Large-Scale-Structure in the Pierre Auger Observatory Data Directions

“The Long Dark Teatime of the Soul[1]”

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Abstract

This thesis presents a method of analysis of Pierre Auger Observatory Cosmic Ray (CR) directions. I look for evidence of large-scale-structure within these CR directions. I have associated directional events by virtue of the angular proximity of their arrival directions, and within three energy ranges around the Greisen-Zatsepin-Kusmin (GZK) energy limit. I design graph theoretical algorithms to grow minimum spanning trees for these directional events and then 'cut' the trees along certain galactic longitudes and latitudes into 'branches', where I expect the galactic magnetic fields or cosmic ray point-sources to exhibit behaviours or patterns, which can be interpreted by branch features. 1,200 simulated CR directions in each energy range provide some statistical context for the Pierre Auger Observatory branch features which may be considered significant. The thesis is a preliminary study of a method of analysis of 'regions of interest' which later may be optimized with a full statistical 'tuning' analysis.

Statement of Originality

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree. I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968. I also give permission for the digital version of my thesis to be made available on the web, via the Universitys digital research repository, the Library catalogue and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

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