dataset. The results show that the model with C=1.1, ε= 0.02, and σ= 0.9 presents the lowest value of RMSE (= 318.85 kN) and the closest value of R to 1. In fact, the optimal model predicts the bearing capacity of the training samples with RMSE= 394.38 kN and R= 0.979. Similarly, these indices for the testing set are RMSE= 318.85 kN and R= 0.972. Moreover, the graph of the measured versus predicted capacity for this model, illustrated in Figure 2, shows the concentration of the points along the perfect fit lines, confirming the excellent performance of the SVM in modelling the axial load-carrying capacity of piles.

4 SUGGESTIONS FOR FUTURE DATA COLLECTION

As discussed in Section 2, only support vectors contribute to the regression function in the SVM; accordingly, only a portion of the training samples forms the final function. Support vectors are those samples that their prediction errors, by the regression function, are greater than ε, the allowable error in the loss function. In other words, a model has the greatest uncertainty with regard to the support vectors and, since the SVM algorithms seek to uncover the underlying relationship between inputs and outputs based on these samples, the uncertainty may be reduced by adding more samples that have characteristics similar to the support vectors. In fact, the input intervals in which the model has the greatest uncertainty can be identified by analysing the support vectors and in this way the intervals that have the greatest frequency of support vectors require further data collection (Oommen and Baise, 2010).

To examine the support vectors, each variable is divided into intervals of almost equal numbers of training samples and the frequency of the support vectors in each interval is determined (Oommen and Baise, 2010). As a result, in this study each variable is divided into 6 intervals of almost 14 or 15 samples. According to the explanation above, those intervals that have a considerable frequency or percentage of support vectors cause uncertainty and the model is less likely to accurately predict the axial loading-capacity of pile cases with similar properties. Consequently, it is recommended to collect more data with similar properties, in order to improve the generalisation ability of the model. Figure 3 shows the results of the statistical analysis in terms of frequency and percentage of support vectors in each interval. Since the length of each variable depends on the frequency of the samples, the length of the intervals varies widely, so a logarithmic scale is used for the horizontal axis in Figure 3.Due to space restrictions, the graphs for 4 variables, including q_{c2}, q_{c3}, f_{s2}, f_{s3}, are not presented in Figure 3.

As illustrated in Figure 3, the frequency of the support vectors in each interval for all the variables is generally between 60% to 80%; however, there are some exceptions. For example, all the samples with $3.17<q_{c1}<4.26$ are support vectors but only 42% of the samples with a perimeter less than 101 cm are made up of support vectors. This considerable frequency of support vectors in the intervals might suggest that the model has great uncertainties with regard to the training set. The SVM model, however, is able to predict the axial load-carrying capacity of piles with excellent accuracy.

A thorough examination of Figure 3 leads to more detailed suggestions for future data collection. With regard to the pile tip cross-sectional area, when $590<A_{tip}<1320$ and greater than 3080 cm^2, more than 70% of samples are support vectors, so it is recommended that more data that have such a cross-sectional area be added to the training set. Similarly, since for $L_{embed}<8.5$ and greater than 21.3 m, support vectors make up a great proportion of the samples, adding more samples of these sizes may improve model performance. For perimeters between 101 and 114 cm, 80% of the training

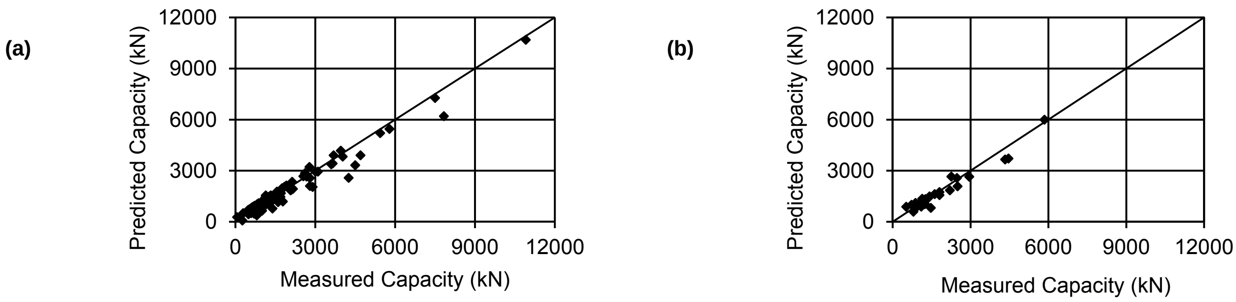

Figure 2. Measured vs. predicted capacity for the optimum SVM model: (a) training set, (b) testing set.

Table 2: Guidelines on data collection for CPT results

Variable	Interval	Percentage of support vectors
q_{c1}	$3.17<q_{c1}<4.26$	100
	$7.3<q_{c1}<15.07$	84.6
q_{c2}	$0.32<q_{c2}<1.55$	85.7
	$8.6<q_{c2}<30.71$	100
q_{c3}	$0.27<q_{c3}<2.46$	85.7
	$11.9<q_{c2}<32.59$	85.7
f_{s1}	$23.21<f_{s1}<45$	85.7
f_{s2}	$2.12<f_{s2}<27.33$	78.6
	$85.6<f_{s2}<102.41$	85.7
f_{s3}	$34.3<f_{s3}<68.82$	86
	$111.51<f_{s3}<138.71$	78.6
q_{ctip}	$9.7<q_{ctip}<14.2$	78.6
	$14.2<q_{ctip}<27.11$	85.7

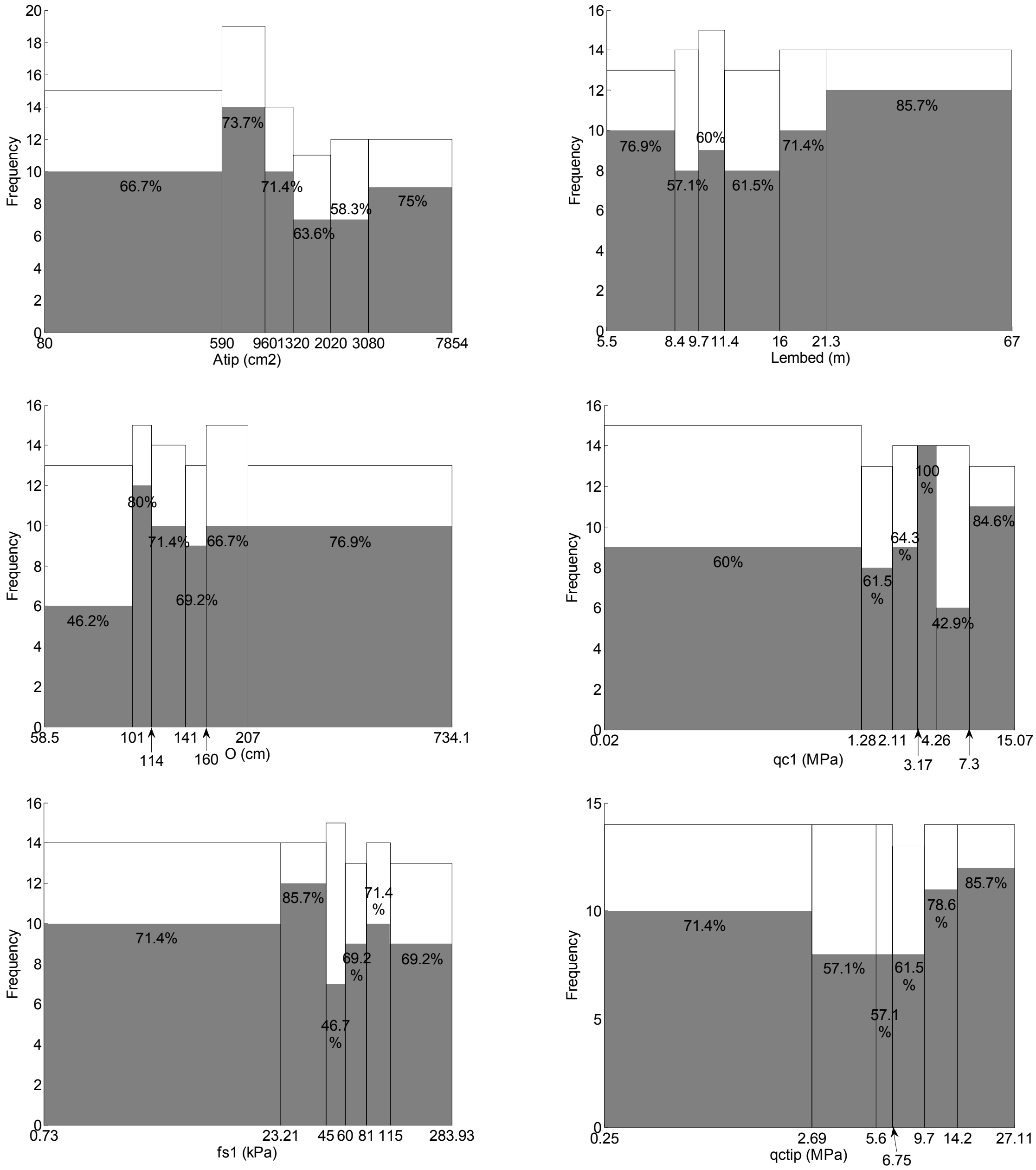

Figure 3. Support vectors' frequency in each interval for different input variables

samples are support vectors; however, for values less than 101 cm the model has the lowest uncertainty in relation to the training samples. For 3.17 <q_{c1}< 4.26 MPa all the training samples are among support vectors and adding more data in this interval may improve the model's performance. Similar analyses have been conducted on the graphs of all the CPT-related variables and the results are summarised in Table 2. Collecting more data that fall in the intervals that are mentioned in this table may increase the generalisation ability of the future models.

Although these recommendations for the further data are clear and direct, implementing this approach is not straightforwardnor simple. In fact, more than 10 intervals for 10 variables have been suggestedfor providing additional data and, in some cases, the range of the intervals is very wide (e.g. 14.2<q_{ctip}<27.11 or 7.3<q_{c1}<15.07). Therefore, to implement this approach a large number of samples, which falls within these intervals, is required. Obviously, this is impractical.

Furthermore, this approach provides no details as to which of these variables causes the data uncertainty; in fact, the contributory variables are intricately related to each other and it is unlikely that only one variable is the root of uncertainty in a sample. A detailed analysis of the support vectors demonstrates that,for the majority of support vectors, there are at least two variables that fall in the intervals with more than 75% frequency of support vectors. As is shown in Figure 4, there are at least 15 samples that have 3 variables in intervals with more than 75% of support vectors.

A probable solution that might address this problem might be prioritising data collection; that is, identifying the most important variables that contribute to the problem and assign them priority in the process of further data collection. In order to identify the relative significance of each input variable with respect to ultimate pile load-capacity prediction, the method proposed by Liong et al. (2000) has been utilised. In this method the change in the model output is measured by varying each of the input variables at a constant rate, and the sensitivity of the model is calculated as follows:

$$S\,(\%) = \frac{100}{r} \sum_{j=1}^{r} \left(\frac{\%\ change\ in\ output}{\%\ change\ in\ input} \right)_j \qquad (7)$$

where, r indicates the number of data. In this paper, a constant rate of 20% (Samui, 2008a) is considered as the change in input for each variable and a sensitivity analysis has been carried out with respect to cross-sectional area of pile tip, perimeter of the pile in contact with soil, embedded length of pile and CPT results (i.e. q_c, f_s and q_{ctip}). Finally, the impact factor of each input variable with respect to the other pile capacity variables is calculated and the results are summarised in Table 3.

It can be seen that the soil properties in the form of CPT measurements, with a combined impact factor of 56.2% (the sum of last 3 rows in Table 3), have the most significant effect on pile load-carrying capacity. Pile embedded length, the cross-sectional area of pile tip and pile perimeter are, respectively, the other most significant factors that influence pile capacity. As a result, a very simple implication of this analysis might be that, for future data collection purposes, priority be given to those samples that their CPT results fall in the intervals, mentioned in the previous section.

5 CONCLUSION

In this study a dataset of 108 samples, including CPT results along the length of piles, full-scale pile load test results and pile properties for each sample, has been employed to examine the performance of support vector machine algorithms in the prediction of the ultimate load carrying capacity of axially-loaded piles. The developed non-linear model, which incorporates a radial basis kernel function, delivers predictions on testing samples with excellent accuracy (R= 0.972 and RMSE= 318.85 kN).

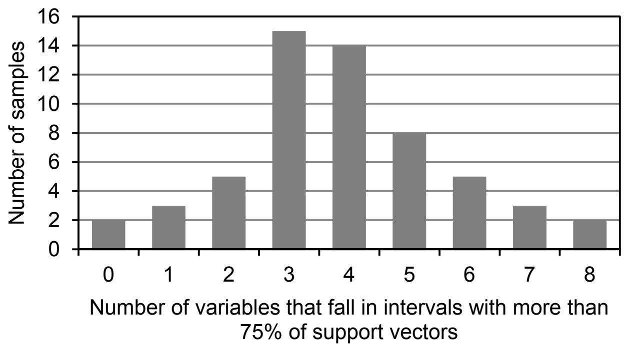

Figure 4.The number of support vectors against the number of variables in the intervals with more than 75% of support vectors frequency

Table 3: Results of sensitivity analysis

Input parameter	Sensitivity (%)
A_{tip}	14.77
O	12.20
L_{embed}	16.79
q_{cav}	23.41
f_{sav}	22.31
q_{ctip}	10.52

Moreover, this study examined a sensitivity analysis and incorporates its results to improve Oommen and Baise's approach (2010) to providing suggestions for further data collection. Their approach relies on the concept of support vectors in order to recommend intervals of the variables that fuelling the training set with samples in such intervals might improve the performance of the model for future studies. However, in a problem like pile load capacity, where the variables are intricately related to one another, this method makes no distinction between the importance of the variables. As a result, intervals of data with wide ranges are generally suggested for every variable which hardly act as a hint for users in collecting new samples. In an attempt to refine the Oommen and Baise's approach, by adopting a sensitivity analysis, priorities can be made to assist the user to acquire additional data in a more effective way. In fact, the results of the sensitivity analysis list the variables that have the most contributory share in the SVM model and then users can search for new samples in order of priority. In other words, in this study, since the results of the CPTs have the most important impact on the model, focusing on collecting more data within the intervals suggested for the CPT results at the first step, might improve the performance of future SVM models for the prediction of pile capacity.

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7 NOTATION

A_{tip}	Cross sectional area of the pile tip	q_{ctip}	Average cone tip resistance below pile tip
b	bias	R	Coefficient of correlation
C	Penalty parameter	r	The number of samples
f_s	Cone sleeve friction in CPT	S	Sensitivity
f_{sav}	Average cone sleeve friction in CPT	w	Weight vector
f_{s1}, f_{s2}, f_{s3}	Average cone sleeve friction along the segments of the embedded length of pile	w_0	Optimal weight vector
L_{embed}	Embedded length of pile	x	Input vectors
$L(\alpha, \alpha^*)$	Lagrange function	x_r, x_s	Support vectors
O	Perimeter of pile	y	Output vectors
P_u	Ultimate bearing capacity	α, α^*	Lagrange multipliers
q_c	Cone tip resistance in the CPT	σ	Width of the RBF kernel
q_{c1}, q_{c2}, q_{c3}	Average cone tip resistance along the segments of the embedded length of pile	ε	Allowable error in the loss function
q_{cav}	Average cone tip resistance in the CPT		